Optimal monitoring to implement clean technologies when pollution is random*

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Abstract

We analyze a model where firms chose a production technology which, together with some random event, determines the final emission level. We consider the coexistence of two alternative technologies: a "clean" technology, and a "dirty" technology. The environmental regulation is based on taxes over reported emissions, and on penalties over unreported emissions. We show that the optimal inspection policy is a cut-off strategy, for several scenarios concerning the observability of the adoption of the clean technology and the cost of adopting it. We also show that the optimal inspection policy induces the firm to adopt the clean technology if the adoption cost is not too high, but the cost levels for which the firm adopts it depend on the scenario.

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1 Introduction

Environmental Agencies (EAs) face the important challenge of encouraging and compelling compliance with environmental laws and regulations. For this aim, they often design a deterrence policy based on inspections. This paper contributes to the literature that analyzes the optimal inspection policy taking into account firms' strategic behavior.[1] We build and analyze a model where firms choose a production technology which, together with some random event, determines the final emission level. That is, we explicitly take into account the random nature of pollution and its effects on the optimal inspection policy.

Although firms can limit emissions of pollutants by deciding the production technology, by adjusting the mix of outputs and inputs, and through the use of abating technologies, this control is often not precise. Many factors such as weather, equipment failures, and human error may cause realized emissions to differ from intended emissions.

We consider the coexistence of two alternative technologies: a "clean" technology, in the sense that its expected level of emissions is low, and a "dirty" technology, whose expected level of emissions is high. For both technologies, the realized emission level is random and it is privately observed by the firm. The environmental regulation is based on taxes over reported emissions, monitoring, and penalties over unreported emissions. The firm reports an emission level and pays the taxes associated to it. The true emission level can only be observed (and made verifiable) by the EA after an inspection.

The EA is interested in the expected emission level. Hence its two concerns are whether the firm adopts the clean technology or not, and to achieve its goal at the lowest cost. We analyze the optimal monitoring of one firm when the EA takes into account the random nature of pollution: bad luck may cause a high level of emissions even when the firm adopts the clean technology while good luck may diminish emissions level of a firm that uses the dirty technology. We distinguish three cases.

First, we assume that the EA knows the firm's cost of adopting the technologies but the technology chosen is not verifiable. We show that the inspection policy on the emission level that induces the firm to adopt the clean technology at the lowest cost is a cut-off

[1] Cohen (1999) and Sandmo (2000) provide two recent and extensive reviews of the literature.

strategy where all the reports under the cut-off are inspected with the same probability and reports over this cut-off are not audited. Second, we analyze situations in which both the technology adopted by the firm and its cost are non-verifiable. In this case, the EA is forced to use the same monitoring policy for all types of firm. We show that firms with low adoption costs will be induced to switch to the clean technology while high-cost firms will keep the dirty one. The optimal policy is then also a cut-off policy consisting on the one that would be designed for the "marginal" firm as if its emissions distribution was an average between the clean and the dirty technology. Third, we consider the case where the technology adopted by the firm is observable, but the cost encountered by the firm is not. In this situation, the firm will be inspected (through a cut-off rule) only if it is producing with the dirty technology.

In the three cases, the optimal inspection policy induces the firm to adopt the clean technology if the adoption cost is not too high. We compare the conditions under which the firm adopts the clean technology with the benchmark case where the EA has all the information about the firm (first-best). When the technology adopted is private information for the firm, the optimal monitoring policy induces the firm to choose the clean technology for a smaller set of parameters than the first best. In contrast, when the cost is private information for the firm while the technology adopted is verifiable, the firm may produce with the clean technology for a larger set of parameters than in the first best.

Several papers have considered that pollution emissions frequently produce stochastic environmental damages.[2] But they have studied different aspects from our paper. Some authors have analyzed the advantages and disadvantages of introducing self-reporting (whereas in our paper is assumed to be in place) on the emission level in situations where emissions are random. In particular, Innes (1999) analyzes a model where there are expost benefits of cleaning-up if an environmental accident (high level of pollution) occurs. In his model, firms choose the level of care (that can be interpreted as the choice

[2] For example, the damage from a given amount of effluent released in a river depends on features which vary temporally, such as seasonal fluctuations in water volume, temperature and turbidity. The effect of airborne emissions on air quality depends on prevailing atmospheric conditions, such as thermal structure, circulation, pressure, and humidity.

of a technology), and this care affects the probability of an accident. Innes shows that when there is no self-reporting a firm will engage in clean-up only if audited, while the firm always cleans-up when self-reporting is in place. Malik (1993) compares the case with and without self-reporting in a situation where collecting penalties and taxes is costly and the monitoring technology is imperfect (including both types I and II of errors). In this framework, self-reporting does not necessarily reduce regulation costs because of costly sanction.[3] Hamilton and Requate (2006) analyze the choice between emission caps and environmental quality standards when emissions are random. They show that when firms invest in abatement equipment, an emission standard induces over-investment relative to the socially optimal resource allocation, while under-investment tends to occur under an ambient environmental policy.

The model analyzed in this paper also contrasts with most of the models that study the optimal inspection policy, since they assume that the firm decides directly its (non-random) emission level (see, for example, Harford, 1978 and 1987, Sandmo, 2000, and Macho-Stadler and Pérez-Castrillo, 2006). In Macho-Stadler and Pérez-Castrillo (2006), we show that the EA optimal strategy induces a corner solution, in the sense that there are always firms that do not comply with the environmental objective and others that do comply but all of them evade the environmental taxes. Concerning the optimality of the use of environmental taxes, Macho-Stadler (2006) shows that it is less costly to achieve any level of compliance through taxes than using standards or tradable permits.

Finally, some previous papers have analyzed how the regulatory regime via emissions taxes or standards may affect firms' adoption of emissions abatement technology (see, for example, Downing and White, 1986, Milliman, 1989, and Tarui and Polasky, 2005). Our paper is complementary to these contributions as we show how to design a monitoring policy, in environments where emissions cannot be identified without inspection, to maximize firms' adoption at the lowest cost.

The paper is organized as follows. In Section 2, we introduce the model and analyze a firm's report given its technology and the inspection policy. Section 3 deals with the optimal policy that induces a single firm to switch to the clean technology, under three different scenarios concerning the observability of the change in technology and the cost

[3] See also Kaplow and Shavell (1994) and Livernois and McKenna (1999).

of switching technologies. In Section 4, we conclude and discuss the optimal inspection policy when the EA faces a family of firms. All proofs are in the Appendix.

2 The Firm's report under emission taxes

We model situations where firm's emissions are random, but they are influenced by the firms' choice of technology. A firm's level of emissions (or damages) e is distributed in the interval $[\underline{e}, \overline{e}]$ according to the distribution function $F(e; E)$, where E denotes the production technology chosen by the firm. We assume that $F(\cdot; E)$ is continuously differentiable and that $f(e; E) = \partial F(e; E)/\partial e > 0$ on $[\underline{e}, \overline{e}]$. The cost of the technology E is sunk.

We assume that emissions are taxed according to a linear schedule, with marginal tax rate t. Therefore, the emissions costs of a firm that produces a level of emissions e, and pays the taxes corresponding to e (i.e., there is perfect monitoring of emissions) are te. Therefore, the ex-ante expected costs of the firm given the technology E are:

$$C(E) = t \int_{\underline{e}}^{\overline{e}} e dF(e; E).$$

We will consider situations where both technology and emissions levels are firm's private information. However, emissions can be assessed if the firm is monitored by the EA. The firm is asked to send a report $z \in [\underline{e}, \overline{e}]$ on its emissions level, once the emissions are realized. The firm may choose a report z that does not coincide with the true emissions level e.

The EA has two instruments to dissuade the firm from cheating about their emissions: monitoring and penalties. We denote by $\alpha(z)$ the probability that the EA will audit the emissions of the firm when it reports a level of emissions z. The strategy $\alpha(\cdot)$ followed by the EA is decided previous to the choice of the technology E, that is, we assume that the EA is able to commit to its monitoring strategy. If the firm is monitored and its level of emissions is found to be higher than its report, then a penalty is imposed to the firm. For simplicity, we assume that the penalty is linear in the underreported emissions. We also assume that the marginal penalty rate, denoted θ, is exogenous. Parameter θ includes the taxes due to the EA, hence $\theta > t$.

The firm's expected costs when the emissions are e, the report is z and the monitoring strategy is $\alpha(\cdot)$ are:

$$c(e, z; \alpha(\cdot)) = tz + \alpha(z)\theta[e - z].$$

The timing of the decisions is as follows. First, the EA decides on the monitoring strategy $\alpha(\cdot)$. Second, the firm chooses the technology E at a certain cost. Emissions are realized according to the density function $f(e; E)$. Third, after having observed the realized emissions e, the firm decides on the report z and pays the taxes tz. The firm is monitored with probability $\alpha(z)$. If it is audited and it has underreported, then the firm pays the penalty $\theta[e - z]$.

The firm chooses z to minimize its costs $c(e, z; \alpha(\cdot))$, as a function of the realized emissions e. That is, at the last stage, the firm chooses $z(e)$. We denote $c(e; \alpha(\cdot)) = c(e, z(e); \alpha(\cdot))$ firm's expected costs when its emissions level is e and it makes the report that minimizes its costs.

We start with two results that provide useful information concerning firm's behavior with respect to the report.

Lemma 1 *(i) A Firm never reports more than their emissions.*

(ii) If $\alpha(z) > t/\theta$, then a firm never reports z when $e > z$.

(iii) When its emissions level is e, then a firm reports honestly only if $\alpha(z) \geq t/\theta$ for all $z \in [\underline{e}, e)$.

The intuition behind Lemma 1 is the following. Given the tax rate t and the penalty rate θ, a monitoring probability of t/θ is enough to spur honest behavior. Therefore, a firm never submits a report z lower than its real emission e if reporting z leads to inspection with a probability higher than t/θ. On the other hand, the firm will not report honestly if it can submit a report $z < e$ that is monitored with a probability lower than t/θ.

According to Lemma 1, the EA will not have incentives to inspect any report with a probability higher than t/θ, since monitoring is costly. Therefore, t/θ is an upper bound for the optimal monitoring probability.

Proposition 1 *Given the monitoring policy $\alpha(\cdot)$, if the report $z(e)$ minimizes firm's costs when the emissions level is e, then:*

$$\alpha(z(e)) \text{ is nonincreasing in } e, \text{ and} \tag{1}$$

$$c(e;\alpha(\cdot)) = c(\underline{e};\alpha(\cdot)) + \theta \int_{\underline{e}}^{e} \alpha(z(x))dx. \tag{2}$$

Moreover, if (1) and (2) hold, then $z(e)$ *minimizes firm's expected costs over the set of all possible equilibrium reports, i.e.,* $\{z|z = z(e^o)$ *for some* $e^o \in [\underline{e}, \overline{e}]\}$ *when the emissions level is* e*.*

For any given report, the penalty that the firm pays if it is caught underreporting increases with its realized pollution level. Therefore, the higher the emission level, the more incentives the firm has to chose reports with low monitoring probability. This explains that $\alpha(z(e))$ is nonincreasing in e. As to the expected costs, equation (2) states that the cost borne by the firm when its emissions are e is the integral of the monitoring probability of every level below e. This equation is also explained by the firm's possibility of underreporting. By inspecting with probability $\alpha(z(x))$, the EA makes the firm pay an expected penalty of $\theta\alpha(z(x))$ when its emission level is x. But this similarly affects the firm's expected costs when it underreports for any emission higher than x, since $z(x)$ is always a possible report. Hence, equation (2) provides the expected cost borne by the firm when its emission level is e.

Note that, although the tax rate t does not explicitly appear in equation (2), it plays a role as it sets the upper bound for the probability $\alpha(.)$. The rate t is only important for those emission levels for which the firm reports honestly. For example, if the report $z(e)$ is such that $\alpha(z(e)) = t/\theta$ for all $e \leq \hat{e}$ and $\alpha(z(e)) < t/\theta$ otherwise, then we can write

$$c(e;\alpha(\cdot)) = c(\underline{e};\alpha(\cdot)) + t\,[\hat{e} - \underline{e}] + \theta \int_{\hat{e}}^{e} \alpha(z(x))dx.$$

We can use Proposition 1 to compute firm's expected costs of using the technology E:

Proposition 2 *Given the monitoring policy* $\alpha(\cdot)$*, if the report strategy* $z(\cdot)$ *minimizes firm's costs for all emissions levels, then:*

$$C(E;\alpha(\cdot)) = c(\underline{e};\alpha(\cdot)) + \theta \int_{\underline{e}}^{\overline{e}} \alpha(z(e))\,[1 - F(e;E)]\,de. \tag{3}$$

In this section, we have analyzed the firm's strategic behavior concerning its report, once it knows the pollution level. We have computed the firm's expected cost due to the environmental policy of taxes, inspection, and penalties. We have developed the analysis for an exogenous monitoring policy. In the next section, we characterize the optimal monitoring policy from the EA's point of view.

3 Optimal monitoring

We analyze a situation where two production technologies are possible: E^D and E^C. Technology E^C is a cleaner but also more expensive technology than E^D (subscript C stands for "clean" and D for "dirty"). We assume that the firm is initially producing according to E^D and we denote by Δ the cost of switching from the dirty technology to the clean one.[4] On the other hand, the clean technology has lower average emissions, i.e., $\int_{\underline{e}}^{\overline{e}} edF(e;E^C) < \int_{\underline{e}}^{\overline{e}} edF(e;E^D)$.[5]

In this paper, we assume that the environmental policy is based on taxes over reported emissions, monitoring, and penalties. For example, we do not consider the possibility that the Government or the EA might give the firm a subsidy if it switches to a clean technology, or that it imposes a fixed penalty to firms keeping the dirty technology. When the technology adopted by the firm is not verifiable (i.e., only the firm knows the expected level of pollution of the technologies), the previous policies based on fixed subsidies or penalties cannot be implemented, as they require the EA to be able to check whether a change to a clean technology has taken place. Similarly, these policies are not possible in those environments where "clean" or "dirty" refer to the care that firms take with respect to the maintenance of the existing technology or to avoiding mistakes. In this sense, we interpret that a firm uses a clean technology when it devotes (monetary and human) resources to the good functioning of its equipment, while a firm produces according to a dirty technology when it does not care much about the correct running of the equipment, thus leading to higher expected level of emissions. On the other hand, when the EA can easily check whether a firm has adopted a more environmentally friendly technology (or whether it is using the technology trying to minimize pollution), a fixed reward or penalty

[4] We can also consider situations where the firm is not using any of the two technologies and it has to chose one of them. In this case, Δ is interpreted as the difference in costs of the technologies, i.e., the cost to adopt the former instead of the later.

[5] In our framework, the emissions from both technologies are equally difficult to inspect. Some authors have analyzed technologies that can affect the observability of firms' emissions. Heyes (1993) considers a model where firms may invest in decreasing "inspectability". Millock *et al.* (2002) studies a choice of technology that affects the verifiability of emission: adopting the technology allows nonpoint sources to become point sources.

can be optimal. Therefore, our analysis applies to those situations where, due to political, technical, or moral hazard constraints, a policy based on fixed subsidies or penalties is not possible.

Given the policy announced by the Government and the EA involving taxes over reported emissions, monitoring, and penalties over unreported emissions, the firm will choose the clean technology if and only if its total expected costs are lower than using the dirty technology, that is, if $C(E^C;\alpha(\cdot)) + \Delta \leq C(E^D;\alpha(\cdot))$. This inequality can be written as the following incentive constraint:

$$\Delta \leq \theta \int_{\underline{e}}^{\overline{e}} \alpha(z(e)) \left[F(e;E^C) - F(e;E^D)\right] de. \tag{4}$$

The monitoring policy decided by the EA strongly influences the choice between E^C and E^D. We normalize the cost of an inspection to 1, and we look for the optimal monitoring policy, that is, the policy that minimizes EA's monitoring costs.

It might be the case that the firm chooses technology E^D for any possible monitoring strategy. Indeed, if the difference in cost Δ is very large, the firm may prefer paying all the expected taxes corresponding to the emissions induced by E^D rather than adopting the clean technology. In what follows, we will assume that the set of functions $\alpha(\cdot)$ that lead the firm to choose E^C is not empty, which is equivalent to state that the toughest policy ($\alpha(z) = t/\theta$ for all z) leads the firm to use the clean technology.

Assumption 1: $\Delta < t \int_{\underline{e}}^{\overline{e}} \left[F(e;E^C) - F(e;E^D)\right] de.$

Although part of the analysis of the optimal policy is developed without assumptions concerning the distribution functions $F(e;E^C)$ and $F(e;E^D)$, the complete characterization of the policies will require further assumptions. In particular, we will assume that the density functions $f(e;E^C)$ and $f(e;E^D)$ are linear. Also, to help notation, we will normalize $[\underline{e},\overline{e}] = [0,1]$.

Assumption 2: *$f(e;E^C) = a + 2\left[1-a\right]e$, $f(e;E^D) = b + 2\left[1-b\right]e$, for all $e \in [0,1]$, where $a, b \in (0,2)$, and $a > b$.*

Note that the property $F(1;E^C) = F(1;E^D) = 1$ characterizes the slope of the linear functions $f(1;E^C)$ and $f(1;E^D)$, once we choose the independent terms a and b. Moreover, the idea that E^C is a cleaner technology than E^D is reflected in the inequality $a > b$. Also note that although Assumption 2 is restrictive, it allows the flexibility of dealing with distribution functions $F(e; E^C)$ and $F(e; E^D)$ that may be linear ($a = 1$ or $b = 1$) concave ($a > 1$ or $b > 1$), or convex ($a < 1$ or $b < 1$). On the other hand, it is a strong assumption that is helpful to identify a simple monitoring policy. We will comment later on the properties of the optimal monitoring policy in more general setups.

In the next subsection, we assume that both the firm and the EA know the cost Δ and we will characterize the policy that the EA puts in place if it wants to induce the firm to adopt technology E^C. That is, we look for the cheapest policy, in terms of monitoring costs, to achieve E^C for a given Δ. In subsection 3.2, we relax the assumption that the EA knows Δ and look for the optimal monitoring policy when Δ is firm's private information. Finally, in subsection 3.3 we will analyze the scenario where the firm has private information concerning Δ but the EA can check whether the firm has adopted the clean technology.

3.1 Optimal monitoring to achieve a clean technology when the cost Δ is public information

We assume that the EA is concerned about inducing the firm to adopt the clean technology. In this section, we consider a situation where the EA observes the cost Δ, but is uninformed about the technology that the firm adopts and about the realized emission level. The EA receives the report z form the firm. The optimization problem of the EA, that minimizes monitoring costs, when it wants the firm to adopt technology E^C is program $[P]$ below:

$$\underset{(\alpha(z))_{z\in[\underline{e},\overline{e}]}}{Min}\int_{\underline{e}}^{\overline{e}}\alpha\left(z(e)\right)dF(e;E^{C})$$

$$\text{s.t.: } \alpha(z(e)) \text{ is nonincreasing in } e$$

$$\alpha(z(e))\in[0,t/\theta] \text{ for all } e\in[\underline{e},\overline{e}]$$

$$\int_{\underline{e}}^{\overline{e}}\alpha(z(e))dF(e;E^{C})\leq B$$

$$z(e) \text{ minimizes } c(e,z;E^{C};\alpha(\cdot)) \text{ for all } e\in[\underline{e},\overline{e}]$$

$$\Delta\leq\theta\int_{\underline{e}}^{\overline{e}}\alpha(z(e))\left[F(e;E^{C})-F(e;E^{D})\right]de.$$

We can simplify program $[P]$ as follows. We do not take into account the constraint that $z(e)$ minimizes $c(e,z;E^{C};\alpha(\cdot))$, and we denote the function $\alpha(z(e))$ as $\beta(e)$. Once we identify $\beta(e)$, we will use Proposition 1 to decompose the function $\beta(e)$ into the optimal monitoring function $\alpha(z)$ and the report function $z(e)$. The optimal $\beta(\cdot)$ solves the following program, that we will denote $[P']$:

$$\underset{(\beta(e))_{e\in[\underline{e},\overline{e}]}}{Min}\int_{\underline{e}}^{\overline{e}}\beta(e)dF(e;E^{C})$$

$$\text{s.t.: } \beta(e) \text{ is nonincreasing in } e$$

$$\beta(e)\in[0,t/\theta] \text{ for all } e\in[\underline{e},\overline{e}]$$

$$\int_{\underline{e}}^{\overline{e}}\beta(e)dF(e;E^{C})\leq B$$

$$\Delta\leq\theta\int_{\underline{e}}^{\overline{e}}\beta(e)\left[F(e;E^{C})-F(e;E^{D})\right]de.$$

Next Proposition states an important general property of the solution to program $[P']$:

Proposition 3 *Under Assumption* 1 *and for any distribution function* $F(.)$*, there exists a solution* $\beta(\cdot)$ *to* $[P']$ *that takes on at most one value different from* 0 *and* t/θ*.*

Given Proposition 3 and $\beta(e)$ nonincreasing in e, there exist $d\in(0,t/\theta)$, e_1 and e_2, with $\underline{e} \leq e_1 \leq e_2 \leq \overline{e}$, such that the optimal function $\beta(e)$ has the following shape:

$$\begin{aligned}\beta(e) &= t/\theta \text{ for all } e \in [\underline{e}, e_1], \\ \beta(e) &= \gamma \text{ for all } e \in (e_1, e_2), \\ \beta(e) &= 0 \text{ for all } e \in [e_2, \overline{e}].\end{aligned}$$

Proposition 3 shows that the optimal monitoring policy is very simple independently on the shape of the distribution functions. Proposition 4 goes a step forward and shows that, under Assumptions 1 and 2, the optimal policy is quite simple. To state this Proposition, let us define the function $h(e)$ as follows:

$$h(e) \equiv f(e; E^C) - \frac{F(e; E^C) - F(e; E^D)}{\int_{\underline{e}}^{e} [F(x; E^C) - F(x; E^D)]\, dx} F(e; E^C).$$

The function $h(e)$ plays an important role in the proof of Proposition 4, and allow us to define a cut-off level. It is easy to check that, under Assumption 2, $h(e)$ is first negative and then positive. We denote by e^* the cut-off level such that $h(e) < 0$ if $e < e^*$ and $h(e) > 0$ if $e > e^*$, that is, e^* is defined by $h(e^*) = 0$.[6] Easy computations show that e^* is an increasing function of a.

Proposition 4 *Suppose Assumptions* 1 *and* 2 *hold.*

(a) If $\Delta < t \int_{\underline{e}}^{e^*} \left[F(e; E^C) - F(e; E^D)\right] de$, *then a solution* $\beta(e)$ *to* $[P']$ *is*

$$\begin{aligned}\beta(e) &= \widehat{\gamma} \text{ for all } e \in [\underline{e}, e^*), \\ \beta(e) &= 0 \text{ for all } e \in [e^*, \overline{e}],\end{aligned}$$

where $\widehat{\gamma} < t/\theta$ *is defined by:*

$$\widehat{\gamma}\theta \int_{\underline{e}}^{e^*} \left[F(e; E^C) - F(e; E^D)\right] de = \Delta. \tag{5}$$

(b) If $\Delta \geq t \int_{\underline{e}}^{e^*} \left[F(e; E^C) - F(e; E^D)\right] de$, *then a solution* $\beta(e)$ *to* $[P']$ *is*

$$\begin{aligned}\beta(e) &= t/\theta \text{ for all } e \in [\underline{e}, \widehat{e}), \\ \beta(e) &= 0 \text{ for all } e \in [\widehat{e}, \overline{e}],\end{aligned}$$

[6] Under Assumption 2, $e^* = 3/4$ when $a = 1$, $e^* = \frac{-2a+\sqrt{4a^2+6a(1-a)}}{2(1-a)} \in (0,1)$ when $a \neq 1$.

where $\widehat{e} \geq e^*$ *is defined by:*

$$t \int_{\underline{e}}^{\widehat{e}} \left[F(e; E^C) - F(e; E^D)\right] de = \Delta.$$

The optimal monitoring policy is very simple. We here highlight its main characteristics. First, the EA will always monitor, at least, the reports corresponding to all the emission levels lower than the cut-off value e^*. Note that the cut-off e^* is usually high; for the intermediate case $a = 1$, $e^* = 3/4$. Second, the probability of monitoring is the same for all the reports subject to audit. Third, as long as the incentive problem is not very acute, in the sense that adopting the clean technology is not very costly, the EA will only monitor when the realized emission level is lower than e^*. Finally, when the incentive problem is very severe, the monitoring probability is the highest possible, among the sensible ones, (i.e., $\beta = t/\theta$) for all the reports subject to audit.

Once we know the optimal function $\beta(e)$, we can use Proposition 1 to state the optimal monitoring policy as a function of the report, $\alpha(z)$, as well as firms' reporting behavior given the optimal monitoring policy, $z(e)$. Proposition 5 characterizes these functions.

Proposition 5 *Suppose Assumptions* 1 *and* 2 *hold.*

(a) If $\Delta < t \int_{\underline{e}}^{e^*} \left[F(e; E^C) - F(e; E^D)\right] de$*, then the following policy* $\alpha^*(z)$ *is optimal:*

$$\alpha^*(z) = \widehat{\gamma} \text{ for all } z \in [\underline{e}, z^*),$$
$$\alpha^*(z) = 0 \text{ for all } z \in [z^*, \overline{e}], \text{ where}$$

$$z^* = \underline{e} + \frac{\Delta}{t} \frac{(e^* - \underline{e})}{\int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)]\, de}.$$

Facing the monitoring policy $\alpha^*(z)$*, the firm's reporting strategy is the following:*

$$z(e) = \underline{e} \text{ for all } e \in [\underline{e}, e^*),$$
$$z(e) = z^* \text{ for all } e \in [e^*, \overline{e}].$$

(b) If $\Delta \geq t \int_{\underline{e}}^{e^*} \left[F(e; E^C) - F(e; E^D)\right] de$*, then the following policy* $\alpha^*(z)$ *is optimal:*

$$\alpha^*(z) = t/\theta \text{ for all } z \in [\underline{e}, \widehat{e}),$$
$$\alpha^*(z) = 0 \text{ for all } z \in [\widehat{e}, \overline{e}].$$

Facing the monitoring policy $\alpha^*(z)$*, the firm's reporting strategy is the following:*

$$z(e) = e \text{ for all } e \in [\underline{e}, \widehat{e}),$$
$$z(e) = \widehat{e} \text{ for all } e \in [\widehat{e}, \overline{e}].$$

We now explain the intuitions behind Propositions 4 and 5. The EA's objective is to dissuade the firm from using the dirty technology at the least possible (monitoring) cost. To "convince" the firm, the EA must choose a monitoring strategy that makes the firm bear high expected environmental costs (also taking into account the penalties) if it uses the dirty technology, and low expected costs if it produces according to the clean one.

A dirty technology has a higher probability to produce high emission levels than a clean technology. For the case of linear density functions over the interval $[0, 1]$ (Assumption 2), the clean technology has higher density for $e \in [0, 1/2)$ and lower density for $e \in (1/2, 1]$. Therefore, in terms of dissuasion, the EA would find it beneficial to make the firm pay as much as possible (and that can be achieved by monitoring with high probability) when realized emissions are high and as little as possible when realized emissions are low. However, the EA does not observe the realized emission level, only the firm does. The EA only receives the firm's report.

When emissions are not public information, equation (2) in Proposition 1 states that the cost borne by the firm when the emission level is e^o is the integral of the monitoring probability of every level below e^o. That is, increasing the probability of monitoring the report corresponding to a level e affects in the same way the cost suffered for every emission level higher than e. Hence, monitoring the report corresponding to a high emission level, say $e' > 1/2$, has good incentive consequences concerning the decision to use a clean technology, as it affects the cost borne for every realized emission $e \geq e'$. On the other hand, monitoring the report corresponding to a low emission level, say $e'' < 1/2$, has mixed incentive consequences since it affects the cost associated to both high (every $e > 1/2$) and low (every $e \in [e', 1/2)$) emission levels.

The difficulty is that, from equation (1) in Proposition 1, the EA is constraint to use a monitoring probability nonincreasing in the emission level. That is, if the EA wants to monitor the (firm's optimal) report corresponding to a certain level of emissions e^o, then it is forced to monitor the reports corresponding to all the levels $e < e^o$ with, at least, the same frequency.

To understand how the EA solves the previous difficulty, consider also that Δ is small in such a way that inducing the firm to switch to the clean technology is easy (Region (a) in Proposition 4). Could it make sense for the EA to monitor only the reports corresponding to low emission levels? The answer is no. The EA does better monitoring reports chosen by a larger range of emission levels (including levels higher than $1/2$) with lower probability. The cost paid by the higher emission levels will be the same, while the cost borne by the lower emission levels will be lower, which gives the firm more incentives to adopt the clean technology (more precisely, it will allow the EA to save on monitoring costs). Is it optimal for the EA to set a full flat policy (i.e., $e^* = \overline{e}$)? The answer to this question is also negative because monitoring the report corresponding to emission levels very close to $\overline{e}$ only affects the payment of a very small interval of emissions.

In the case where the density function $f(e; E^C)$ is uniform, i.e., $a = 1$, the trade-off leads to a flat policy consisting in auditing the reports corresponding to every $e < 3/4 = e^*$ with the same (small) probability. When $f(e; E^C)$ is not uniform, the argument is more complex, as switching monitoring probabilities from one level to the other has consequences in terms of monitoring costs. This is why when the distribution function $f(e; E^C)$ is decreasing, it is optimal to state an even flatter technology ($e^* > 3/4$), while the opposite happens when $f(e; E^C)$ is increasing.[7]

The previous discussion also allows to comment on the generality of the results with

[7] It is worth comparing our context with situations in which the objective of the agency is to raise the largest amount of taxes, for a given technology. In such latest situations, the agency is much less interested in focusing in high-emission levels. For example, in the tax evasion literature it is assumed that the distribution of income is given and the objective of the enforcement agency is to maximize the collected revenues (taxes plus penalties). In this case, the optimal policy consists in auditing all the taxpayers reporting incomes lower than a certain cut-off income with a probability high enough so that those reports will happen to be truthful, while the taxpayers earning higher incomes will report the cut-off income and will not be subjet to audit. The main intuition for this result is the one we have provided in the main text: putting pressure over the report corresponding to an emission level increases the revenue collected from every higher level. That is, it is beneficial to concentrate the monitoring in the lowest levels of income (with the maximum probability t/θ). Some papers in the tax evasion literature are Reinganum and Wilde (1985), Scotchmer (1987), Sánchez and Sobel (1993), and Macho-Stadler and Pérez-Castrillo (1997).

respect to the shape of the distribution functions. First, according to our arguments, monitoring every single emission with some probability (i.e., $e_2 = \overline{e}$) is not optimal for general distribution functions. Second, the property that the monitoring policy is flat for quite a wide range of emissions can be stated under quite reasonable hypotheses. For example, assume that $F(e; E^C) > F(e; E^D)$ for all $e \in (\underline{e}, \overline{e})$, $F(e; E^C) - F(e; E^D)$ is first increasing and then decreasing in e, and $F(e; E^C)$ is concave in e. Under these necessary conditions, it is possible to prove that there exists a cut-off value $e^{\#}$ that lies in the region of emissions where $F(e; E^C) - F(e; E^D)$ is decreasing such that $\beta(e)$ is constant for all $e < e^{\#}$. In particular, the reports corresponding to all emission levels $e < e^{\#}$ are monitored with a low probability when the cost of adopting the clean technology is low.

On the other hand, it seems more difficult to propose general necessary conditions to establish the precise form of the optimal monitoring strategy for higher emission levels. Although we know that the highest levels are never monitored, it is difficult to prove more general results.

Next, Corollary 1 states the monitoring cost $ECost(\Delta)$ of the implementation of the clean technology as a function of the parameters of the model.

Corollary 1 *Suppose Assumptions* 1 *and* 2 *hold.*
(I) Expected monitoring costs ECost are the following:

(Ia) If $\Delta < t\int_{\underline{e}}^{e^*} \left[F(e; E^C) - F(e; E^D)\right] de$, *then:*

$$ECost(\Delta) = \frac{\Delta}{\theta} \frac{F(e^*; E^C)}{\int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)]\, de}.$$

(Ib) If $\Delta \geq t\int_{\underline{e}}^{e^*} \left[F(e; E^C) - F(e; E^D)\right] de$, *then:*

$$ECost(\Delta) = \frac{t}{\theta} F(\widehat{e}; E^C).$$

(II) Expected monitoring costs are increasing in the difference Δ *and they are decreasing with the penalty rate* θ*; they are higher the less clean is technology* E^C *and the less dirty is technology* E^D*. Finally, expected costs are increasing in the ratio* t/θ *in Region (b).*

We now explain the comparative statics in Corollary 1. First, the higher the cost Δ for the firm to switch the to clean technology, the higher the monitoring cost required to give it incentives to adopt E^C. We can easily check that:

$$\frac{\partial ECost}{\partial \Delta} = \frac{f(e_2; E^C)}{\theta \left[F(e_2; E^C) - F(e_2; E^D)\right]},$$

where $e_2 = e^*$ in Region (a) and $e_2 = \widehat{e}$ in Region (b). Second, a higher penalty rate θ makes it easier to "convince" the firm, hence it decreases the EA's cost. Third, the larger (in terms of expected pollution) the difference between the two technologies, the more the EA's monitoring can target the dirty technology, which also decreases monitoring costs. Finally, an increase in the tax rate t forces the EA to increase the monitoring probability if it wants the firm to be honest when the level of pollution is low (which is the optimal policy in Region (b)). Therefore, the monitoring costs increase with t. That is, a though policy in terms of penalty rate and (in Region (b)) a soft policy in terms of tax rate help in keeping low monitoring costs.

3.2 Optimal policy when the cost Δ is not observable by the EA

We now address the EA's optimal policy when the cost Δ of adopting the clean technology is the firm's own private information. We model this situation as follows. The firm knows Δ while the EA only has statistical information about it. The EA believes that the parameter Δ is distributed according to the density function $g(\Delta)$ over the interval $\left[0, \overline{\Delta}\right]$; we denote by $G(\Delta)$ the distribution function of Δ. The EA cares about expected pollution, hence its concern is whether the firm chooses the clean or the dirty technology. Given that the only instrument in hands of the EA is the monitoring probability, the policy is anonymous, i.e., every type of firm is subject to the same monitoring policy.[8]

Inspection of the incentive compatibility constraint (4) makes it clear that incentives to switch to the clean technology are strictly decreasing with the switching cost. That is, for a given monitoring policy, if a firm with parameter Δ adopts the clean technology, it will also adopt it if its parameter is $\Delta' < \Delta$. Therefore, any policy $\alpha(.)$ will induce the

[8] We can also see the analysis developed in this and next subsection as the study of the optimal monitoring policy when the EA faces a family of firms characterized by the cost parameter Δ. The EA has some beliefs about the distribution of the adoption cost in the family of firms, beliefs that are reflected in the function $G(\Delta)$. However, it does not know the adoption cost of any particular firm. Next propositions and corollaries have an immediate interpretation in this context.

firm to adopt E^C if its parameter lies in an interval $[0, \Delta^n]$, for some $\Delta^n \in [0, \overline{\Delta}]$.[9]

Next proposition characterizes the policy that minimizes monitoring costs when the EA wants the firm to switch to E^C if Δ lies in the interval $[0, \Delta^n]$. The policy is qualitative the same as the one stated in Proposition 5, although the cut-off levels are different. The precise value for the parameters e^n, z^n, $\widehat{e}^n$, and $\widehat{\gamma}^n$ that appear in Proposition 6 are given in the Appendix. They do not correspond to the optimal cut-off levels whenever the EA would like to give incentives to switch technology to a firm with parameter Δ^n. That is, the homogeneous monitoring policy does not coincide with the optimal policy for the "marginal firm" Δ^n. It would correspond to a firm with adoption costs of Δ^n, whose incentives are given by the difference between the distribution functions $F(e; E^C)$ and $F(e; E^D)$, but whose actual emissions are given by the (average) distribution function $G(\Delta^n)F(e; E^C) + [1 - G(\Delta^n)]\, F(e; E^D)$ instead of $F(e; E^C)$.

Proposition 6 *Suppose the firm's cost parameter* Δ *is distributed according to* $G(\Delta)$*, it is firm's private information, and assumptions* 1 *and* 2 *hold. If the EA wants the firm to adopt* E^C *if* $\Delta \in [0, \Delta^n]$ *and cannot observe the technology choice:*

(a) If $\Delta^n < t \int_{\underline{e}}^{e^n} \left[F(e; E^C) - F(e; E^D)\right] de$*, then the following policy* $\alpha^n(z)$ *is optimal:*

$$\begin{aligned} \alpha^n(z) &= \widehat{\gamma}^n \text{ for all } z \in [\underline{e}, z^n)\,, \\ \alpha^n(z) &= 0 \text{ for all } z \in [z^n, \overline{e}]\,. \end{aligned}$$

(b) If $\Delta^n \geq t \int_{\underline{e}}^{e^n} \left[F(e; E^C) - F(e; E^D)\right] de$*, then the following policy* $\alpha^n(z)$ *is optimal:*

$$\begin{aligned} \alpha^n(z) &= t/\theta \text{ for all } z \in [\underline{e}, \widehat{e}^n)\,, \\ \alpha^n(z) &= 0 \text{ for all } z \in [\widehat{e}^n, \overline{e}]\,. \end{aligned}$$

The policy $\alpha^n(z)$ stated in Proposition 6 requires monitoring all reports below a cut-off value (e^n or $\widehat{e}^n$ depending on the region) with the same probability, that is, a large range of (low) reports are monitored with a uniform probability, while high reports are never monitored. The intuitions behind the optimality of this policy are similar to the one discussed after Propositions 4 and 5.

[9]The letter n in Δ^n stands for (technology adoption) *non* verificable. In next subsection, the adoption is supposed verifiable and we will use Δ^v.

The expected monitoring cost of the policy $\alpha^n(z)$ depends on the interval $[0, \Delta^n]$ of types of firms that the EA wants to adopt E^C. The larger the interval, i.e., the higher Δ^n, the higher the expected cost $ECost^n([0, \Delta^n])$ when the adoption of the technology is not observable. Using the envelop theorem in program $[P^M]$ in the proof of Proposition 6, we can deduce that:[10]

$$\begin{aligned}\frac{\partial ECost^n([0, \Delta^n])}{\partial \Delta^n} &= \gamma g(\Delta^n) \left[F(e_2; E^C) - F(e_2; E^D)\right] \\ &\quad + \frac{\left[G(\Delta^n) f(e_2; E^C) + [1 - G(\Delta^n)] f(e_2; E^D)\right]}{[F(e_2; E^C) - F(e_2; E^D)]\, \theta}, \end{aligned} \tag{6}$$

where $e_2 = e^n$ and $\gamma = \widehat{\gamma}^n$ in Region (a) and $e_2 = \widehat{e}^n$ and $\gamma = t/\theta$ in Region (b). To explain expression (6), note that an increase in the cut-off level Δ^n has two effects on the monitoring costs. First, for a firm with a higher switching cost to adopt E^C, the monitoring probability must increase. This affects the firm independently on its type and is reflected in the second term in the right-hand side of (6). Second, there are types of firms that were keeping E^D before the increase in the cut-off and are adopting E^C after the change. A firm using E^C is monitored more often (although its expected payment is lower) than if it keeps E^D (this is due to the property that the monitoring probability should be non-decreasing in realized emission, see Proposition 1). Both effects go in the same direction: inducing more proportion of firms to adopt E^C increases the monitoring costs.

How is the optimal Δ^{n*} decided? If the firm's cost Δ was public information (and the firm's technology verifiable), the Government (or the EA) would weight benefits of adopting technology E^C due to the reduction in pollution against costs of adoption, Δ. This balance would determine the optimal Δ^* below which the firm should (from a social point of view) adopt E^C. When the firm has private information about Δ, then the Government also takes into account the monitoring cost. One natural form for the Government's welfare function is:

$$B(G(\Delta^n)) - ECost^n([0, \Delta^n]) - \kappa \int_0^{\Delta^n} \Delta g(\Delta) d\Delta,$$

where $B(G(\Delta^n)$ is an increasing and concave function measuring the benefits due to the

[10] The optimal solution of program $[P^M]$ always involves $e_1 = \underline{e}$.

firms' adoption of E^C when the switching cost is lower than Δ^n and κ is the weight the Government gives to firms' profits.

Given that $ECost^n([0, \Delta^n])$ is increasing in Δ^n, it is immediate that the optimal decision in this case will involve a level $\Delta^n < \Delta^*$, that is, the expected level of pollution will be higher than the first-best level of pollution:

Corollary 2 *Suppose the cost parameter Δ and the technology adopted are the firm's own private information. Then, the optimal monitoring policy induces the firm to adopt technology E^C for an interval of parameters $[0, \Delta^{n*}]$ that is smaller than the first-best interval $[0, \Delta^*]$.*

3.3 Optimal monitoring when the technology adopted by the firm is verifiable but Δ is not

In this subsection, we study the environments where the EA can easily verify the technology adopted by the firm. However, it does not know the adoption costs Δ.

Given that the EA is not concerned about the environmental taxes raised, the optimal policy in this case involves not monitoring at all the firm if it decides to switch to E^C. Therefore, the firm can "buy" immunity from environmental taxes by adopting the clean technology. For similar reasons as in the previous subsection, for any given monitoring policy (that will only be applied to the firm if it keeps E^D) the firm adopts E^C if its parameter Δ lies in an interval $[0, \Delta^v]$. What is the optimal monitoring policy for the firm when it adopts E^D? It needs to give incentives for the firm to switch to E^C even when its costs are Δ^v and the distribution of emissions of those firms that are monitored is $F(e; E^D)$. Therefore:

Proposition 7 *Suppose the firm's cost parameter Δ is distributed according to $G(\Delta)$, it is firm's private information, and assumptions 1 and 2 hold. If the EA wants the firm to adopt E^C if $\Delta \in [0, \Delta^v]$ and can observe the technology choice:*

(i) If the firm adopts E^C, it is not monitored.

(ii) If the firm adopts E^D, is audited according to the policy found in Proposition 5 for a firm with adoption costs of Δ^v.

The monitoring policy will only be applied to the firm if it uses E^D, which happens when its parameter lies in the interval $\left(\Delta^v, \overline{\Delta}\right]$. Moreover, the policy applied is the one that would be optimal if the EA would face a firm with "known" adoption cost of Δ^v. Therefore, the expected monitoring costs $ECost^v([0, \Delta^v])$ to achieve firm's adoption of E^C for switching costs in $[0, \Delta^v]$ when the technology used by the firm is verifiable, are:

$$ECost^v([0, \Delta^v]) = [1 - G(\Delta^v)] \int_{\underline{e}}^{e^*} \widehat{\gamma} f(e; E^D) = [1 - G(\Delta^v)] \frac{\Delta^v F(e^*; E^D)}{\theta \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)]\, de},$$

when the parameters lie in Region (a) of Proposition 5, i.e., $\Delta < t \int_{\underline{e}}^{e^*} \left[F(e; E^C) - F(e; E^D)\right] de$. In Region (b):

$$ECost^v([0, \Delta^v]) = [1 - G(\Delta^v)] \frac{tF(\widehat{e}; E^D)}{\theta}.$$

Consider Region (a) (the qualitative properties in Region (b) are similar). It is immediate that:

$$\frac{\partial ECost^v([0, \Delta^v])}{\partial \Delta^v} = [[1 - G(\Delta^v)] - g(\Delta^v)\Delta^v] \frac{F(e^*; E^D)}{\theta \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)]\, de}.$$

As it was the case in the previous section, an increase in Δ^v has two effects on the monitoring costs. The monitoring probability must increase to "convince" the firm more often to adopt E^C. But, the probability that the firm is monitored is lower, as it switches to E^C more often. That is, there is an effect (the positive term in the previous equation) that makes the monitoring cost increase, while another effect (the negative term) goes in the sense of decreasing monitoring costs. In fact, there are environments where there is too much adoption of clean technology compared with the first-best situation.[11]

Corollary 3 *Suppose the cost parameter Δ is the firm's own private information and that the adoption of the technology is verifiable. Then, the optimal monitoring policy induces the firm to adopt technology E^C for an interval of parameters $[0, \Delta^{v*}]$ that may be larger or shorter than the first-best interval $[0, \Delta^*]$.*

[11] For example, there is too much adoption if $g(\Delta)$ is uniform and $\Delta^* > \overline{\Delta}/2$.

4 Conclusion

We have considered a situation where the environmental policy is based on taxes over reported emissions, monitoring, and penalties. We have assumed that emissions are firm's private information and that they depend on a firm's decision (adopting the clean or the dirty technology) and some random elements. The added value of our paper lies in the analysis of the optimal monitoring policy when this random characteristic is present. We have developed the analysis in different scenarios depending on whether the technology adopted by the firm is verifiable or not, and on whether the EA knows the cost of adopting the clean technology. In all the cases, the optimal policy is a cut-off policy, where all reports below the threshold are inspected with the same probability, while reports above the threshold are not monitored. We have also shown that if the adoption of the technology is firms' private information, too few firms will adopt the clean technology under the optimal monitoring policy. However, this is not necessarily true if the EA can check the technology adopted.

5 Appendix

Proof of Lemma 1. First, reporting more than the emissions is never optimal, since the expected payment is always higher. Second, if $e > z$ and $\alpha(z) > t/\theta,$ then $c(e, z; \alpha(\cdot)) = tz + \alpha(z)\theta[e - z] > tz + t[e - z]$, which is the payment the firm would make if it would report e. Therefore, reporting z is not optimal. Finally, by similar reasons, reporting e is not optimal when $\alpha(z) < t/\theta$ for some $z \in [\underline{e}, e)$. ■

Proof of Proposition 1. Consider two emissions levels e_1 and e_2 with $e_1 > e_2$ and the optimal reports corresponding to these levels, $z(e_1)$ and $z(e_2)$. Given that the firm prefers reporting $z(e_1)$ than $z(e_2)$ when the emissions level is e_1, and viceversa, we have:

$$\begin{aligned} c(e_1; \alpha(\cdot)) &= tz(e_1) + \alpha(z(e_1))\theta[e_1 - z(e_1)] \leq tz(e_2) + \alpha(z(e_2))\theta[e_1 - z(e_2)], \\ c(e_2; \alpha(\cdot)) &= tz(e_2) + \alpha(z(e_2))\theta[e_2 - z(e_2)] \leq tz(e_1) + \alpha(z(e_1))\theta[e_2 - z(e_1)]. \end{aligned}$$

These equations imply:

$$\alpha(z(e_1))\theta[e_1 - e_2] \leq c(e_1; \alpha(\cdot)) - c(e_2; \alpha(\cdot)) \leq \alpha(z(e_2))\theta[e_1 - e_2]. \tag{7}$$

First, since $e_1 - e_2 > 0$, (7) requires that $\alpha(z(e_1)) \leq \alpha(z(e_2))$, i.e., $\alpha(z(e))$ is nonincreasing in e. Second, $\alpha(z(e))$ nonincreasing and (7) imply that $c(e; \alpha(\cdot))$ is differentiable in e almost everywhere, with

$$\frac{dc(e; \alpha(\cdot))}{de} = \alpha(z(e))\theta \text{ almost everywhere.}$$

Equation (2) immediately follows.

Finally, assume (1) and (2) hold. Then, a firm with emissions level e reporting $z(e^o)$ has a expected cost of:

$$tz(e^o) + \alpha(z(e^o))\theta[e - z(e^o)] = c(e^o; \alpha(\cdot)) + \alpha(z(e^o))\theta[e - e^o] =$$

$$c(e; \alpha(\cdot)) + \theta \int_e^{e^o} \alpha(z(x))dx + \alpha(z(e^o))\theta[e - e^o] = c(e; \alpha(\cdot)) + \theta \int_e^{e^o} [\alpha(z(x)) - \alpha(z(e^o))]\, dx.$$

Given (1), $\int_e^{e^o} [\alpha(z(x)) - \alpha(z(e^o))]\, dx \geq 0$.

Therefore, $z(e)$ is optimal in $\{z | z = z(e^o) \text{ for some } e^o \in [\underline{e}, \overline{e}]\}$. ■

Proof of Proposition 2. According to equation (2):

$$C(E; \alpha(\cdot)) = \int_{\underline{e}}^{\overline{e}} c(e; \alpha(\cdot))dF(e; E) = c(\underline{e}; \alpha(\cdot)) + \int_{\underline{e}}^{\overline{e}} \left[\theta \int_{\underline{e}}^{e} \alpha(z(x))dx\right] dF(e; E).$$

Integrating by parts, we obtain:

$$\begin{aligned}
\int_{\underline{e}}^{\overline{e}} \left[\int_{\underline{e}}^{e} \alpha(z(x))dx\right] dF(e; E) &= \left[\left[\int_{\underline{e}}^{e} \alpha(z(x))dx\right] F(e; E)\right]_{e=\underline{e}}^{e=\overline{e}} - \int_{\underline{e}}^{\overline{e}} \alpha(z(e))F(e; E)de \\
&= \int_{\underline{e}}^{\overline{e}} \alpha(z(x))dx - \int_{\underline{e}}^{\overline{e}} \alpha(z(e))F(e; E)de.
\end{aligned}$$

Equation (3) immediately follows. ■

Proof of Proposition 3. Consider a solution $\beta^*(\cdot)$ to program $[P']$ and B^* the optimal budget. We claim that $\beta^*(\cdot)$ is also the solution to the program $[P'']$ below:

$$\underset{(\beta(e))_{e\in[\underline{e},\overline{e}]}}{Max} \int_{\underline{e}}^{\overline{e}} \beta(e) \left[F(e; E^C) - F(e; E^D)\right] de$$

$$\text{s.t.: } \beta(e) \text{ is nonincreasing in } e$$

$$\beta(e) \in [0, t/\theta] \text{ for all } e \in [\underline{e}, \overline{e}]$$

$$\int_{\underline{e}}^{\overline{e}} \beta(e)dF(e; E^C) \leq B^*.$$

Indeed, if a function $\beta'(\cdot)$ would exist involving a higher value for the solution, $\beta^*(\cdot)$ would not be the solution to $[P']$: the EA could use $\beta''(\cdot)$ that coincides with $\beta'(\cdot)$ until the lowest emissions level e^o that satisfies

$$\Delta = \theta \int_{\underline{e}}^{e^o} \beta'(e) \left[F(e;E^C) - F(e;E^D)\right] de$$

and $\beta''(e) = 0$ for all $e > e^o$. This policy would be cheaper than $\beta'(\cdot)$, hence it would cost less than B^*, which is not possible.

We can now use known results (see, for example, Step 4 in the proof of Proposition 1 in Sánchez and Sobel, 1991) to state that there exists a solution to $[P'']$ that takes on at most one value different from 0 and t/θ. ■

Proof of Proposition 4. According to Proposition 3, we can rewrite $[P']$ as $[P'']$:

$$\underset{(\gamma,e_1,e_2)}{Min} \left\{ \frac{t}{\theta} F\left(e_1;E^C\right) + \gamma \left[F\left(e_2;E^C\right) - F\left(e_1;E^C\right)\right] \right\}$$

$$\text{s.t.:}\ \frac{\Delta}{\theta} = \frac{t}{\theta} \int_{\underline{e}}^{e_1} \left[F(e;E^C) - F(e;E^D)\right] de + \gamma \int_{e_1}^{e_2} \left[F(e;E^C) - F(e;E^D)\right] de. \quad (8)$$

We start by proving some claims.

Claim 1 : We can restrict attention to policies where $e_2 < \overline{e}$.

To prove Claim 1, consider the set of policies characterized by (e_1, e_2, γ), with $e_1 < \overline{e}$. We do the analysis fixing the level of e_1. The parameter γ is given by (8), that is,

$$\gamma = \frac{1}{\int_{e_1}^{e_2} [F(e;E^C) - F(e;E^D)]\, de} \left[\frac{\Delta}{\theta} - \frac{t}{\theta} \int_{\underline{e}}^{e_1} \left[F(e;E^C) - F(e;E^D)\right] de \right].$$

Therefore, the cost of the policy as a function of e_2 is given by the function $m(e_2)$:

$$m(e_2) \equiv \frac{t}{\theta} F(e_1;E^C) + A \frac{F(e_2;E^C) - F(e_1;E^C)}{\int_{e_1}^{e_2} [F(e;E^C) - F(e;E^D)]\, de},$$

where A is a positive constant that does not depend on e_2 (it is the second factor in the expression for γ). $m'(e_2 = \overline{e})$ is proportional to $f(e_2;E^C) \int_{e_1}^{e_2} \left[F(e;E^C) - F(e;E^D)\right] de$. Hence, $m'(e_2 = \overline{e}) > 0$ given Assumption 2. This implies that, at the optimum, it is always the case that the cost is minimized for a value of e_2 lower than $\overline{e}$.

Claim 2 : *A policy such that* $e_1 = e_2 < e^*$ *is not optimal.*

We consider the policies of the form $\beta(e) = \gamma$ for all $e \in [\underline{e}, \widetilde{e})$ and $\beta(e) = 0$ for all $e \in [\widetilde{e}, \overline{e}]$, for which (8) holds. In this class of policies, we consider a marginal change in $\widetilde{e}$, accompanied by the corresponding change in γ so that (8) still holds, i.e.,

$$\frac{\partial \gamma}{\partial \widetilde{e}} = -\frac{\gamma \left[F(\widetilde{e}; E^C) - F(\widetilde{e}; E^D)\right]}{\int_{\underline{e}}^{\widetilde{e}} \left[F(e; E^C) - F(e; E^D)\right] de}.$$

The cost of any policy in this class is $\gamma F(\widetilde{e}; E^C)$. Hence, the change in cost due to the proposed marginal change is $F(\widetilde{e}; E^C)\partial\gamma + \gamma f(\widetilde{e}; E^C)\partial\widetilde{e} = h(\widetilde{e})\gamma\partial\widetilde{e}$. By Assumption 2, $h(\widetilde{e}) < 0$ given that $\widetilde{e} < e^*$. Therefore, a marginal increase in $\widetilde{e}$ would reduce the cost. Therefore, a policy with $\gamma = t/\theta$ (i.e., $e_1 = e_2$) cannot be optimal since there is room to increase $\widetilde{e}$ and decrease γ in a profitable way, which proves Claim 2.

Claim 3 : *A policy such that* $e_1 < e_2$ *is not optimal when* $e_1 < e^*$.

We follow a similar path as in Claim 2. Consider the class of policies of the form $\beta(e) = \gamma'$ for all $e \in [\underline{e}, e_1)$, $\beta(e) = \gamma$ for all $e \in [e_1, e_2)$, and $\beta(e) = 0$ for all $e \in [e_2, \overline{e}]$, with $\gamma' > \gamma$, for which equation (8) holds (where we substitute t/θ by γ'). We want to show that $\gamma' = t/\theta$ cannot be optimal within this class of policies (hence, it cannot be optimal in general). A marginal change in e_1 accompanied by the corresponding change in γ' so that equation (8) holds, must satisfy:

$$\frac{\partial \gamma'}{\partial e_1} = -\frac{(\gamma' - \gamma)\left[F(e_1; E^C) - F(e_1; E^D)\right]}{\int_{\underline{e}}^{e_1} \left[F(e; E^C) - F(e; E^D)\right] de}.$$

Given that the cost of the policy is $\gamma' F\left(e_1; E^C\right) + \gamma\left[F\left(e_2; E^C\right) - F\left(e_1; E^C\right)\right]$,the proposed marginal change in e_1 will result in a change in costs of $h(e_1)\left(\gamma' - \gamma\right)\partial e_1$.

By the same reasons as in Claim 2, a marginal increase in e_1 would decrease the costs whenever $e_1 < e^*$ and $\gamma' > \gamma$. In particular, the policy where $\gamma' = t/\theta$ cannot be optimal, since there is room to decrease γ' and increase e_1, which lowers the cost of the monitoring.

Claim 4 : *A policy such that* $e_1 = e_2 > e^*$ *and* $\gamma < t/\theta$ *is not optimal.*

The proof is the same as the proof of Claim 2. The difference is that now $h(\widetilde{e})$ is positive since $\widetilde{e} > e^*$ Therefore, decreasing $\widetilde{e}$ and increasing γ (when this change is possible, i.e., when $\gamma < t/\theta$) decreases the costs of the policy.

Claim 5 : *A policy such that* $e_1 < e_2$ *is not optimal when* $e_1 \geq e^*$.

To prove this Claim, we consider Program $[P'']$ stated at the beginning of the proof of Proposition 3. By contradiction, suppose that the optimal e_1 is an interior solution (we already now that $e_2 < \overline{e}$). Denoting $\lambda \geq 0$ the Lagrange multiplier of (8) in $[P'']$, the first order conditions of the Lagrange function with respect to e_1 and e_2 must hold:

$$\frac{\partial \mathcal{L}}{\partial e_1} = \left[\frac{t}{\theta} - \gamma\right]\left[f\left(e_1; E^C\right) - \lambda\left[F(e_1; E^C) - F(e_1; E^D)\right]\right] = 0, \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial e_2} = \gamma\left[f\left(e_2; E^C\right) - \lambda\left[F(e_2; E^C) - F(e_2; E^D)\right]\right] = 0. \tag{10}$$

Given $\gamma > 0$ and $\gamma < t/\theta$, from (9) and (10), it follows that:

$$\frac{f\left(e_1; E^C\right)}{F(e_1; E^C) - F(e_1; E^D)} = \frac{f\left(e_2; E^C\right)}{F(e_2; E^C) - F(e_2; E^D)}. \tag{11}$$

Under Assumption 2, equation (11) is written as:

$$\frac{a + 2\left[1 - a\right] e_1}{\left[a - b\right]\left[e_1 - e_1^2\right]} = \frac{a + 2\left[1 - a\right] e_2}{\left[a - b\right]\left[e_2 - e_2^2\right]},$$

i.e., $[a + 2[1 - a] e_1] e_2^2 - [a + 2[1 - a] e_1^2] e_2 + a[e_1 - e_1^2] = 0$. Easy calculations show that, when $e_1 \geq e^*$ the previous equality does not have any solution (in e_2) in the interval $(e_1, 1]$.

We now complete the proof of the proposition. Claims 3 and 5 allow to state that the optimal policy has only two regions. Hence, it has the following form: $\beta(e) = \widehat{\gamma}$ for all $e \in [\underline{e}, \widehat{e})$ and $\beta(e) = 0$ for all $e \in [\widehat{e}, \overline{e}]$, where, given Claims 1 and 2, $\widehat{e} \in [e^*, \overline{e})$. Finally, Claim 4 leaves as the unique candidate the policy proposed in Proposition 4. ■

Proof of Proposition 5. (a) We first prove that, given $\alpha^*(z)$, $z(e)$ is the optimal firms' strategy. It is easy to check that $\widehat{\gamma} < t/\theta$ implies that firms either will report $z = \underline{e}$ or $z = z^*$, any other possible report is dominated. The expected costs of a firm with emissions level e are lower reporting $\underline{e}$ than z^* if:

$$t\underline{e} + \widehat{\gamma}\theta\left[e - \underline{e}\right] < tz^* = t\underline{e} + \frac{\Delta\left(e^* - \underline{e}\right)}{\int_{\underline{e}}^{e^*}\left[F(e; E^C) - F(e; E^D)\right] de},$$

i.e., given the characterization of $\widehat{\gamma}$,

$$\frac{\Delta\left[e - \underline{e}\right]}{\int_{\underline{e}}^{e^*}\left[F(e; E^C) - F(e; E^D)\right] de} < \frac{\Delta\left(e^* - \underline{e}\right)}{\int_{\underline{e}}^{e^*}\left[F(e; E^C) - F(e; E^D)\right] de},$$

or $e < e^*$.

Since $z(e)$ is optimal for the firms given $\alpha^*(z)$, the policy $\alpha^*(z)$ achieves the policy $\beta(e)$ found in Proposition 4, hence, it is optimal under Assumptions 1 and 2.

(b) In this case, it is immediate to check that firms' strategy is optimal given $\alpha^*(z)$ and that the policy $\alpha^*(z)$ is then optimal. ■

Proof of Corollary 1. The proof follows easily from Proposition 5. ■

Proof of Proposition 6. Given Δ^n, the EA solves the following program:

$$\underset{(\beta(e))_{e\in[\underline{e},\overline{e}]}}{Min} \ B$$

$$\text{s.t.: } \beta(e) \text{ is nonincreasing in } e$$

$$\beta(e) \in [0, t/\theta] \text{ for all } e \in [\underline{e}, \overline{e}]$$

$$G(\Delta^n)\int_{\underline{e}}^{\overline{e}} \beta(e)dF(e;E^C) + [1-G(\Delta^n)]\int_{\underline{e}}^{\overline{e}} \beta(e)dF(e;E^D) \leq B$$

$$\Delta^n = \theta\int_{\underline{e}}^{\overline{e}} \beta(e)\left[F(e;E^C) - F(e;E^D)\right]de.$$

Following the same steps as in Proposition 4, there exists a solution to the previous program that takes on at most one value γ different from 0 and t/θ. Also, the policy minimizing monitoring costs must solve program $[P^M]$ below:

$$\underset{(\gamma,e_1,e_2)}{Min}\left\{\frac{t}{\theta}F\left(e_1;E^M\right) + \gamma\left[F\left(e_2;E^M\right) - F\left(e_1;E^M\right)\right]\right\}$$

$$\text{s.t.: } \frac{\Delta^n}{\theta} = \frac{t}{\theta}\int_{\underline{e}}^{e_1}\left[F(e;E^C) - F(e;E^D)\right]de + \gamma\int_{e_1}^{e_2}\left[F(e;E^C) - F(e;E^D)\right]de. \qquad (12)$$

where we have denoted $F(e;E^M) \equiv G(\Delta^n)F(e;E^C) + [1-G(\Delta^n)]\,F(e;E^D)$. We note that the distribution function $F(e;E^M)$ is the cumulative distribution function of a linear density function $f(e;E^M) = a^n + 2\,[1-a^n]\,e$, where $a^n = G(\Delta^n)a + [1-G(\Delta^n)]\,b$. We denote

$$h^n(e) \equiv f(e;E^M) - \frac{F(e;E^C) - F(e;E^D)}{\int_{\underline{e}}^{e}\left[F(x;E^C) - F(x;E^D)\right]dx}F(e;E^M).$$

Under Assumption 2, $h^n(e)$ is first negative and then positive. We denote by e^n the cut-off level such that $h^n(e^n) = 0$.[12] It is easily checked that $e^n < e^*$.

From now on, we can follow the same steps as in Claims 1 to 5 in the proof of Proposition 4, where we have to consider Δ^n instead of Δ, e^n instead of e^*, and $h^n()$ instead of $h()$. The claims lead to the following unique candidate policy:

[12] More precisely $e^o = \frac{-2a^o+\sqrt{4a^{o2}+6a^o[1-a^o]}}{2[1-a^o]} \in (0,1)$ when $a^o \neq 1$.

(a) If $\Delta^n < t\int_{\underline{e}}^{e^n}\left[F(e;E^C)-F(e;E^D)\right]de$, then :

$$\begin{aligned}\beta^n(e) &= \widehat{\gamma}^n \text{ for all } e\in[\underline{e},e^n)\,,\\ \beta^n(e) &= 0 \quad \text{for all } e\in[e^n,\overline{e}]\,, \text{ with}\end{aligned}$$

$$\widehat{\gamma}^n\theta\int_{\underline{e}}^{e^n}\left[F(e;E^C)-F(e;E^D)\right]de =^n .$$

(b) If $\Delta^n \geq t\int_{\underline{e}}^{e^n}\left[F(e;E^C)-F(e;E^D)\right]de$, then:

$$\begin{aligned}\beta^n(e) &= t/\theta \text{ for all } e\in[\underline{e},\widehat{e}^n)\,,\\ \beta^n(e) &= 0 \quad \text{for all } e\in[\widehat{e}^n,\overline{e}]\,, \text{ with}\end{aligned}$$

$$t\int_{\underline{e}}^{\widehat{e}^n}\left[F(e;E^C)-F(e;E^D)\right]de = \Delta^n.$$

Given the previous function $\beta^n(e)$, we follow the same steps as in the proof of Proposition 5 to show that the function $\alpha^n(z)$ corresponds to $\beta^n(e)$. The cut-off value z^n that appears in the Proposition corresponds to the report that makes a firm whose realized emission is e^n indifferent between reporting 0 (and being monitored with probability $\widehat{\gamma}^n$) and reporting z^n and avoiding monitoring. That is, z^n is characterized by $t\underline{e}+\widehat{\gamma}^n\theta\left[e^n-\underline{e}\right]=tz^n$. ■

References

[1] M.A. Cohen, Monitoring and Enforcement of Environmental Policy, in Folmer H. and Tiltenberg T. (Eds), "*The International Year-book of Environmental Resource Economics 1999/2000*", Edward Elgar Publishing (1999).

[2] P. B. Downing and L. J. White, Innovation in Pollution Control, *Journal of Environmental Economics and Management* 13, 18-29 (1986)

[3] L. Kaplow and S. Shavell, Optimal Law Enforcement with Self-Reporting of Behavior, *Journal of Political Economy* 102 (3), 583-606 (1994).

[4] S. F. Hamilton and T. Requate, Emission Caps versus Ambient Standards when Damage from Pollution is Stochastic, mimeo (2006).

[5] J. D. Harford, Firm Behavior under Imperfectly Enforceable Pollution Standards and Taxes, *Journal of Environmental Economics and Management* 5, 26-43 (1978).

[6] J. D. Harford, Self-reporting of Pollution and the Firm's Behavior under Imperfectly Enforceable Regulation, *Journal of Environmental Economics and Management* 14 (3), 293-303 (1987).

[7] E. Helland, The Enforcement of Pollution Control Laws: Inspection, Violations and Self-reporting, *Review of Economics and Statistics* 80 (1), 141-153 (1998).

[8] A. Heyes, Environmental Enforcement when Inspectability is Endogenous, *Environmental and Ressource Economics* 4, 479-494 (1993).

[9] R. Innes, Remediation and Self-reporting in Optimal Law Enforcement, *Journal of Public Economics* 72, 379-393 (1999).

[10] L. Kaplow and S. Shavell, Optimal Law Enforcement with Self-reporting Behavior, *Journal of Political Economy* 102 (3), 583-606 (1994).

[11] J. Livernois and C.J. McKenna, Trust or Consequences. Enforcing Pollution Standards with Self-reporting, *Journal of Public Economics* 71 (3), 415-440 (1999).

[12] I. Macho-Stadler, Environmental Regulation: Choice of Instruments under Imperfect Compliance, forthcoming in *Spanish Economic Review*, 2006.

[13] I. Macho-Stadler and D. Pérez-Castrillo, Optimal Auditing with Heterogeneous Income Sources, *International Economic Review* 38 (4), 951-968 (1997).

[14] I. Macho-Stadler and D. Pérez-Castrillo, Optimal Enforcement Policy and Firms' Emissions and Compliance with Environmental Taxes, *Journal of Environmental Economics and Management* 51, 110-131 (2006).

[15] A. Malik, Self-reporting and the Design of Policies for Regulating Stochastic Pollution, *Journal of Environmental Economics and Management* 24, 241-257 (1993).

[16] S. R. Milliman and R. Prince, Firm Incentives to Promote Technological Change in Pollution, *Journal of Environmental Economics and Management* 17, 292-296 (1989).

[17] K. Millock, D. Sunding and D. Zilberman, Regulating Pollution with Endogenous Monitoring, *Journal of Environmental Economics and Management* 44, 221-241 (2002).

[18] J. Reinganum and L. Wilde, Income Tax Compliance in a Principal-Agent Framework, *Journal of Public Economics* 26, 1-18 (1985).

[19] I. Sánchez and J. Sobel, Hierarchical Design and Enforcement of Income Tax Policies, UCSD Discussion paper 91.02 (1991).

[20] I. Sánchez and J. Sobel, Hierarchical Design and Enforcement of Income Tax Policies, *Journal of Public Economics* 50, 345-369 (1991).

[21] A. Sandmo, *The Public Economics of Environment*, Oxford University Press (2000).

[22] A. Sandmo, Efficient Environmental Policy with Imperfect Compliance, *Environmental and Resource Economics* 23, 85-103 (2002).

[23] S. Scotchmer, The Economic Analysis of Taxpayer Compliance, *American Economic Review*, Papers and Proceedings 77, 229-233 (1987).

[24] N. Tarui and S. Polasky, Environmental Regualtion with Technology Adoption, Learning and Strategic Behavior, *Journal of Environmental Economics and Management* 50, 447-467 (2005).