Financial Intermediation in a Model of Growth Through Creative Destruction.

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Abstract

This paper presents an endogenous growth model in which the research activity is financed by intermediaries that are able to reduce the incidence of researcher's moral hazard. It is shown that financial activity is growth promoting because it increases research productivity. It is also found that a subsidy to the financial sector may have larger growth effects than a direct subsidy to research. Moreover, due to the presence of moral hazard, increasing the subsidy rate to R&D may reduce the growth rate. I show that there exists a negative relation between the financing of innovation and the process of capital accumulation. Concerning welfare, the presence of two externalities of opposite sign steaming from financial activity may cause that the no-tax equilibrium provides an inefficient level of financial services. Thus, policies oriented to balance the effects of the two externalities will be welfare improving.

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1 Introduction

The renewed interest on growth and their determinants has pointed at the financial structure as one of the key factors in the development of nations. This paper introduces a financial sector in one of the more recent models of growth, the one first presented in Howitt and Aghion (1998). This framework allows us to explicitly model how the R&D activity is financed by means of contracts designed to reduce the incidence of researcher's moral hazard. As a consequence, the financial sector will have real effects on the economy.

Analyzing the interaction between financial and economic activity has been the aim of a rather prolific literature. The first remarkable reference is the work of Schumpeter at the beginning of the twentieth century. He suggested that financial institutions are important for economic activity because they evaluate and finance entrepreneurs in their research and development projects. Similarly, development economists like Gurley and Shaw (1955), Goldsmith (1969), and McKinnon (1973) defended the idea that financial development encourages growth because it increases the level of investment and improves its allocation. In addition, they argued that faster growing economies require higher amounts of financial services and that the richer the economy, the sooner it is able to pay for financial superstructures. Unfortunately, a lack of formal analysis is common to all these papers on development. This is probably because previous to the formulation of a rigorous framework on the relationship between finance and growth it was necessary to develop further the theory of economic growth.[1]

Neoclassical exogenous growth theory did not offer the appropriate frame of reference because financial variables could only have level effects. The appearance of the first works on endogenous growth determined the starting point of the literature on growth and finance. Classic references of this first line of research are Greenwood and Jovanovic (1990), Bencivenga and Smith (1991, 1993), Levine (1991, 1992) and Saint Paul (1992). They used the basic Ak framework combined with credit market models of financial intermediation. In these papers, financial markets are considered as institutions intended to provide services of risk pooling and collection of information about borrowers. They also facilitate the flow of resources from savers to investors in the presence of market imperfections. Most of the papers on this area introduce several devices to fight against adverse selection, moral hazard or liquidity shocks in order to make intermediaries arise endogenously. The role of intermediation is thus, to reduce the inefficiency caused by these imperfections. Consequently, financial institutions promote growth because their activity implies a more efficient

[1] This point was already stated by Pagano (1993).

allocation of resources. With respect to the backward link from growth to finance suggested by empirical evidence, they follow the basic argument of earlier work. Namely, that there exists a fixed component in the cost of financial services and that some limit of wealth must be trespassed before the establishment of a financial structure is affordable.

New developments in the theory of economic growth have led to another line of research. Grossman and Helpman (1991b) and Romer (1990) suggested that economic growth comes mainly from the invention and development of new products rather than from the accumulation of physical or human capital. Recovering the Schumpeterian view of the role of financial institutions in economic activity, some authors tried to explain how financing of innovation can affect the growth process. Good exponents of this literature are King and Levine (1993a), De la Fuente and Marín (1996) and Blackburn and Hung (1998). Using this new framework they introduce informational frictions in the credit market, providing a rationale for the appearance of intermediaries. King and Levine consider financial intermediaries that act as evaluators of prospective entrepreneurs and as providers of insurance for innovators. However they do not introduce incentive problems. This type of problems can arise because risk averse innovators will try to get full insurance. That is, they will try to get the same payment no matter whether they innovate or not. If this payment is positive, researchers do not have incentives to innovate, especially, if to innovate they must exert effort. The papers by De la Fuente and Marín, and Blackburn and Hung take this moral hazard problem into account though from different perspectives. The first pair of authors provides banks with an imperfect monitoring technology that reveals the innovator's level of effort with a certain probability, while Blackburn and Hung use the costly state verification paradigm, that is, that innovators have incentives to declare that they have not been successful so as to avoid payment. At some cost, investors can verify the result of the project. The common message of this group of papers is that financing of innovation is crucial for economic growth, and that the more efficient is the financial sector the faster the economy will grow. Concerning the feedback effects of growth on finance, these models provide a natural link without recurring to fixed costs assumptions. De la Fuente and Marín argue that growth causes changes in factor prices which increase the return to information gathering and hence favor financial intermediation activities.

The above growth models used by the latter line of research ignore capital accumulation as a source of growth. Aghion and Howitt (1998) argue that they ignore capital accumulation because it is assumed that labor is the only input into research and that labor is inelastically supplied. Therefore, a rise in capital intensity will have two opposite effects. On one hand, it will make payoffs to innovation greater but on the other hand, it will increase labor's productivity, making the input to research more expensive. These two effects cancel each other out so that capital accumulation leaves innovative activity unaffected and thus, it cannot influence long run growth.[2] However, it is arguable that the only source of growth is innovation and, accordingly, Aghion and Howitt propose another model of creative destruction with capital accumulation. They assume that research is produced out of labor and intermediate inputs. In their model, both R&D activities and capital accumulation determine growth and moreover, they are complementary. Growth cannot go on forever if there were no innovation because diminishing returns would reduce investment while without capital accumulation the rising cost of capital would choke off innovation.

This paper explicitly models the contractual relationship between the researcher and the provider of funds for the project in a model of endogenous technological change in the spirit of Howitt and Aghion (1998). Financial intermediaries are endowed with a monitoring technology that allows them to force researchers to exert a higher level of effort than the one they would choose in the absence of monitoring. Hence, research productivity is determined in the credit market and thus, may be affected by financial variables. In particular, the promotion of financial activities will enhance the economy's growth performance. That is, subsidies to financial intermediation will increase R&D productivity moving the economy to a faster growing balanced growth path. In addition, a subsidy to financial intermediation may be more effective than a direct subsidy to research. The latter policy induces a higher research intensity that rises the growth rate. However, the tax change reduces researchers' incentives to exert effort, which implies higher monitoring costs and a lower R&D productivity. This undercuts the positive growth effects of the research subsidy to the point that for a high enough subsidy rate, the growth effect can become negative.

It is also shown that there exists a negative relationship between the equilibrium level of financial services and capital accumulation. The intuition for this comes from the fact that a policy that promotes financial activity will increase research productivity and thus, reduce the incentives to accumulate capital due to the business stealing effect.

The effect of financial activity on research productivity causes two external effects of opposite sign. On one hand, its positive effect on the productivity of the research project will spillover to the other sectors of the economy and it will increase their productivity. On the other hand, the increase in R&D productivity will raise the arrival rate of innovations and consequently, the probability that an incumbent producer is replaced by the latest innovator. The higher probability of being replaced and thus, of losing the flow of profits, discourages capital accumulation. This is the so-called business stealing effect, or creative destruction process. The interaction of these

[2] For details see Aghion and Howitt (1998) pages 99-102.

externalities makes the no-tax equilibrium level of financial services inefficient. Consequently, there exists a role for policies aimed at bringing the provision of financial services closer to its efficient level.

The paper is divided in 6 sections. Section 2 presents the model, sections 3 and 4 study the steady state and the dynamics of the system respectively, section 5 performs the welfare analysis and section 6 concludes the paper.

2 The model

I consider a model of creative destruction with capital accumulation and technological spillovers.[3] In the basic model without intermediation, capital accumulation and investment in R&D are the key variables for long run growth. In the present model however, they are not the only ones. This is due to the fact that research productivity is no longer an exogenous parameter. It will be determined by the amount of resources devoted to the financial sector of the economy. The availability of financial services increases the success probability of projects and, hence, the productivity of research. Thus, financial activities will also be relevant for the determination of long run growth.

2.1 Consumers

There is a representative consumer who maximizes the present value of utility

$$V(C_t) = \int_0^\infty \ln(C_t) e^{-\rho t} dt. \tag{1}$$

I use the logarithmic functional form for simplicity. As usual C_t is consumption at date t and ρ is the rate of discount of consumption.

2.2 Final good sector

The consumption good is produced in a competitive market out of labor and intermediate goods. Labor is represented by a continuous mass of individuals L, and it is assumed to be inelastically supplied. Intermediate goods are produced by a continuum of sectors of mass 1, being m_{it} the supply of sector i at date t. The production function is a Cobb-Douglas with constant returns on intermediate goods and efficiency units of labor

$$Y_t = L^{1-\alpha} \int_0^1 A_{it} m_{it}^\alpha di,$$

[3] The growth model is based on the work of Howitt and Aghion (1998).

where Y_t is final good production and A_{it} is the productivity coefficient of each sector. I assume equal factor intensities to simplify calculations.

2.3 Intermediate goods

The intermediate sector has a monopolistic structure. In order to become the monopolist producer of an intermediate good, the entrepreneur has to buy the patent of the latest version of the product. This patent gives him the right to produce the good until an innovation occurs and the monopolist is displaced by the owner of the new technology.

The only input in the production of intermediate goods is capital. In particular, it is assumed that A_{it} units of capital are needed to produce one unit of intermediate good i at date t. As we will see, this assumption is necessary in order to obtain stability. The evolution of each sector's productivity coefficient, A_{it} is determined in the research sector.

Capital is hired in a perfectly competitive market at the rental rate ζ_t. Hence, the cost of one unit of intermediate good is $A_{it}\zeta_t$. On the other hand, the equilibrium price of the intermediate good, $p(m_{it})$ will be its marginal product

$$p(m_{it}) = \alpha L^{1-\alpha} A_{it} m_{it}^{\alpha-1},$$

where m_{it} is production of intermediate good i at date t. Thus, the monopolist's profit maximization problem is the following:

$$\begin{aligned} \pi_{it} &= \max_{m_{it}} [p(m_{it})m_{it} - A_{it}\zeta_t m_{it}] \\ s.t. \quad p(m_{it}) &= \alpha L^{1-\alpha} A_{it} m_{it}^{\alpha-1}, \end{aligned}$$

from where we obtain the profit-maximizing supply and the flow of profits as

$$\begin{aligned} m_{it} &= L\left(\frac{\alpha^2}{\zeta_t}\right)^{\frac{1}{1-\alpha}} \\ \pi_{it} &= \alpha(1-\alpha)L^{1-\alpha} A_{it} m_{it}^{\alpha}. \end{aligned}$$

Thanks to the assumption of equal factor intensity, supply of intermediate goods is equal in all sectors, $m_{it} = m_t$. Thus, the aggregate demand of capital is equal to $\int_0^1 A_{it} m_t di$. Let $A_t = \int_0^1 A_{it} di$, be the aggregate productivity coefficient. Then, equilibrium in the capital market requires demand to equal supply

$$A_t m_t = K_t,$$

or equivalently, the flow of intermediate output must be equal to capital intensity k_t,

$$m_t = \frac{K_t}{A_t} \equiv k_t.$$

With this notation we can express the equilibrium rental rate in terms of capital intensity

$$\zeta_t = \alpha^2 L^{1-\alpha} k_t^{\alpha-1}. \tag{2}$$

2.4 Research sector

Innovations are produced using the same technology of the final good. Hence, it needs physical capital (embodied in the intermediate goods) apart from labor to be produced. Technology is assumed to be increasingly complex and hence further innovations will require higher investments. Accordingly, if N_t is the amount invested in research, the Poisson arrival rate of innovations will be $\lambda_t n_t$, where $n_t = \frac{N_t}{A_t^{\max}}$ is the productivity adjusted level of research and λ_t is research productivity. The total amount of investment in research is divided by $A_t^{\max}$ in order to take into account the effect of increasing technological complexity since $A_t^{\max}$ is the leading edge coefficient that represents the aggregate state of knowledge. We approximate aggregate technological development by the productivity coefficient of the most advanced technology in the economy. When an innovation occurs, the productivity coefficient of that sector jumps discontinuously to $A_t^{\max}$. The leading edge coefficient grows gradually, at a rate that depends on the aggregate flow of innovations. The flow of profits to a monopolist who started producing at t, $\alpha(1-\alpha) L^{1-\alpha} A_t^{\max} m_t^{\alpha}$, is the payoff to innovators if they succeed. Because this payment does not depend on the sector, the level of research will be the same across sectors and the aggregate flow of innovations is thus $\lambda_t n_t$. We will assume that $A_t^{\max}$ grows at a rate proportional to this aggregate flow of innovations

$$\frac{\dot{A}_t^{\max}}{A_t^{\max}} = \sigma \lambda_t n_t, \quad \sigma > 0.$$

It can be proved (see Appendix A) that the long-run cross-sectoral distribution of the relative productivity parameters, $a_{it} = \frac{A_{it}}{A_t^{\max}}$, is time invariant and equal to

$$H(a) = a^{\frac{1}{\sigma}}, \quad 0 \leq a \leq 1. \tag{3}$$

To simplify, it is assumed that the initial distribution of a is also $H(a)$.

Consider the arbitrage equation of the research sector. This equation establishes the equality between the expected value of an innovation and its cost. The value of an innovation at t, V_t, must be the present value of the future flow of profits to the incumbent producer until a new technology displaces the monopolist. This flow of profits is $(1-\alpha)\alpha A_t^{\max} L^{1-\alpha} k_t^{\alpha}$, so the present value is given by

$$V_t = \int_t^{\infty} e^{-\int_t^{\tau}[r_s+\lambda_s n_s]ds}(1-\alpha)\alpha A_t^{\max} L^{1-\alpha} k_{\tau}^{\alpha} d\tau.$$

The expected marginal revenue of the innovation must equal its marginal cost. The cost of one unit of research in terms of output is 1. Therefore, since $n_t = \frac{N_t}{A_t^{\max}}$, the cost of one unit of research intensity is $A_t^{\max}$. I assume that there is a proportional tax on innovation that increases its cost.[4] Thus, the marginal cost of increasing research intensity is $(1+\tau_n)A_t^{\max}$ units of output, where τ_n is the tax to innovative activity. Hence, the research arbitrage condition may be written as

$$1+\tau_n = \lambda_t \frac{(1-\alpha)\alpha L^{1-\alpha} k_t^{\alpha}}{r_t + \lambda_t n_t}. \tag{4}$$

Equation (4) gives the research intensity as a function of capital intensity and the endogenously determined arrival rate of innovations, λ_t. Thus, the equilibrium level of research is a function of capital intensity and, indirectly, of financial intensity.[5]

2.5 Capital market

Capital is used as a factor of production in the intermediate goods sector. We have seen that equilibrium in the capital market requires the rental rate to satisfy equation (2). The owner of a unit of capital will obtain ζ_t for it. This amount must be enough to cover the cost of capital. This includes the rate of interest (r_t), the depreciation rate (δ), and the tax rate on capital accumulation (τ_k). Hence, the capital market arbitrage equation is

$$r_t + \delta + \tau_k = \alpha^2 L^{1-\alpha} k_t^{\alpha-1}, \tag{5}$$

which establishes a decreasing relationship between the interest rate and capital intensity.

2.6 Financing of research

Financial intermediaries channel savings both for its use as capital in production and to finance research projects. I assume that each intermediary has access to deposits at the market determined rate of interest. There is no risk of bankruptcy because they hold a perfectly diversified portfolio of production loans and research financing contracts.

[4] Perhaps, this is better understood if we consider a negative tax, i.e. a subsidy. The subsidy would reduce the cost of innovation.

[5] The arrival rate of innovations, or R&D productivity, is positively related to monitoring intensity.

No imperfection is introduced in the provision of production loans. However, I will consider some degree of informational asymmetry in the design of research financing contracts. In particular, I assume that researchers have no funds to invest in the project and, therefore, they have to look for external finance. The limited liability constraint implies that there will exist a potential problem of moral hazard on the part of the researcher. The funds needed for the project will be provided by intermediaries which are endowed with a monitoring technology that allows them to increase the effort of the researcher. Moreover, I assume that the intensity with which the intermediary monitors the researcher determines the additional effort that the former can force the latter to exert, as in Besanko and Kanatas (1993). It is assumed that there exists a one-to-one relationship between effort and probability of success. Therefore, the monitoring services of the financial intermediaries determine R&D productivity.

Consider a research project that requires an initial investment of one unit of output and that will yield a return v with probability λ. Given the research sector outlined in the previous section, the return per unit of output invested, v, must be equal to $\frac{V}{A^{\max}}$. The researcher obtains the funds from the intermediary and in exchange she will pay a fix amount p in case of success and nothing otherwise.[6]

The expected profits for the researcher are given by

$$\lambda(v-p)-D(\lambda),$$

where $D(\lambda)$ is the disutility caused by the effort necessary to obtain a probability of success equal to λ. We will assume that it takes the following form, which is borrowed from the work of Besanko and Kanatas (1993):

$$D(\lambda)=\frac{\lambda^2}{2\beta}.$$

If the researcher received no monitoring at all, the level of effort he would exert would be $\lambda_0=\beta(v-p)$. This *no-monitoring level of effort* is implementable at no cost for the intermediary. However, if the intermediary wishes to impose a higher level of effort, he will have to face a cost which I assume increasing and convex in the difference between the desired level of effort and λ_0.[7] In particular, I assume that in order to obtain a success probability of λ, the investment required is given by the following expression:

$$M(\lambda-\lambda_0)=\frac{(\lambda-\lambda_0)^2}{2s},$$

[6] This is a consequence of the limited liability constraint.

[7] See Besanko and Kanatas (1993) for details.

and therefore, the profits of the intermediary can be written as

$$\Pi_I = \lambda p - (1+\tau_f)M(\lambda - \lambda_0) - 1,$$

where τ_f is a tax on the monitoring activities of intermediaries. Notice that imposing taxation on monitoring activities implies that we are assuming that the monitoring costs of the intermediary are observable. Thus we are considering moral hazard only on the part of the researcher. This different treatment can be justified by the nature of the *effort* that intermediaries and researchers do. The disutility caused to the researcher by this effort is non-pecuniary while the monitoring effort of banks can be measured in monetary units, a feature that makes it easier to observe, especially when we are talking about financial intermediaries, one of the most regulated sectors in developed economies. Furthermore, we do not require that monitoring costs are observable to depositors but to the policy maker and to the researcher. In this respect, assuming observability is not as strong as it would be in other contexts.

There exists a large number of intermediaries that compete in the provision of financial services. A researcher will choose one of them on the basis of his supply of financial services since it will determine the probability of success of her project. However, once the researcher chooses an intermediary to finance her project, she will not be able to break this contract and ask another bank for finance. This assumption can be justified by the existence of switching costs or by the reluctancy of research firms to reveal information about their project. In addition, the fact that once the choice is made the researcher cannot turn to another intermediary implies that the bank is placed in a position of power in its relationship with the researcher. In particular, for a given λ, the intermediary will be able to impose the payment that maximizes his profits, i.e.

$$p(v,\lambda) = v - \frac{\lambda\left[\beta(1+\tau_f) - s\right]}{\beta^2(1+\tau_f)}. \tag{6}$$

The fact that the intermediary is able to impose the payment that maximizes his profits does not mean that the researcher is not going to gain with the contract. Indeed, the nature of the limited liability constraint implies that the researcher is always going to obtain a positive payment in expected terms.[8] Notice also that this payment scheme implies a negative relationship between

[8] Recall that the payment is positive in case of success and zero in case of failure, which yields a positive payment in expected terms. In order to guarantee that the expected payment is positive we have to impose some restrictions on the parameters. In particular, we require

$$s < \frac{\beta(1+\tau_f)}{2}.$$

p and λ. This is optimal for the intermediary because p is positively related to the monitoring cost of obtaining a given level of effort. Additionally, if the researcher is subject to an intensive control, she will have to pay less to the intermediary while there is a higher probability that the project succeeds. This may compensate the researcher for the intensive monitoring. In fact, if the relationship between p and λ is given by (6), the expected profits of the researcher become monotonically increasing in λ. Hence, this contract makes monitoring desirable for the researcher, since it will reduce the share of the intermediary in the project's return and increase the probability that the project succeeds. As a consequence, a researcher will choose the intermediary that offers the highest level of monitoring services. Therefore, no λ that implies a positive amount of profits will be an equilibrium since any intermediary can attract all the researchers by marginally increasing the degree of monitoring intensity and hence the probability of success. If the number of intermediaries is sufficiently large to impede agreements that limit competition, in equilibrium bank profits will be zero. Therefore, the equilibrium probability of success will be the highest value of λ that implies zero profits. That is, it is the positive root of

$$\lambda p(v, \lambda) - (1 + \tau_f) M(\lambda - \lambda_0(v, p(v, \lambda))) - 1 = 0$$

which yields a positive relationship between the productivity of research and the value of the project, as expressed by

$$\lambda = \tilde{\lambda}(v). \tag{7}$$

2.7 Equilibrium

Equations (4), (5) and (7) determine partial equilibrium in each market. These equations can be combined in order to obtain the following equilibrium conditions for each market:

(a) Research market equilibrium

$$1 + \tau_n = \lambda_t (v_t - p(v_t, \lambda_t)). \tag{8}$$

(b) Capital market equilibrium

$$r_t + \delta + \tau_k = \alpha^2 L^{1-\alpha} k_t^{\alpha - 1}. \tag{9}$$

(c) Credit market equilibrium

$$\lambda_t = \tilde{\lambda}(v_t). \tag{10}$$

Notice that the research arbitrage condition has been modified to take into account the payment to the intermediary.

Equations (6) and (8) imply the following equilibrium expression for λ:

$$\lambda = \left[\frac{\beta^2(1+\tau_f)(1+\tau_n)}{\beta(1+\tau_f)-s}\right]^{\frac{1}{2}}. \tag{11}$$

Hence, research productivity is time invariant and depends only upon the research and credit markets' structural parameters.

Using (11), equation (10) may be written in the following form:

$$v_t = \frac{\lambda}{\Phi(\tau_f, \tau_n)},$$

where

$$\Phi(\tau_f, \tau_n) = \frac{2\beta^2(1+\tau_f)(1+\tau_n)}{(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]}.$$

Thus, the system formed by equations (8), (9) and (10) can be reduced to the following system:[9]

$$\lambda = \left[\frac{\beta^2(1+\tau_f)(1+\tau_n)}{\beta(1+\tau_f)-s}\right]^{\frac{1}{2}}.$$

$$\frac{\lambda}{\Phi(\tau_f, \tau_n)} = \frac{\alpha(1-\alpha)L^{1-\alpha}k_t^{\alpha}}{r_t+\lambda n_t} \tag{12}$$

$$r_t + \delta + \tau_k = \alpha^2 L^{1-\alpha} k_t^{\alpha-1}, \tag{13}$$

which determines the equilibrium values of k_t and n_t. Notice also that from equations (12) and (13) one can obtain the equilibrium relationship between n_t and k_t as given by

$$n_t = n^d(k_t) = \left(\frac{\Phi(\tau_f, \tau_n)}{\lambda^2}\right)\frac{(1-\alpha)\alpha L^{1-\alpha}k_t^{\alpha}}{1+\tau_n} - \frac{\alpha^2 L^{1-\alpha}k_t^{\alpha-1}-\delta-\tau_k}{\lambda}. \tag{14}$$

With this equilibrium relationship the model can be reduced to a dynamic system of two differential equations in capital and consumption. The law of motion of capital is given by

$$\dot{K}_t = Y_t - C_t - N_t - E_t - \delta K_t,$$

where E_t is the total amount of resources invested in monitoring. If $M(\lambda - \lambda_0)$ is the monitoring cost per unit of output invested in research, then E_t must equal $M(\lambda - \lambda_0)N_t$. Notice that in

[9] Notice that in equation (12) we are just substituting v_t by its expression in equilibrium.

equilibrium $M(\lambda - \lambda_0)$ is a constant. Thus, in order to simplify, let us denote it by $e = M(\lambda - \lambda_0) = \frac{s(1+\tau_n)}{2(1+\tau_f)[\beta(1+\tau_f)-s]}$ so that E_t will be equal to eN_t.

The law of motion for consumption comes from utility maximization

$$\dot{C}_t = (r_t - \rho)C_t.$$

In order to obtain a system with steady state, express all variables in terms of efficiency units [10]

$$\dot{k}_t = L^{1-\alpha}k_t^{\alpha} - c_t - (1+\sigma)(1+e)n_t - (\delta + g_t)k_t \quad (15)$$

$$\dot{c}_t = (r_t - \rho - g_t)c_t, \quad (16)$$

and substitute the equilibrium expressions for r_t, g_t and n_t in equations (15) and (16) to express the system in terms of capital intensity and consumption per efficiency unit

$$\dot{k}_t = L^{1-\alpha}k_t^{\alpha} - c_t - (1+\sigma)(1+e)n^d(k_t) - (\delta + g^d(k_t))k_t$$

$$\dot{c}_t = (\alpha^2 L^{1-\alpha}k_t^{\alpha-1} - \delta - \tau_k - \rho - g^d(k_t))c_t.$$

where

$$g^d(k_t) = \sigma\lambda n^d(k_t).$$

Due to the non-linearity of the system it must be linearized around the steady state in order to analyze the local dynamics. Accordingly, we will study the system at the steady state in the next section.

3 Steady State Analysis

In a steady state all variables grow at a constant rate. If we substitute the equilibrium values $m_{it} = k_t = \frac{K_t}{A_t}$ in the aggregate production function, we obtain the usual Cobb-Douglas functional form at the aggregate level

$$Y_t = (A_t L)^{1-\alpha} K_t^{\alpha}.$$

[10] Note that

$$A_t = \int_0^1 A_{it}di = A_t^{\max}\int_0^1 \frac{A_{it}}{A_t^{\max}}di = A_t^{\max}\int_0^1 ah(a)da = A_t^{\max}E(a) = \frac{A_t^{\max}}{1+\sigma}.$$

Therefore, $\frac{N_t}{A_t} = \frac{(1+\sigma)N_t}{A_t^{\max}} = (1+\sigma)\,n_t$.

This expression implies that the rate of growth of output will be that of the aggregate productivity coefficient and, given that A_t is proportional to the leading edge coefficient, the growth rate of the economy will be

$$g = \sigma\lambda n,$$

where λ and n are constant and determined jointly with k through the equilibrium conditions of research, capital and credit markets.[11] These conditions, evaluated at the steady state, are the following:

$$\frac{\lambda}{\Phi} = \frac{\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{\rho + (1+\sigma)\lambda n}$$

$$\rho + \sigma\lambda n + \delta + \tau_k = \alpha^2 L^{1-\alpha}k^{\alpha-1} \tag{17}$$

$$\lambda = \left[\frac{\beta^2(1+\tau_f)(1+\tau_n)}{\beta(1+\tau_f)-s}\right]^{\frac{1}{2}},$$

from where we obtain

$$n = \left(\frac{\Phi(\tau_f,\tau_n)}{\lambda^2}\right)\frac{\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{(1+\sigma)} - \frac{\rho}{(1+\sigma)\lambda}, \tag{18}$$

and the equation that implicitly determines the steady state value of k, which is the result of plugging (18) into (17)

$$F(k) = \frac{\rho}{(1+\sigma)} + \left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right)\frac{\sigma\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{(1+\sigma)} + \delta + \tau_k - \alpha^2 L^{1-\alpha}k^{\alpha-1} = 0. \tag{19}$$

The steady state growth rate can be expressed in terms of capital intensity using equation (17) to obtain

$$g = \alpha^2 L^{1-\alpha}k^{\alpha-1} - \rho - \delta - \tau_k.$$

The use of implicit differentiation allows us to analyze the effect on k of parameter changes, and to obtain the following comparative statics results:

Proposition 1 *The steady state growth rate increases with subsidies to capital accumulation and to financial activity. The growth rate is increasing (decreasing) in* τ_n *when* $\tau_n > -\frac{s}{2\beta(1+\tau_f)-s}$ $\left(\tau_n < -\frac{s}{2\beta(1+\tau_f)-s}\right)$.

[11] Variables without time suscript denote steady state values.

Proof. See Appendix. ■

Proposition 2 *The steady state growth rate is increasing in σ (the size of innovations), decreasing in ρ and δ and increasing in s (the scale parameter of the monitoring costs) and β (the scale parameter of the disutility of effort).*

Proof. See Appendix. ■

Proposition 1 establishes a marginal positive relation between financial activity and growth. This relation may be understood because a subsidy to financial activity (or equivalently a reduction in τ_f) implies a lower monitoring cost. Thus, monitoring intensity increases. Accordingly, the positive growth effect of this policy is due to the externality that financial activity causes on the accumulation of public knowledge. Promoting financial activity is equivalent to increase the productivity of R&D and thus, to make a better use of the resources allocated to research.

The result obtained for the growth effects of research subsidies reflects the moral hazard problem of R&D. The smaller cost of research represents an increase of the expected return for researchers that does not depend on the effort they exert. It can be shown that a lower τ_n reduces the no monitoring level of effort.[12] This implies a higher monitoring cost and, thus, λ falls. Therefore, even though we expect a positive effect on research intensity, the R&D productivity reduction may be enough to cause a negative effect on the growth rate.

Aghion and Howitt (1998) argue that capital accumulation and innovation are complementary factors for long run growth. To illustrate this assertion, they reduce the capital tax, a measure that directly affects the capital market, and study the reaction of the economy. The reduction of the cost of capital rises the equilibrium value of capital intensity making the flow of profits accruing to a successful innovator grow. Consequently, investment in the research sector will increase. Thus a policy that directly favors capital accumulation also incentives innovation and economic growth. The same argument can be applied in the present model. Therefore, innovation and capital accumulation continue being complementary factors for long run growth. Furthermore, this policy has no negative effects either on λ_0 or on λ. Thus, a subsidy to capital accumulation may be preferable in terms of growth to a direct subsidy to research.

[12] The equilibrium expression for λ_0 is given by

$$\lambda_0 = \left[\frac{(1+\tau_n)\left[\beta\left(1+\tau_f\right)-s\right]}{\left(1+\tau_f\right)}\right]^{\frac{1}{2}}. \tag{20}$$

Thus, the result follows immediately.

We can perform the same experiment on financial activity. Thus, let us reduce the financial tax. The lower monitoring cost stimulates the production of financial services, inducing a rise in the arrival rate of innovations and, consequently, a larger rate of creative destruction. This discourages capital investment because the incumbent monopolist faces a larger probability of being replaced. Thus, the effect on capital accumulation is negative. That is, a policy that incentives financial activity will make the economy grow faster even though it will discourage capital investment. Therefore, capital and financial intensity should be considered substitutive factors for long run growth. Notice that this negative effect of research financing on capital accumulation undercuts the growth effects of intermediation promoting policies.

At the no-tax equilibrium a marginal reduction of any of the three taxes would increase the growth rate. In order to identify the most effective policy, the tax changes are made equivalent in terms of the amount of resources generated for the government budget. The budget constraint of the government is given by

$$\tau_n N_t + \tau_k K_t + \tau_f E_t = T,$$

where T is the lump-sum transference or tax used to balance the budget when we introduce a policy change. In order to make two policy changes equivalent, the change induced on T must be the same. Therefore, to compare the growth effects of τ_k, τ_f and τ_n, we must compare the following expressions:

$$\left|\frac{dg}{dT}\right|_{\substack{dT=K_t d\tau_k \\ \tau_f=\tau_k=\tau_n=0}} = \frac{dg}{d\tau_k}\frac{d\tau_k}{dT} = \frac{dg}{d\tau_k}\frac{1}{K_t}$$

$$\left|\frac{dg}{dT}\right|_{\substack{dT=E_t d\tau_f \\ \tau_f=\tau_k=\tau_n=0}} = \frac{dg}{d\tau_f}\frac{d\tau_f}{dT} = \frac{dg}{d\tau_f}\frac{1}{E_t}$$

$$\left|\frac{dg}{dT}\right|_{\substack{dT=N_t d\tau_n \\ \tau_f=\tau_k=\tau_n=0}} = \frac{dg}{d\tau_n}\frac{d\tau_n}{dT} = \frac{dg}{d\tau_n}\frac{1}{N_t},$$

which allow us to establish the following propositions:

Proposition 3 *At the no-tax equilibrium, the growth effect of τ_f is stronger than the growth effect of τ_n, i.e.,* $\frac{dg}{d\tau_f}\frac{1}{E_t} < \frac{dg}{d\tau_n}\frac{1}{N_t}$.

Proof. See Appendix. ■

Proposition 4 *At the no-tax equilibrium, the growth effect of* τ_f *is stronger than the growth effect of* τ_k*, i.e.* $\frac{dg}{d\tau_f}\frac{1}{E_t} < \frac{dg}{d\tau_k}\frac{1}{K_t}$*, whenever*

$$\alpha\,(1-\alpha)\,L^{1-\alpha}k^{\alpha} < \frac{\lambda}{\Phi}\left(\frac{2\,[\beta - s]}{s}\right)\rho. \tag{21}$$

Proof. See Appendix. ■

Proposition 3 implies that, at the no-tax equilibrium, subsidizing the financial sector will be more growth promoting than directly subsidizing research. Similarly, Proposition 4 implies that the financial tax may have larger effects on growth than the capital tax. Therefore, there exist situations in which subsidizing financial activity is the most effective policy in order to improve the growth performance of the economy. Notice that in the case of Proposition 4, condition (21) is expressed in terms of k which is an endogenous variable. Consequently, it could happen that the condition is never satisfied. However, by means of calibration, it is relatively easy to find sets of parameters for which the condition is satisfied. Notice also that the effectiveness of the financial tax depends upon s, the scale parameter for monitoring costs. A small s means a large monitoring cost and a low monitoring intensity, e. Therefore, the lower the s, the smaller the relative amount of resources allocated to financial services in equilibrium and the stronger the marginal effect we can induce on monitoring intensity. To sum up, this result proposes the use of subsidies or tax cuts to financial activity as an alternative instrument to promote innovation without the moral hazard problems of direct research subsidies.

4 Dynamics

After analyzing the behavior of the economy at its long run equilibrium, the system can now be linearized so as to study the dynamics of the model around the steady state. Recall that the system is formed by the following equations:

$$\begin{aligned}\dot{k}_t &= L^{1-\alpha}k_t^{\alpha} - c_t - (1+\sigma)(1+e)n^d(k_t) - (\delta + g^d(k_t))k_t \\ \dot{c}_t &= (\alpha^2 L^{1-\alpha}k_t^{\alpha-1} - \delta - \tau_k - \rho - g^d(k_t))c_t.\end{aligned}$$

The linearized system is obtained computing the Jacobian of the system and evaluating it at the steady state. In order to simplify notation let us express the system as follows

$$\begin{aligned}\dot{k}_t &= \varphi(k_t, c_t; \tau_k, \tau_f, \tau_n) \\ \dot{c}_t &= \phi(k_t, c_t; \tau_k, \tau_f, \tau_n).\end{aligned}$$

Then the derivatives needed are the following:

$$\begin{aligned}
\varphi_k(k,c) &= \alpha L^{1-\alpha}k^{\alpha-1} - (1+\sigma)(1+e)n^{d\prime}(k_t) - (\delta+g) - k(g^{d\prime}(k)) \\
\varphi_c(k,c) &= -1 \\
\phi_k(k,c) &= c(-\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2} - g^{\prime d}(k)) \\
\phi_c(k,c) &= 0.
\end{aligned}$$

With this notation the linearized system will be

$$\begin{aligned}
\dot{k}_t &= \varphi_k(k,c)(k_t-k) - (c_t-c) \\
\dot{c}_t &= \phi_k(k,c)(k_t-k).
\end{aligned}$$

The determinant of the matrix of the system is equal to the function $\phi_k(k,c)$ evaluated at the steady state, which can be proved to be negative. Therefore the system presents local saddle path stability. For future reference, let λ_1 be the negative eigenvalue and λ_2 the positive one.

5 Welfare analysis

Now that we have characterized the dynamics of the system we can analyze the welfare implications of changes in tax parameters.

From equation (1) we can express utility at the steady state in terms of the stationary level of consumption and the long-run growth rate

$$V_s(c,g) = \int_0^\infty \ln(cA_t)e^{-\rho t}dt = \frac{\ln(cA_0)}{\rho} + \frac{g}{\rho^2}.$$

The change in steady state welfare is a combination of the change in steady state consumption and the change in steady state growth

$$\frac{\partial V_s(c,g)}{\partial \tau_i} = \frac{1}{\rho c}\frac{\partial c}{\partial \tau_i} + \frac{1}{\rho^2}\frac{\partial g}{\partial \tau_i} \qquad \text{for } i = k, f, n. \tag{22}$$

This measure of welfare is valid to compare two situations of long run equilibrium. However, it does not consider the periods of transition during which the economy moves from one equilibrium to another. In order to reflect the transition we must analyze the effect on lifetime utility. Rewrite equation (1) to obtain the following expression for lifetime utility as a function of the different tax rates (τ_i where $i = k, f, n.$):

$$V(\tau_i) = \frac{\ln(A_0)}{\rho} + \int_0^\infty \left[\int_0^t g_s(\tau_i)ds\right] e^{-\rho t}dt + \int_0^\infty \ln(c_t(\tau_i))e^{-\rho t}dt$$

where $g_t(\tau_i)$ and $c_t(\tau_i)$ are the time paths of the growth rate and the level of consumption per efficiency unit after a change in one of the tax parameters. The effect on utility will thus be given by the effects on the paths of growth and consumption. I will obtain first the effect on the paths of consumption and capital intensity and then use the latter to get the effect on the path of the growth rate.

Let $c = p(k, \tau_i)$ be the saddle path of the system which can be interpreted as the graph of a policy function relating consumption and capital. Then, we know that its slope p_k, is positive and equal to $\frac{\phi_k}{\lambda_1}$. Substituting the policy function into the law of motion of k, the equilibrium dynamics of the system can be characterized by a single differential equation which describes the evolution of the state variable along the stable manifold.

$$\dot{k} = \varphi(k, c) = \varphi(k, p(k, \tau_i)) = \Psi(k, \tau_i).$$

The solution to this equation, $k_t(\tau_i)$, gives the equilibrium value of k as a function of time and the tax parameter. Using $k_t(\tau_i)$ in the policy function we would obtain the time path of c

$$c_t(\tau_i) = p(k_t(\tau_i), \tau_i).$$

To calculate the change in welfare we need the derivative of the whole time path of c with respect to τ_i

$$\frac{dc_t(\tau_i)}{d\tau_i} = p_k \frac{dk_t(\tau_i)}{d\tau_i} + p_{\tau_i}, \tag{23}$$

where p_{τ_i} is the derivative of the policy function with respect to the tax or graphically, the shift in the saddle path caused by the policy change.

In order to compute $\frac{dk_t(\tau_i)}{d\tau_i}$, notice that $k_t(\tau_i) = k(t, \tau_i)$ must satisfy identically the original equation

$$\dot{k}(t, \tau_i) \equiv \varphi\left(p(k(t, \tau_i), \tau_i), k(t, \tau_i), \tau_i\right),$$

differentiate both sides with respect to τ_i

$$\dot{k}_{\tau_i} = \frac{dk_{\tau_i}}{dt} = \left[\varphi_c p_k + \varphi_k\right] k_{\tau_i} + \varphi_c p_{\tau_i} + \varphi_{\tau_i}.$$

Hence k_{τ_i} satisfies a linear differential equation. Moreover, when we start from a steady state, the coefficients of this equation are constant and we can write

$$\dot{k}_{\tau_i} = \lambda_1 k_{\tau_i} - p_{\tau_i} + \varphi_{\tau_i}.$$

The general solution is given by

$$k_{\tau_i}(t) = \exp(\lambda_1 t)\, k_{\tau_i}(0) + (1 - \exp(\lambda_1 t)) k_{\tau_i}(\infty).$$

Since k is a predetermined variable, the change at the date of the policy change $k_{\tau_i}(0)$ must be zero. The long run effect, $k_{\tau_i}(\infty) = \lim_{t\to\infty} k_{\tau_i}(t)$, is in fact the derivative of the steady state value of k with respect to the tax parameter, and can be expressed as

$$k_{\tau_i}(\infty) = \frac{p_{\tau_i} - \varphi_{\tau_i}}{\lambda_1}.$$

The equilibrium time path of the derivative of k with respect to τ_i is thus given by

$$k_{\tau_i}(t) = (1 - \exp(\lambda_1 t)) \left[\frac{p_{\tau_i} - \varphi_{\tau_i}}{\lambda_1}\right],$$

that is, k will gradually reach its new steady state value at a rate equal to the negative eigenvalue.

Substitute now in equation (23) to obtain the final expression for the derivative of the time path of consumption with respect to the tax parameter

$$\frac{dc_t(\tau_i)}{d\tau_i} = p_k(1 - \exp(\lambda_1 t)) \left[\frac{p_{\tau_i} - \varphi_{\tau_i}}{\lambda_1}\right] + p_{\tau_i}.$$

As before, we can identify the immediate change and the long run effect

$$\begin{aligned} \frac{dc_0(\tau_i)}{d\tau_i} &= p_{\tau_i}, \\ \frac{dc_\infty(\tau_i)}{d\tau_i} &= p_k \left[\frac{p_{\tau_i} - \varphi_{\tau_i}}{\lambda_1}\right] + p_{\tau_i}, \end{aligned}$$

where the first represents the necessary jump of consumption to get on the new saddle path and the second is the effect on the steady state value of consumption. Thus, consumption will initially jump to the new saddle path and then it will approach its new steady state value at a rate equal to λ_1.

The derivative of the growth rate and consumption per efficiency unit at date t are given by

$$\frac{dg_t(\tau_i)}{d\tau_i} = \frac{dg^d(k)}{dk}(1 - \exp(\lambda_1 t)) \frac{\partial k}{\partial \tau_i} + \frac{\partial g^d(k)}{\partial \tau_i} \tag{24}$$

$$\frac{dc_t(\tau_i)}{d\tau_i} = \frac{\partial c}{\partial \tau_i} - p_k \exp(\lambda_1 t) \frac{\partial k}{\partial \tau_i}. \tag{25}$$

Notice that the derivatives of g^d are evaluated at the steady state because we consider the stationary equilibrium as the situation before the tax change.

Expressions (24) and (25) allow us to write the change in welfare as follows:

$$\frac{\partial V(\tau_i)}{\partial \tau_i} = \frac{\partial V_s(\tau_i)}{\partial \tau_i} + \left[\frac{\frac{\rho - \lambda_1}{\rho} \frac{dg^d(k)}{dk} + \frac{(1-\alpha)\zeta}{k}}{\lambda_1 (\rho - \lambda_1)}\right] \frac{\partial k}{\partial \tau_i}. \tag{26}$$

Equations (22) and (26) give the general expressions for the effect of the three taxes on the different measures of welfare. Let us see now the specific results for each policy.

5.1 Tax on capital

The effect on welfare of the capital tax is given by

$$\frac{\partial V_s(c,g)}{\partial \tau_k} = \frac{1}{\rho c}\frac{\partial c}{\partial \tau_k} + \frac{1}{\rho^2}\frac{\partial g}{\partial \tau_k}$$

$$\frac{\partial V(c,g)}{\partial \tau_k} = \frac{\partial V_s(c,g)}{\partial \tau_k} + \left[\frac{\left(\frac{\rho-\lambda_1}{\rho}\right)\frac{dg^d(k)}{dk} + \frac{(1-\alpha)\zeta}{k}}{\lambda_1\left(\rho-\lambda_1\right)}\right]\frac{\partial k}{\partial \tau_k}, \tag{27}$$

where the first expression represents the effect on welfare if the transition is excluded. Both the expression in square brackets in equation (27) and $\frac{\partial k}{\partial \tau_k}$ are negative. Therefore, the effect on welfare using the second measure will always be larger than the effect if we use the first measure.

Proposition 1 shows that $\frac{\partial g}{\partial \tau_k}$ is negative. However, the effect on consumption is ambiguous. The derivative of consumption with respect to the capital tax is given by

$$\frac{\partial c}{\partial \tau_k} = \frac{k}{1+\frac{\Phi}{\lambda}\left(\frac{\sigma}{1+\sigma}\right)k}\left(-\frac{1}{\alpha} + \frac{(1+e)\Phi}{\lambda^2} - \frac{\rho+\tau_k}{(1-\alpha)\,\zeta} + \frac{\Phi}{\lambda}\left(\frac{\sigma}{1+\sigma}\right)k\right).$$

The functional form of this derivative implies that for large enough values of steady state capital intensity, the derivative will be positive while it may be negative for smaller values of k. Since the relationship between k and the capital tax is negative, this suggests that for negative or small values of τ_k we might expect a positive effect on consumption while for large values of the tax, $\frac{\partial c}{\partial \tau_k}$ may become negative. Therefore, we may roughly represent the relationship between consumption and the capital tax as an inverted U-shaped curve whose maximum shifts right or left depending on the structural characteristics of the economy. In summary, there may exist a consumption maximizing value of τ_k but whether it is a subsidy or a tax depends upon the economy considered. These results can also be applied to the relationship between welfare and this tax. I have calibrated the model for a usually accepted set of parameters obtaining that in every case, the welfare maximizing rate of this policy instrument was a subsidy.[13] Consequently, in economies with a positive capital tax rate, a tax reduction will generally cause a welfare improvement.

[13] The set of parameters used includes $\rho = 0.02$, $\delta = 0.05$, $\sigma = \ln(1.1)$ and $L = 1$. The values of β and s were chosen so that the resulting steady state values of the growth rate and the probability of success lay in a reasonable interval. The computer program used for calibration is available upon request.

5.2 Tax on financial services

The welfare derivatives for the financial tax are

$$\frac{\partial V_s(c,g)}{\partial \tau_f} = \frac{1}{\rho c}\frac{\partial c}{\partial \tau_f} + \frac{1}{\rho^2}\frac{\partial g}{\partial \tau_f}$$

$$\frac{\partial V(c,g)}{\partial \tau_f} = \frac{\partial V_s(c,g)}{\partial \tau_f} + \left[\frac{\left(\frac{\rho-\lambda_1}{\rho}\right)\frac{dg^d(k)}{dk} + \frac{(1-\alpha)\zeta}{k}}{\lambda_1(\rho-\lambda_1)}\right]\frac{\partial k}{\partial \tau_f},$$

and given that $\frac{\partial k}{\partial \tau_f}$ is positive, the effect on welfare of this tax will always be smaller if we consider the transition.

As before, we know that the derivative of the growth rate with respect to this tax is negative. The effect on consumption is given by

$$\begin{aligned}\frac{\partial c}{\partial \tau_f} &= (1-\alpha)\,\zeta\frac{\partial k}{\partial \tau_f}\left(\frac{1+\alpha}{\alpha} - \frac{(1+e)\Phi}{\lambda^2} + \frac{\rho+\tau_k}{(1-\alpha)\,\zeta}\right) + \\ &+ \left[-\frac{\partial}{\partial \tau_f}\frac{\Phi\,(1+e)}{\lambda^2}\right]\alpha\,(1-\alpha)\,L^{1-\alpha}k^{\alpha} + \rho\left[\frac{\partial}{\partial \tau_f}\frac{1+e}{\lambda}\right]. \end{aligned} \tag{28}$$

In order to simplify the analysis, the range of values of the tax parameters is restricted so that we can give an unambiguous sign to this derivative. To this end, we will not consider values of the capital tax rate below $-\rho$ nor subsidy rates to the research sector above $\frac{5}{7}$. Under these assumptions, we can establish the following proposition:

Proposition 5 *If $\tau_k > -\rho$ and $\tau_n > -\frac{5}{7}$, the derivative of steady state consumption per efficiency unit with respect to the financial tax is positive.*

Proof. See Appendix ∎

Consequently, a marginal change in the financial tax will cause opposite effects on growth and consumption, depending the final welfare change on which effect dominates. Obviously, the value of the discount rate is determinant for the sign of $\frac{\partial V_s(c,g)}{\partial \tau_f}$. This derivative will be positive whenever $\frac{\partial c}{\partial \tau_f} + \frac{c}{\rho}\frac{\partial g}{\partial \tau_f}$ is positive. A small ρ means that consumers weight more heavily the growth effect of the tax. Thus, if ρ is small enough, welfare will increase with reductions of the financial tax. Notice also that for a given discount rate, increases in τ_f make steady state consumption per efficiency unit grow. Therefore, we may expect positive effects on welfare for low values of the tax though they may disappear as the tax rate increases. Hence, we also find the inverted U-shaped curve representing the relationship between welfare and the financial tax.

A calibration of the model gives a rough idea of how can financial policies improve welfare. At the no tax equilibrium and for the same set of parameters used before, I obtain the following results:

Table 1

Calibration for $\rho = 0.02$ *at the no-tax equilibrium.*

$\boldsymbol{\alpha}$	$\frac{\partial V_s}{\partial \tau_f}$	$\frac{\partial V}{\partial \tau_f}$
0.80	-0.014	-0.031
0.75	-0.010	-0.023
0.70	-0.005	-0.015
0.65	-0.002	-0.007
0.60	0.001	-0.002
0.55	0.004	0.002
0.50	0.005	0.004
0.45	0.005	0.005
0.40	0.005	0.005
0.35	0.003	0.003

A negative sign of the welfare derivative means that the optimal policy is to reduce the financial tax. Conversely, a positive entry implies that the optimal policy is a tax increase. This calibration suggests that financial services will be underprovided in a relatively capital intensive economy while in less capital intensive economies, a reduction of its provision could increase welfare. Recall that the financial sector has real effects on the economy only because it can modify the productivity of research. A high α means a relatively high equilibrium value of k which in turn implies a high research intensity. Therefore, a policy that favors monitoring and thus, increases the productivity of research, will have larger growth effects in an economy with a relatively higher research intensity. This larger growth effect will be able to compensate for the reduction in steady state consumption per efficiency unit. On the contrary, if α is small, so is equilibrium research intensity and thus, the higher productivity in this case will not be able to induce a large enough increase in the growth rate.

5.3 Tax on research activity

The welfare derivatives for the research tax are

$$\frac{\partial V_s(c,g)}{\partial \tau_n} = \frac{1}{\rho c}\frac{\partial c}{\partial \tau_n} + \frac{1}{\rho^2}\frac{\partial g}{\partial \tau_n}$$

$$\frac{\partial V(c,g)}{\partial \tau_n} = \frac{\partial V_s(c,g)}{\partial \tau_n} + \left[\frac{\frac{\rho-\lambda_1}{\rho}\frac{dg^d(k)}{dk} + \frac{(1-\alpha)\zeta}{k}}{\lambda_1(\rho-\lambda_1)}\right]\frac{\partial k}{\partial \tau_n},$$

and as with the financial tax, the fact that $\frac{\partial k}{\partial \tau_n}$ is positive makes the effect on welfare of this tax smaller if we consider the transition.

The derivative of steady state consumption per efficiency unit is given by the following expression:

$$\begin{aligned}\frac{\partial c}{\partial \tau_n} &= (1-\alpha)\zeta\frac{\partial k}{\partial \tau_n}\left(\frac{1+\alpha}{\alpha} - \frac{(1+e)\Phi}{\lambda^2} + \frac{\rho+\tau_k}{(1-\alpha)\zeta}\right) + \\ &\quad + \left[-\frac{\partial}{\partial \tau_n}\frac{\Phi(1+e)}{\lambda^2}\right]\alpha(1-\alpha)L^{1-\alpha}k^{\alpha} + \rho\left[\frac{\partial}{\partial \tau_n}\frac{1+e}{\lambda}\right].\end{aligned}$$

The effect of the research tax on consumption is established in the next proposition:

Proposition 6 *If $\tau_n > -\frac{s}{2\beta(1+\tau_f)-s}$ and $\tau_k > -\rho$, the derivative of steady state consumption per efficiency unit with respect to the research tax is positive.*

Proof. See Appendix. ■

Given that the effect on growth of this tax is negative, the final effect on welfare will depend upon the discount rate.[14] As with the financial tax, if ρ is small enough, welfare may increase with a reduction of research taxation. In general though, we expect the typical inverted-U relationship in the sense that increases of the research tax may initially improve welfare though further increases could finally harm it.

If the government were considering whether to subsidize the research or the financial sector, we know that the financial tax will have larger effects on growth and in this sense it would be preferable.[15] However, we must consider also the effect on consumption. We would like to have

[14] I will restrict the rest of the welfare analysis of this tax to $\tau_n > -\frac{s}{2\beta(1+\tau_f)-s}$, because the sign of the derivative of consumption for smaller values of τ_n is ambiguous.

[15] In what follows, I assume that the initial situation is the no-tax equilibrium. Therefore, the effect on growth of the two subsidies is positive being the financial tax more effective.

the result that the effect on consumption of the financial subsidy is smaller since consumption will be reduced. However, we find the opposite result. That is, a financial subsidy will cause a larger reduction in steady state consumption per efficiency unit than a research subsidy. Consequently, whether one policy is preferable to the other in terms of welfare will depend upon the discount rate of the economy. A calibration of the model for $\rho = 0.02$, yields the following results:

Table 2

Welfare effects of τ_f and τ_n

α	$\frac{\partial V_s}{\partial \tau_f}\frac{1}{e}$	$\frac{\partial V_s}{\partial \tau_n}$	$\frac{\partial V}{\partial \tau_f}\frac{1}{e}$	$\frac{\partial V}{\partial \tau_n}$
0.80	−14.0	5.7	−30.9	5.7
0.75	−10.0	5.4	−22.9	5.4
0.70	−5.0	5.1	−15.0	5.1
0.65	−2.0	4.8	−7.5	4.8
0.60	1.0	4.3	−1.6	4.3
0.55	4.0	3.8	2.4	3.8
0.50	5.0	3.1	4.6	3.1
0.45	5.0	2.4	5.2	2.4
0.40	5.0	1.6	4.7	1.6
0.35	3.0	0.8	3.3	0.8

Notice that the sign of the welfare derivative with respect to the research tax is positive in every case. This means that a subsidy (a marginal reduction of the tax) would reduce welfare. In other words, the positive growth effect is not enough to compensate for the negative effect on steady state consumption per efficiency unit. Therefore, if the government wishes to increase welfare, the appropriate policy is a research tax increase. With respect to the other policy instrument, the financial tax, the effect on welfare of the latter is larger when α is either very large or very small. Thus, if we consider $\alpha = 0.75$ as a proxy for the capital intensity of a developed economy, a policy that promotes the financing of research projects by intermediaries dominates a direct subsidy to research both in terms of growth and welfare.

6 Conclusions

Innovation is nowadays recognized as one of the most important factors of economic growth. However, the presence of informational asymmetries and the difficult appropriation of R&D's external effects cause inefficiencies that may reduce the private production of innovation. This paper analyses the consequences on economic growth of the activity of financial intermediaries that try to reduce the incidence of moral hazard on research. There exists moral hazard because in the absence of monitoring, researchers choose the amount of effort that maximizes their expected utility, a smaller level of effort than the one that would maximize the expected value of the project. The no-monitoring level of effort is smaller because the researcher receives only a part of the value of the innovation while the rest goes to the intermediary. However, the intermediary is provided with a monitoring technology that enables him to impose a higher effort. The monitoring intensity will determine the amount of effort affordable and the probability of success of the research project. This paper shows that a policy that incentives monitoring is able to improve the growth performance of the economy due to its positive effect on R&D productivity. Furthermore, it is shown that directly subsidizing research may reduce the growth rate of the economy. The negative effect on growth of a research subsidy may appear because it accentuates the incidence of moral hazard. As a consequence, this paper proposes subsidies to capital accumulation and to financial activity as alternative growth promoting policies. The advantage of these policies with respect to the research subsidy is that they do not see their effects undercut by a reduction of R&D productivity.

A subsidy to financial activity increases the growth rate of the economy. However, its effect on steady state consumption per efficiency unit is negative. Therefore, the actual value of the discount rate will determine the sign of the welfare effect in each case. Nevertheless, for a typical value of the discount rate, it is obtained that financial services will be underprovided in relatively capital intensive economies while they will be overprovided in less capital intensive economies. This may be due to the interaction of two externalities of opposite sign. On the one hand, the positive effect of financial activities on R&D productivity makes the whole economy more productive since the growth rate of aggregate productivity depends positively on the arrival rate of innovations. However, the magnitude of this positive effect depends upon the relative importance of the research sector which in turn is determined by capital intensity. Thus, the more capital intensive the economy, the greater this effect will be. On the other hand, a higher probability of success due to a more intense monitoring implies a higher probability of replacement for the incumbent producer. This discourages capital accumulation. Whether the reduction in the equilibrium level of capital causes a large or a small effect depends upon the initial situation of the economy. If capital intensity was relatively low, the initial equilibrium level of capital is relatively small and a further reduction will have large negative effects on the economy. On the contrary, if the economy was in an equilibrium with a large level of capital per efficiency unit, a reduction will not represent a big damage. Thus, the positive externality is stronger when capital intensity is high, while the negative externality has larger effects when the economy is less capital intensive. Therefore, policies aimed at balancing the effects of the two externalities will be welfare improving.

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A Proofs of propositions

Proof that $H(a)$ is the limiting distribution of relative productivities.
(Adapted from Aghion and Howitt (1998))

Let $F(\cdot, t)$ denote the cumulative distribution of the absolute productivity parameters, A, across sectors at date t. Pick any $A > 0$ and let it be the leading edge coefficient at $t_0 \geq 0$. Define $\Phi(t) = F(A, t)$. Then

$$\begin{aligned} \Phi(t_0) &= 1 \\ \frac{d\Phi(t)}{dt} &= -\Phi(t)\lambda_t n_t \text{ for all } t \geq t_0. \end{aligned} \tag{29}$$

Equation (29) gives the rate at which the fraction of sectors with a productivity coefficient smaller than A falls. This rate is given by the flow of innovations occurred in the sectors behind A, i.e. $\Phi(t)\lambda_t n_t$. The solution to this differential equation is

$$\Phi(t) = e^{-\int_{t_0}^{t} \lambda_s n_s ds} \text{ for all } t \geq t_0.$$

Recall that

$$\frac{dA_t^{\max}}{dt} = \sigma A_t^{\max} \lambda_t n_t$$

and that $A = A_{t_0}^{\max}$, therefore

$$\frac{A}{A_t^{\max}} = e^{-\sigma \int_{t_0}^t \lambda_s n_s ds},$$

or equivalently

$$\Phi(t) = \left(\frac{A}{A_t^{\max}}\right)^{\frac{1}{\sigma}}.$$

Define a to be the relative productivity $\frac{A}{A_t^{\max}}$. By construction, $\Phi(t)$ is the fraction of sectors in which the productivity coefficient is less than A. Hence, the last equation establishes that this fraction is given by equation (3) at date t if a is the relative productivity at t of a sector that innovated on or after date t_0. If t is large enough, this will include almost all values of a between 0 and 1. ■

Proof of Proposition 1. The signs of the derivatives of the growth rate depend upon the signs of the derivatives of the steady state capital intensity. Consider equation (19) which defines the steady state values of k. Straightforward differentiation yields

$$\begin{aligned}
\frac{\partial F(k)}{\partial k} &= \alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2}\left[1+\frac{\sigma}{(1+\sigma)}\frac{\Phi}{\lambda}k\right] \\
\frac{\partial F(k)}{\partial \tau_k} &= 1 \\
\frac{\partial F(k)}{\partial \tau_f} &= \frac{\partial}{\partial \tau_f}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right)\frac{\sigma\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{(1+\sigma)} \\
\frac{\partial F(k)}{\partial \tau_n} &= \frac{\partial}{\partial \tau_n}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right)\frac{\sigma\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{(1+\sigma)},
\end{aligned} \tag{30}$$

where

$$\frac{\partial}{\partial \tau_f}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right) = \frac{\Phi e}{\lambda(1+\tau_n)}\frac{(1+\tau_n)\,s-2\left[\beta(1+\tau_f)-s\right]}{(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]},$$

expression which is negative for the range of values assumed for the parameters. The sign of the derivative in (30) depends upon $\frac{\partial}{\partial \tau_n}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right)$ given by

$$\frac{\partial}{\partial \tau_n}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right) = \left(\frac{\Phi}{2\lambda(1+\tau_n)}\right)\frac{2\left[\beta(1+\tau_f)-s\right]-(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]}{(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]}.$$

This derivative is negative if and only if $\tau_n > -\frac{s}{2\beta(1+\tau_f)-s}$. Therefore,

$$\frac{\partial k}{\partial \tau_k} = -\frac{\frac{\partial F(k)}{\partial \tau_k}}{\frac{\partial F(k)}{\partial k}} < 0,$$

$$\frac{\partial k}{\partial \tau_f} = -\frac{\frac{\partial F(k)}{\partial \tau_f}}{\frac{\partial F(k)}{\partial k}} > 0,$$

$$\frac{\partial k}{\partial \tau_n} = -\frac{\frac{\partial F(k)}{\partial \tau_n}}{\frac{\partial F(k)}{\partial k}} \geq 0 \qquad \text{for } \tau_n \geq -\frac{s}{2\beta(1+\tau_f)-s} \text{ and}$$

$$\frac{\partial k}{\partial \tau_n} = -\frac{\frac{\partial F(k)}{\partial \tau_n}}{\frac{\partial F(k)}{\partial k}} < 0 \qquad \text{for } \tau_n < -\frac{s}{2\beta(1+\tau_f)-s}.$$

Given the signs of the derivatives of k with respect to the different taxes, the effects on growth can be obtained recalling that the following equation must hold in equilibrium:

$$g = \alpha^2 L^{1-\alpha} k^{\alpha-1} - \rho - \delta - \tau_k.$$

Consequently, the derivative of the growth rate with respect to the capital tax is given by

$$\frac{\partial g}{\partial \tau_k} = -(1-\alpha)\,\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{\partial k}{\partial \tau_k} - 1,$$

or equivalently

$$\frac{\partial g}{\partial \tau_k} = \frac{-\left(\frac{\sigma}{1+\sigma}\right)\frac{\Phi(\tau_f,\tau_n)}{\lambda} k}{\left[1 + \left(\frac{\sigma}{1+\sigma}\right)\frac{\Phi(\tau_f,\tau_n)}{\lambda} k\right]},$$

which is unambiguously negative. Therefore, the growth rate depends negatively on the capital tax and thus, a subsidy increase or a reduction of the tax would enhance growth.

The derivatives of the growth rate with respect to the financial tax and to the innovation tax are

$$\frac{\partial g}{\partial \tau_f} = -(1-\alpha)\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{\partial k}{\partial \tau_f},$$

and

$$\frac{\partial g}{\partial \tau_n} = -(1-\alpha)\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{\partial k}{\partial \tau_n}.$$

Given the signs of the derivatives of k we have previously obtained, the corresponding results of Proposition 1 follow. ■

Proof of Proposition 2. The derivative of k with respect to σ is given by the following expression:

$$\frac{\partial k}{\partial \sigma}=\frac{-\lambda n}{(1+\sigma)\,\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2}\left[1+\frac{\sigma}{(1+\sigma)}\frac{\Phi(\tau_f,\tau_n)}{\lambda}k\right]},$$

which is negative. Thus, capital intensity at the steady state is negatively related to σ. In consequence, the derivative of g with respect to σ is positive.

The other two results are immediate since the derivative of g with respect to δ is equal to the derivative with respect to τ_k and the derivative of k with respect to ρ satisfies

$$\frac{\partial k}{\partial \rho}=\left(\frac{1}{1+\sigma}\right)\frac{\partial k}{\partial \tau_k}.$$

Therefore, if the derivative of g with respect to τ_k is negative, so is the derivative of g with respect to ρ.

Regarding the effect on the growth rate of changes in s and β, notice that

$$\frac{\partial F(k)}{\partial s}=\left(\frac{\sigma\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{1+\sigma}\right)\frac{\partial\left(\frac{\Phi}{\lambda}\right)}{\partial s},$$

and

$$\frac{\partial F(k)}{\partial \beta}=\left(\frac{\sigma\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{1+\sigma}\right)\frac{\partial\left(\frac{\Phi}{\lambda}\right)}{\partial \beta},$$

where

$$\frac{\partial}{\partial s}\left(\frac{\Phi}{\lambda}\right)=\left(\frac{\Phi}{\lambda}\right)\frac{[2\beta(1+\tau_f)-(3+\tau_n)s]}{2\left[\beta(1+\tau_f)-s\right]\left[(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]\right]}$$

$$\frac{\partial}{\partial \beta}\left(\frac{\Phi}{\lambda}\right)=\left(\frac{\Phi}{\lambda\beta}\right)\frac{[\beta(1+\tau_f)-2s]+(1+\tau_n)\frac{\left[\beta(1+\tau_f)\left[2\beta(1+\tau_f)-3s\right]+2s^2\right]}{2\left[\beta(1+\tau_f)-s\right]}}{\left[(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]\right]},$$

are both positive. Therefore, $\frac{\partial F(k)}{\partial s}$ and $\frac{\partial F(k)}{\partial \beta}$ are also positive, which implies that $\frac{\partial k}{\partial s}$ and $\frac{\partial k}{\partial \beta}$ are negative. Therefore, the derivatives of the growth rate with respect to these parameters are both positive. ■

Proof of Proposition 3. $\frac{dg}{d\tau_f}\frac{1}{E_t}<\frac{dg}{d\tau_n}\frac{1}{N_t}$ holds if and only if $\frac{dg}{d\tau_f}\frac{1}{e}<\frac{dg}{d\tau_n}$. At the no tax equilibrium this inequality is given by the following expression:

$$-\frac{(1-\alpha)\alpha^2L^{1-\alpha}k^{\alpha-2}}{e}\frac{\partial k}{\partial \tau_f}<-(1-\alpha)\alpha^2L^{1-\alpha}k^{\alpha-2}\frac{\partial k}{\partial \tau_n},$$

or equivalently

$$\frac{1}{e}\frac{\partial k}{\partial \tau_f} > \frac{\partial k}{\partial \tau_n}.$$

This inequality holds whenever

$$\frac{1}{e}\frac{\partial}{\partial \tau_f}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right) < \frac{\partial}{\partial \tau_n}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right).$$

Evaluating both derivatives at the no-tax equilibrium and simplifying we obtain that the condition for the inequality to hold is

$$s < \frac{4}{7}\beta.$$

The parameters involved in the last expression (s and β) must be positive and satisfy the following condition:

$$s < \frac{\beta}{2}(1+\tau_f),$$

which is necessary to guarantee a positive expected value of the project for the researcher. Therefore, at the no-tax equilibrium, the growth effect of τ_f is larger than the growth effect of τ_n.

■

Proof of Proposition 4. The growth effect of τ_f is larger in absolute value than the growth effect of τ_k when $\frac{dg}{d\tau_f}\frac{1}{E_t} < \frac{dg}{d\tau_k}\frac{1}{K_t}$ which at the steady state is equivalent to require that $\frac{dg}{d\tau_f}\frac{1}{(1+\sigma)en} < \frac{dg}{d\tau_k}\frac{1}{k}$. Evaluating both derivatives at the no-tax equilibrium and simplifying yields the desired expression, i.e., $\alpha\left(1-\alpha\right)L^{1-\alpha}k^{\alpha} < \frac{\lambda}{\Phi}\frac{2[\beta-s]}{s}\rho$. ■

Proof of Proposition 5. The derivative of c with respect to τ_f is given by equation (28). In order to obtain positive values of steady state consumption, we assume that the parameters are such that $\frac{1+\alpha}{\alpha} - \frac{(1+e)\Phi}{\lambda^2} > 0$. Under this assumption, the first term of this expression is positive and so is the second. However, the last term may be positive or negative depending on the actual values of τ_f and τ_n. Nevertheless, from equation (14) we can express this derivative as follows:

$$\frac{\partial c}{\partial \tau_f} = \left(1-\alpha\right)\zeta\frac{\partial k}{\partial \tau_f}\left(\frac{1+\alpha}{\alpha} - \frac{(1+e)\Phi}{\lambda^2} + \frac{\rho+\tau_k}{\left(1-\alpha\right)\zeta}\right) + \tag{31}$$

$$+\left[-\frac{\partial}{\partial \tau_f}\left(\frac{\Phi\left(1+e\right)}{\lambda^2}\right)\right]\frac{\lambda^2}{\Phi}\left(1+\sigma\right)n + \rho\left[\frac{\partial}{\partial \tau_f}\left(\frac{1+e}{\lambda}\right) - \frac{\partial}{\partial \tau_f}\left(\frac{\Phi\left(1+e\right)}{\lambda^2}\right)\frac{\lambda^2}{\Phi}\right],$$

where the first term is positive because $\frac{\partial k}{\partial \tau_f}$ is positive, $\rho + \tau_k$ is positive under the assumptions of the proposition and we had previously assumed that the parameters must be such that $\frac{1+\alpha}{\alpha} > \frac{(1+e)\Phi}{\lambda^2}$ in order to guarantee a positive level of consumption in equilibrium.

The second term of (31) will be positive whenever $\frac{\partial}{\partial \tau_f}\left(\frac{\Phi(1+e)}{\lambda^2}\right)$ is negative. This derivative is given by the following expression, which is negative when $\tau_n > -\frac{5}{7}$:

$$\frac{\partial}{\partial \tau_f}\left(\frac{\Phi(1+e)}{\lambda^2}\right) = \left(\frac{\Phi e}{\lambda^2}\right) \frac{2\beta(1+\tau_f)^2 - (1+\tau_n)\left[4\beta(1+\tau_f) - s\right] - 2\left[2\beta(1+\tau_f) - s\right]}{(1+\tau_f)\left[(1+\tau_n)\left[2\beta(1+\tau_f) - s\right] + 2\left[\beta(1+\tau_f) - s\right]\right]}.$$

The third term of (31) may be expressed as follows:

$$\frac{\rho e}{(1+\tau_f)}\left[\frac{2(\tau_f - \tau_n)}{(1+\tau_n)} + \frac{(1+\tau_n)\left[4\beta(1+\tau_f) - s\right] + 2\left[2\beta(1+\tau_f) - s\right] - 2\beta(1+\tau_f)^2}{(1+\tau_n)\left[2\beta(1+\tau_f) - s\right] + 2\left[\beta(1+\tau_f) - s\right]}\right]. \tag{32}$$

For $\tau_n > -\frac{5}{7}$ and $\tau_f \geq \tau_n$, this expression is positive. However, if $\tau_f < \tau_n$ the sign of the whole expression is not so obvious. When $\tau_f < \tau_n$, the second term of expression (32) is increasing in s. Therefore, it will approach its minimum value when s goes to zero. This implies that

$$\frac{(1+\tau_n)\left[4\beta(1+\tau_f) - s\right] + 2\left[2\beta(1+\tau_f) - s\right] - 2\beta(1+\tau_f)^2}{(1+\tau_n)\left[2\beta(1+\tau_f) - s\right] + 2\left[\beta(1+\tau_f) - s\right]} > \frac{2(1+\tau_n) + 2 - (1+\tau_f)}{(2+\tau_n)},$$

or equivalently that the term in brackets of equation (32) is larger than $\frac{(1+\tau_f)(3+\tau_n)}{(1+\tau_n)(2+\tau_n)}$ which is positive for all values of τ_f and τ_n between -1 and 1.

In summary, it has been shown that the three terms are positive for the range of values of τ_n and τ_k considered. Therefore, the derivative in (31) is positive. ■

Proof of Proposition 6. The derivative of steady state consumption per efficiency unit with respect to the research tax is given by the following expression:

$$\frac{\partial c}{\partial \tau_n} = (1-\alpha)\zeta \frac{\partial k}{\partial \tau_n}\left(\frac{1+\alpha}{\alpha} - \frac{(1+e)\Phi}{\lambda^2} + \frac{\rho + \tau_k}{(1-\alpha)\zeta}\right) +$$

$$+\left[-\frac{\partial}{\partial \tau_n}\left(\frac{\Phi(1+e)}{\lambda^2}\right)\right]\alpha(1-\alpha)L^{1-\alpha}k^{\alpha} + \rho\left[\frac{\partial}{\partial \tau_n}\left(\frac{1+e}{\lambda}\right)\right], \tag{33}$$

where the first term is positive since we have imposed $\frac{1+\alpha}{\alpha} > \frac{(1+e)\Phi}{\lambda^2}$. The second term is also positive since

$$\frac{\partial}{\partial \tau_n}\left(\frac{\Phi(1+e)}{\lambda^2}\right) = \left(\frac{\Phi}{\lambda^2}\right) \frac{s - (1+\tau_f)\left[2\beta(1+\tau_f) - s\right]}{(1+\tau_f)\left[(1+\tau_n)\left[2\beta(1+\tau_f) - s\right] + 2\left[\beta(1+\tau_f) - s\right]\right]},$$

is negative. However, the last term has an ambiguous sign. The derivative in brackets may be expressed as

$$\frac{\partial}{\partial \tau_n}\left(\frac{1+e}{\lambda}\right) = \frac{e-1}{2\lambda(1+\tau_n)}.$$

Thus, the sum of the second and third term of (33) yields

$$\frac{-\frac{\Phi}{\lambda^2}\alpha\left(1-\alpha\right)L^{1-\alpha}k^{\alpha}\left[s-\left(1+\tau_f\right)\left[2\beta\left(1+\tau_f\right)-s\right]\right]}{\left(1+\tau_f\right)\left[\left(1+\tau_n\right)\left[2\beta\left(1+\tau_f\right)-s\right]+2\left[\beta\left(1+\tau_f\right)-s\right]\right]}+\rho\frac{e-1}{2\lambda\left(1+\tau_n\right)}. \tag{34}$$

Next, use (18) in order to write expression (34) as follows:

$$\frac{\Phi n(1+\sigma)\left[\frac{2\beta\left(1+\tau_f\right)-s}{\left(1+\tau_f\right)}-\frac{s}{\left(1+\tau_f\right)^2}\right]}{2\beta^2\left(1+\tau_f\right)\left(1+\tau_n\right)}+\frac{\rho\frac{\Phi}{\lambda}\left[\frac{\left(1+\tau_n\right)\left[2\beta\left(1+\tau_f\right)-s\right]}{\beta\left(1+\tau_f\right)-s}-2\right]\left[\frac{2\left[\beta\left(1+\tau_f\right)-s\right]}{\left(1+\tau_n\right)}+\frac{s}{\left(1+\tau_f\right)}\right]}{8\beta^2\left(1+\tau_f\right)\left(1+\tau_n\right)}$$

The first term is positive while the sign of the second term is determined by

$$\frac{\left(1+\tau_n\right)\left[2\beta\left(1+\tau_f\right)-s\right]}{\beta\left(1+\tau_f\right)-s}-2,$$

expression that happens to be positive for $\tau_n > -\frac{s}{2\beta\left(1+\tau_f\right)-s}$. ■