LINKING CONFLICT TO INEQUALITY AND POLARIZATION[1]

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ABSTRACT

In this paper we study a behavioral model of conflict that provides a basis for choosing certain indices of dispersion as indicators for conflict. We show that the (equilibrium) level of conflict can be expressed as an (approximate) linear function of the Gini coefficient, the Herfindahl-Hirschman fractionalization index, and a specific measure of polarization due to Esteban and Ray.

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1. Introduction

In this paper we study a behavioral model of conflict that provides a basis for choosing certain indices of dispersion as indicators for conflict. We show that the (equilibrium) level of conflict can be expressed as an (approximate) linear function of the Gini coefficient, the Herfindahl-Hirschman fractionalization index, and a specific measure of polarization due to Esteban and Ray.

Income inequality has been always viewed as closely related to conflict. In the Introduction of his celebrated book "On Income Inequality" Sen (1972) asserts that "the relation between inequality and rebellion is indeed a close one". Early empirical studies on the role of inequality in explaining civil conflict have focussed on the personal distribution of income or of landownership.[2]

Contemporary literature has shifted the emphasis from class to ethnic conflict. Here too the initial presumption has been that ethnic diversity is a key factor for ethnic conflict. Easterly and Levine (1997) used the index of *fractionalization* as a measure of diversity, and the measure has been used in several different empirical studies of conflict (see, e.g., Collier and Hoffler (2004), Fearon and Laitin (2003) and Miguel, Satyanath and Sergenti (2004). More recently, following on the idea that highly fragmented societies may not be highly conflictual,[3] measures of polarization have also made their way into empirical studies of conflict.[4]

These contributions, while loosely based on theoretical arguments, are essentially empirically motivated in an attempt to identify a statistical regularity. The preference for one particular index or another simply depends on its ability to fit the facts. In contrast, there is to our knowledge no behavioral model explaining why should we expect — to begin with — a relationship between the Gini or the fractionalization indices, and conflict.[5]

In this paper we present a behavioral model of conflict that precisely defines the links between conflict and measures of dispersion, such as inequality and polarization. The model is general, in that it allows for conflict over both divisible private goods and

[2]See, for instance, Nagel (1974), Muller and Seligson (1987), Brockett (1992) or the survey article by Lichbach (1989).

[3]For instance, Horowitz (1985) argues that large cleavages are more germane to the study of conflict, stating that "a centrally focused system [with few groupings] possesses fewer cleavages than a dispersed system, but those it possesses run through the whole society and are of greater magnitude. When conflict occurs, the center has little latitude to placate some groups without antagonizing others."

[4]See Esteban and Ray (1994) and Wolfson (1994) for the earliest development of polarization measures, and Reynal-Querol (2002) for a special case of the Esteban-Ray measure which is then applied to a cross-section study of ethnic conflict by Montalvo and Reynal-Querol (2005). See also the special issue of the *Journal of Peace Research* edited by Esteban and Schneider (2008) entirely devoted to the links between polarization and conflict.

[5]Esteban and Ray (1999) do discuss the possible links between polarization and equilibrium conflict in a model of strategic behavior. Montalvo and Reynal-Querol (2005b) also derive a measure of polarization from a rent-seeking game.

(group-based) public goods.[6] It is also general in that it allows for varying degrees of within-group cohesion, running the gamut from individualistic decisions (as in the voluntary contributions model) all the way to choices imposed by benevolent group leaders. Our main result is that equilibrium conflict can be approximated as a weighted average of a particular inequality measure (the Gini coefficient), the fractionalization index used by Easterly and Levine and others, and a *particular* polarization measure from the class axiomatized by Esteban and Ray (1994). Moreover, the weights depend in a precise way on two parameters: the "degree of publicness" of the prize and the extent of intra-group "cohesion". In particular, our result suggests that if our derived equation were to be taken to the data, the estimated coefficients would be informative regarding these parameters.

While we link the *severity* of conflict to these measures, our paper does not address the issue of conflict *onset*. As discussed in Esteban and Ray (2008a), the knowledge of the costs of open conflict may act as a deterrent. For this reason we argue there that the relationship between conflict onset and the factors determining the intensity of conflict may be non-linear. This issue is not addressed here: we assume that society is in a state of conflict throughout.

We organize this paper as follows. Section 2 provides a very brief presentation of the basic measures of inequality and polarization. Section 3 develops a game-theoretic model of conflict and some of its basic properties. The main result is obtained in Section 4. Section 5 discusses the accuracy of our approximation. Section 6 concludes.

2. Inequality and Polarization

Suppose that population is distributed over m groups, with n_i being the share of the population belonging to group i. Denote by δ_{ij} the "distance" between groups i and j (more on this below). Fix the location of any given group i and compute the average distance to the rest of locations. The Gini index G is the average of these distances as we take each location in the support as a reference point.[7] We write it in unnormalized form[8] as

$$G = \sum_{j=1}^{m} \sum_{i=1}^{m} n_i n_j \delta_{ij}. \tag{1}$$

We haven't been very specific about the distance δ_{ij}. When groups are identified by their income, this is simply the absolute value of the income difference between i and j. However, in principle we could apply this index to distributions over political, ethnic or religious groups. Unfortunately, in most cases where distance is non-monetary the

[6] The specific formulation is borrowed from Esteban and Ray (2001).

[7] The properties of the Gini index are well known. Its first axiomatization is due to Thon (1982).

[8] The Gini is typically renormalized by mean distance; this makes no difference to the current exposition.

available information does not permit a reasonable estimate of δ_{ij}. This is why it is common to assume (sometimes implicitly) that $\delta_{ij} = 1$ for all $i \neq j$ and, of course, $\delta_{ii} = 0$. In that case, (1) reduces to

$$F = \sum_{i=1}^{m} n_i(1 - n_i) \tag{2}$$

This is the widely used Hirschman-Herfindahl *fractionalization index* (Hirschman (1964)). It captures the probability that two randomly chosen individuals belong to different groups. As mentioned before, this measure has been used to link ethnolinguistic diversity to conflict, public goods provision, or growth.[9] At the same time, we know of no behavioral model of conflict that explicitly establishes a link between conflict and inequality (or fractionalization).

Esteban and Ray (1994) introduce the notion of *polarization* as an appropriate indicator for conflict.[10] Their approach is founded on the postulate that group "identification" (proxied by group size) *and* intergroup distances can both be conflictual. Duclos, Esteban and Ray (2004) work with density functions over a space of characteristics to axiomatize a class of polarization measures, which we describe here for discrete distributions:

$$P_\beta = \sum_{i=1}^{m} \sum_{j=1}^{m} n_i^{1+\beta} n_j \delta_{ij}, \text{ for } \beta \in [0.25, 1] \tag{3}$$

An additional axiom, introduced and discussed by Esteban and Ray (1994), pins down the value of β at 1:

$$P \equiv P_1 = \sum_{i=1}^{m} \sum_{j=1}^{m} n_i^2 n_j \delta_{ij}. \tag{4}$$

Because (4) is not derived formally for the model studied in Duclos, Esteban and Ray (2004), we provide a self-contained treatment in the Appendix.

[9]See also Collier and Hoeffler (1998), Alesina, Baqir and Easterly (1999), Ellingsen (2000), Hegre et al. (2001), Alesina et al. (2003) and Fearon (2003) among others.

[10]Foster and Wolfson (1992) and Wolfson (1994, 1997) proposed an alternative measure of polarization specifically designed to capture the "disapearence of the middle class". Later, alternative measures of polarization have been proposed by Wang and Tsui (2000), Chakravarty and Majumder (2001), Zhang and Kanbur (2001), Reynal-Querol (2002), Rodríguez and Salas (2002), and Esteban, Gradín and Ray (2007).

The formal properties of this measure are discussed in detail in Esteban and Ray (1994).[11] It suffices here to focus on the squared term, which imputes a large weight to group identification. This weighting of group size implies that P does not satisfy Dalton's Transfer Principle (or equivalently, compatibility with second-order stochastic dominance of distance distributions). In this fundamental aspect it behaves differently from Lorenz-consistent inequality measures. In particular, P attains its maximum at a symmetric bimodal distribution.

As in the case of fractionalization, a situation of particular relevance is one in which group distances are binary: $\delta_{ij} = 1$ for all $j \neq i$ and $\delta_{ii} = 0$. In this case P reduces to

$$\tilde{P} = \sum_{i=1}^{m} n_i^2 (1 - n_i), \tag{5}$$

This is the measure of polarization proposed by Reynal-Querol (2002).

3. A Model of Conflict

We wish to explore the relationship between the measures described in the previous section and the equilibrium level of conflict attained in a behavioral model in which agents optimally choose the amount of resources to expend in conflict.[12]

3.1. **Public and Private Goods.** Consider a society composed of individuals situated in m groups. Let N_i be the number of individuals in group i, and N the total number of individuals, so that $\sum_{i=1}^{m} N_i = N$. These groups are assumed to contest a budget with *per capita* value normalized to unity. We shall suppose that a fraction λ of this budget is available to produce society-wide public goods. One of the groups will get to control the mix of public goods (as described below), but it is assumed that λ is given. The remaining fraction, $1 - \lambda$, can be privately divided, and once again the "winning" group can seize these resources.[13]

All individuals derive identical linear payoff from their consumption of the private good, but differ in their preference over the public goods available. All the members

[11] Although in Esteban and Ray (1994) and Duclos, Esteban and Ray (2004) groups are identified by their income — and hence δ_{ij} is the income distance between the two groups — the notion and measure of polarization can be naturally adapted to the case of "social polarization". Duclos, Esteban and Ray (2004) consider the case of "pure social polarization", in which income plays no role in group identity or intergroup alienation. For that case they propose (4) as the appropriate polarization measure (pp. 1759) with δ_{ij} interpreted as the alienation felt by an individual of group i with respect to a member of group j.

[12] We build on the model of conflict in Esteban and Ray (1999).

[13] This description may correspond to a conflict for the control of the government. Once in government the group may decide to change the types of public goods provided and the beneficiaries of the various forms of transfers in the budget. But it is not possible to substantially modify the structure of the budget.

of a group share the same preferences. Each group has a mix of public goods they prefer most. Using the private good as numeraire, define u_{ij} to be public goods payoff to a member of group i if a single unit per-capita of the optimal mix for group j is produced. We may then write the *per capita* payoff to group i as $\lambda u_{ii} + (1-\lambda)(N/N_i)$ (in case i wins the conflict) and λu_{ij} (in case some other group j wins).[14]

The parameter λ can also be interpreted as an indicator of the importance of the public good payoff relative to the "monetary" payoff used as numeraire.

We presume throughout that $u_{ii} > u_{ij}$ for all i, j with $i \neq j$.

3.2. **Conflict Resources and Outcomes.** We view conflict as a situation in which there is no agreed-upon rule aggregating the alternative claims of different groups. The success of each group is taken to be probabilistic, depending on the expenditure of "conflict resources" by the members of each group. We now describe this conflict.

Let r denote the resources expended by a typical member of any group. We take such expenditure to involve a isoelastic cost of

$$c(r) = \frac{1}{\theta} r^{\theta}, \tag{6}$$

where we assume that $\theta \geq 2$ (more on this below). Denote by $r_i(k)$ the contribution of resources by member k of group i, and define $R_i \equiv \sum_{k \in i} r_i(k)$. Our measure of societal conflict is the total of all resources supplied by every individual:

$$R = \sum_{i=1}^{m} R_i. \tag{7}$$

Let p_j be the probability that group j wins the conflict. We suppose that

$$p_j = \frac{R_j}{R} \tag{8}$$

for all $j = 1, \ldots, m$, provided that $R > 0$.[15] Thus the probability that group i will win the lottery is taken to be exactly equal to the share of total resources expended in support of alternative i.

[14] Note that there is no exclusion in the provision of public goods. These are always provided to the entire population; only the mix differs depending on which group has control. The implicit assumption is that a scaling of the population requires a similar scaling of public goods output in order to generate the same per-capita payoff. Because we hold the per-capita budget constant (and therefore change total budget with population), this gives us exactly the specification in the main text.

[15] Assign some arbitrary vector of probabilities (summing to one) in case $R = 0$. There is, of course, no way to complete the specification of the model at $R = 0$ while maintaining continuity of payoffs for all groups. So the game thus defined must have discontinuous payoffs. This poses no problem for existence; see Esteban and Ray (1999).

3.3. **Payoffs and Extended Utility.** We may therefore summarize the overall expected payoff to an individual k in group i as

$$
\begin{aligned}
\pi_i(k) &= \sum_{j=1}^{m} p_j \lambda u_{ij} + p_i \frac{(1-\lambda)N}{N_i} - \frac{1}{\theta} r_i(k)^\theta \\
&= \sum_{j=1}^{m} p_j \lambda u_{ij} + p_i \frac{(1-\lambda)}{n_i} - \frac{1}{\theta} r_i(k)^\theta, \qquad (9)
\end{aligned}
$$

where $n_i \equiv N_i/N$ is the population share of group i.

We now turn to a central issue: how are resources chosen? For reasons that will become clear, we wish to allow for a flexible specification in which (at one end) individuals choose r to maximize their own payoff, while (at the other end) there is full intra-group cohesion and individual contributions are chosen to maximize group payoffs. We permit these cases as well as a variety of situations in between by defining a group-i member k's *extended utility* to be

$$U_i(k) \equiv (1-\alpha)\pi_i(k) + \alpha \sum_{\ell \in i} \pi_i(\ell), \qquad (10)$$

where α lies between 0 and 1. When $\alpha = 0$, individual payoffs are maximized. When $\alpha = 1$, group payoffs are maximized.[16] Note that k enters again in the summation term in (10), so the weight on own payoffs is always 1.

One could interpret α as a measure of intragroup concern or altruism among the agents, but this interpretation is not necessary. An equivalent (but somewhat looser) interpretation is that α is some measure of how within-group monitoring, coupled with promises and threats, manage to overcome the free-rider problem of individual contributions. We are comfortable with either interpretation, but formally take it that each individual acts to maximize the expectation of extended utility, as defined in (10).

3.4. **Equilibrium.** The choice problem faced by a typical individual member k of group i is easy to describe: given the vector of resources expended by all other groups and by the rest of the members of the own group, choose $r_i(k)$ to maximize (10). This problem is well-defined provided that at least one individual in at least one other group expends a positive quantity of resources.

[16] This is similar to the description of intra-group altruism adopted in Sen (1964). For a more general specification in the context of intergenerational altruism, see Barro and Becker (1989). It will not matter whether extended utility is defined on other individual's payoffs (the specification here), or their gross expected payoff excluding resource cost, or indeed on others' *extended* utility. (In this last case we would need a contraction property for extended utility to be well-defined.) The results are insensitive to the exact choice.

Some obvious manipulation shows that the the maximization of (10) is equivalent to the maximization of

$$[(1-\alpha)+\alpha N_i]\left[p_i\frac{1-\lambda}{n_i}+\lambda\sum_{j=1}^{m}p_ju_{ij}\right]-\frac{1}{\theta}r_i(k)^\theta-\alpha\sum_{\ell\in i;\ell\neq k}\frac{1}{\theta}r_i(\ell)^\theta$$

by the choice of $r_i(k)$. Simplify this expression by defining, for each i, $\sigma_i\equiv(1-\alpha)+\alpha N_i$, $\Delta_{ii}\equiv 0$, and $\Delta_{ij}\equiv\lambda[u_{ii}-u_{ij}]+(1-\lambda)/n_i$ for all $j\neq i$. Then our individual equivalently chooses $r_i(k)$ to maximize

$$-\sigma_i\sum_{j=1}^{m}p_j\Delta_{ij}-\frac{1}{\theta}r_i(k)^\theta. \tag{11}$$

Continuing to assume that $r_j(\ell)>0$ for some $\ell\in j\neq i$, the solution to the choice of $r_i(k)$ is completely described by the interior first-order condition:

$$\frac{\sigma_i}{R}\sum_{j=1}^{m}p_j\Delta_{ij}=r_i(k)^{\theta-1}, \tag{12}$$

where we use (7) and (8).

An *equilibrium* is a collection $\{r_i(k)\}$ of individual contributions where for every group i and member k, $r_i(k)$ maximizes (11), given all the other contributions.

PROPOSITION **1.** *An equilibrium always exists and it is unique. In an equilibrium, every individual contribution satisfies the first-order condition* (12). *In particular, in every group, members make the same contribution:* $r_i(k)=r_i(\ell)$ *for every* i *and* $k,\ell\in i$.

Proof. First observe that in any equilibrium, $R_j>0$ for some group j.[17] But this means that every member of every group *other* than j must satisfy (12). This proves that in equilibrium, $R_i>0$ for all i, and that for every group i and $k\in i$, (12) is satisfied. In particular, we see that $r_i(k)=r_i(\ell)$ for every i and $k,\ell\in i$.

Call this common value r_i. Multiply both sides of (12) by r_iN_i and use (8) to see that

$$\sigma_i\sum_{j=1}^{m}p_ip_j\Delta_{ij}=N_ir_i^\theta,$$

and now define $v_{ij}\equiv\sigma_i\Delta_{ij}/N_i$ for all i to obtain the system

$$\sum_{j=1}^{m}p_ip_jv_{ij}=r_i^\theta \tag{13}$$

for all i. This is precisely the system described in Proposition 3.1 of Esteban and Ray (1999), with s in place of p and $c(r)\equiv(1/\theta)r^\theta$. Under the assumption that $\theta\geq 2$, the

[17]If this is false, then $R_i=0$ for all i so that each group has a success probability given by the arbitrary probability vector specified in footnote 15. For at least one group, say j, this probability must be strictly less than one. But any member of j can raise this probability to 1 but contributing an infinitesimal quantity of resources, a contradiction.

proof of Proposition 3.2 applies entirely unchanged to show that the system (13) has a unique solution. ■

When $\theta = 2$, so that the cost function is quadratic, we can express the equilibrium of the conflict game in particularly crisp form. For each i, the equilibrium condition (13) can now be written as

$$\sum_{j=1}^{m} p_j v_{ij} n_i^2 = p_i \rho^2,$$

where $\rho \equiv R/N$ is "per-capita conflict". Denote by W the $m \times m$ matrix with $n_i^2 v_{ij}$ as representative element. Then the equilibrium probability vector $\mathbf{p}$ and per-capita conflict level ρ must together solve

$$W\mathbf{p} = \rho^2 \mathbf{p},$$

so that ρ^2 is the unique positive eigenvalue of the matrix W and the equilibrium vector of win probabilities $\mathbf{p}$ is the associated eigenvector on the m-dimensional unit simplex.

4. Polarization, Inequality and Conflict

In this section, we establish our central result, one that links equilibrium conflict to a linear combination of the distributional measures discussed earlier.

It will be useful to isolate the deviation of group win probabilities p_i from group population share n_i. Let γ_i stand for the ratio of these two objects: $\gamma_i \equiv p_i/n_i$. Note that γ_i is not only an endogenous variable, it is (typically) unobservable as well. It refers to the *individual* contribution r_i, relative to the other r_j's. If there were no differences in individual behavior across groups, γ_i would equal 1 for all groups and win probabilities would simply be equal to group population shares. For this reason, we shall refer to the γ_i's as "behavioral correction factors".

Proposition **2.** *Suppose that we make the approximation assumption that every behavioral correction factor equals one. Then the per-capita cost of conflict is a linear function of the three distributional measures F, G, and P:*

$$\rho^\theta = \left(\frac{R}{N}\right)^\theta \approx \omega_1 + \omega_2 G + \alpha[\lambda P + (1-\lambda)F], \tag{14}$$

where $\omega_1 \equiv (1-\lambda)(1-\alpha)(m-1)/N$ and $\omega_2 \equiv \lambda(1-\alpha)/N$. In particular, when population is large, per-capita conflict is proportional to a convex combination of only P and F, provided that group cohesion $\alpha > 0$.

This stark result expresses equilibrium conflict in a behavioral model as a linear combination of three familiar distributional indices; the Gini coefficient, the Herfindahl-Hirschman fractionalization index, and the Esteban-Ray polarization measure with coefficient $\beta = 1$ (see (4)). Moreover, the weights on the combination tell us when each measure is likely to be a more important covariate of conflict. Specifically, the weights associated to each of these three indices depend on the degree of publicness of the prize, as proxied by λ, and on the level of intra-group cohesion, as proxied by α. They also depend on overall population.

In particular, as population grows large, the weight on the "intercept term" as well as the Gini coefficient converges to zero. Conflict becomes roughly proportional to population, and the ratio of the two only depends on polarization and fractionalization, provided group cohesion α is positive. This last restriction is easy to understand. If $\alpha = 0$, then free-rider considerations become dominant, and conflict *per capita* dwindles to zero for large populations.

The merit of a decomposition such as (14) depends on whether it yields a deeper and more intuitive understanding of the factors influencing conflict beyond the abstractions of a specific model. We would claim that our decomposition does accomplish this to some degree. It seems reasonable to classify the main forces driving conflict into three categories: group size, what the group is fighting for and the degree of overall cohesion of (or commitment to) the cause of the group. What groups are fighting for will determine how alienated groups will feel from each other. How cohesive a group is will determine the extent of within-group identification. These are precisely the two ingredients emphasized in Esteban and Ray (1994) and Duclos, Ray and Esteban (2004) as the main determinants of conflict.

Suppose that we observe a situation of conflict in which all groups fight for the control of an excludable private good (such as the revenue from valuable natural resources). Then the only feature distinguishing the different groups is their size. There is no "primordial" inter-group alienation relevant to this conflict. In that case we should expect that the distribution of group sizes will be the most relevant explanatory factor for conflict. Any measure designed to capture inter-group "distances" should have little to say here. Indeed, the decomposition above with full privateness — $\lambda = 0$ — leaves group fractionalization as the sole relevant indicator for conflict.

At the other extreme, full publicness brings out the natural differences in group preferences over public goods. Now fractionalization plays no role. Only the measures reflecting inter-group alienation remain: the Gini and the polarization index. The relative weighting that each enjoys now depends on the degree of group cohesion or identification. When $\alpha = 0$, there is no group identification at all and only the Gini coefficient matters. When $\alpha = 1$ it is polarization that comes to the forefront.

What is remarkable about this result, though, is that precisely *these* three measures — and *only* these three — are highlighted by our model of conflict (and that they enter in this convenient linear fashion). It is the simplicity of this relationship which is the main contribution of the paper.

At the same time, this extremely simple structure depends on the approximation that all behavioral correction factors equal unity. Our first task below is to provide a formal proof of the Proposition that shows exactly where the approximation lies. Our second task is to judge the accuracy of the approximation by computing the exact solution for conflict (without the restriction on correction factors) and compare this with the approximate solution described in Proposition 2. We shall do this numerically.

Proof of Proposition 2. Recall the equilibrium condition (13), which we write as

$$\sum_{j=1}^{m} p_i p_j \frac{\sigma_i \Delta_{ij}}{n_i N} = r_i^\theta = \frac{p_i^\theta \rho^\theta}{n_i^\theta},$$

where $\rho \equiv R/N$. Multiply both sides by $p_i^{1-\theta} n_i^\theta$ and use the fact that $p_i = \gamma_i n_i$ to obtain

$$\begin{aligned} p_i \rho^\theta &= \sum_{j=1}^{m} p_i^{2-\theta} p_j n_i^{\theta-1} \frac{\sigma_i \Delta_{ij}}{N} \\ &= \sum_{j=1}^{m} \gamma_i^{2-\theta} \gamma_j n_i n_j \frac{\sigma_i \Delta_{ij}}{N}. \end{aligned}$$

Adding over all i we conclude that

$$\rho^\theta = \sum_{i=1}^{m} \sum_{j=1}^{m} \gamma_i^{2-\theta} \gamma_j n_i n_j \frac{\sigma_i \Delta_{ij}}{N}. \tag{15}$$

Recall that $\sigma_i = (1-\alpha) + \alpha N_i$, that $\Delta_{ii} = 0$, and that $\Delta_{ij} = \lambda \delta_{ij} + (1-\lambda)/n_i$ for all $i \neq j$, where δ_{ij} is defined in the statement of the proposition. Opening up the terms σ_i and Δ_{ij}, and setting all correction factors to their approximation of 1, we see that

$$\rho^\theta \approx \sum_{i=1}^{m} \sum_{j \neq i} n_i n_j \left[\frac{1-\alpha}{N} + \alpha n_i \right] \left[\lambda \delta_{ij} + \frac{1-\lambda}{n_i} \right].$$

Expanding these terms, we obtain the desired result. ∎

5. Accuracy of the Approximation

The use of approximations is standard in economics. For instance, we use GNP as a proxy of social welfare or the Gini index of the distribution of personal income as a proxy for the level of equality. In both cases these measures abstract from the effects of endogenous individual choices, such as labor effort or consumption decisions (among other things). Yet, we find them useful indicators for the complex variables they intend to capture. Hoping for a good approximation by sacrificing the behavioral correction factors is exactly in the same spirit.

At the same time, the explicit nature of the model means that the accuracy of this approximation can be examined, and our exercise would be seriously incomplete if we did not do so. It appears to be difficult to do so analytically, though we do not rule out the possibility.

Before we proceed to a more detailed discussion, we should note that there are questions for which the discrepancy between p_i and n_i (or the divergence of the behavioral correction factor from unity) is of first order interest. For example, Esteban and Ray (1999) study the "activism" of "extremist" groups (those that are positioned at one end of a line in preference space), defining activism precisely by the ratio of p_i to n_i. Or consider the well-known Pareto-Olson thesis, which argues that small groups have a higher ratio of p_i to n_i. These are important issues in their own right, but all the same it is legitimate to ask whether neglecting them can significantly alter the structural relationship asserted in Proposition 2.

5.1. **Variation in the Correction Factors.** While a fully analytical approach appears to be out of reach, some preliminary remarks may be useful. Recall (15), which we use as the basis for our approximation result. We have

$$\begin{aligned}\rho^\theta &= \sum_{i=1}^m \sum_{j \neq i} \gamma_i^{2-\theta}\gamma_j n_i n_j \left[\frac{1-\alpha}{N} + \alpha n_i\right]\left[\lambda\delta_{ij} + \frac{1-\lambda}{n_i}\right] \\ &\approx \alpha \sum_{i=1}^m \sum_{j \neq i} \gamma_i^{2-\theta}\gamma_j \left[\lambda n_i^2 n_j \delta_{ij} + (1-\lambda) n_i n_j\right], \end{aligned} \tag{16}$$

where the second approximate equality takes the limit as N becomes large. A quick consultation of the equilibrium condition (13) tells us, moreover, that r_i becomes insensitive to the exact value of α as N grows large, keeping population *proportions* in each group constant.[18] This means that the same is true of the correction factors γ_i, and we can conclude that for large populations, ρ^θ is roughly proportional to α.

Meanwhile, the same is true of our approximation, which states that

$$\rho^\theta \approx \alpha[\lambda P + (1 - \lambda F)]$$

for large N. It follows that the relative accuracy of our approximation is independent of α when the population is large (as long as α is positive). This is why in the simulations below we shall fix α at one positive value (0.5) in the case of large populations. The specific value of α may well matter, however, when population is "small".

More significantly, the accuracy of our approximation may depend on the mix of public and private goods (the value of λ). To see this a bit more explicitly, focus on the case of contests and quadratic cost functions. Recall the first order condition (12) and manipulate it slightly to write

$$r_i \rho = \left[\frac{1-\alpha}{N} + \alpha n_i\right] \Delta_i(1 - p_i),$$

where $\Delta_i \equiv \lambda + (1-\lambda)/n_i$. Now send N to infinity and manipulate some more to obtain

$$p_i = \frac{\alpha n_i^2 \Delta_i}{\rho^2 + \alpha n_i^2 \Delta_i} \tag{17}$$

[18] To see this, observe that σ_i/N_i converges to 1 as $N_i \to \infty$, and that α appears nowhere else in the system described by (13).

for all i, where ρ^2 must solve

$$\sum_{j=1}^{m} \frac{\alpha n_j^2 \Delta_j}{\rho^2 + \alpha n_j^2 \Delta_j} = 1.$$

This last equation confirms that ρ^2 is linearly homogeneous in α; i.e., that $\rho^2 = \alpha\hat{\rho}^2$, where $\hat{\rho}$ is equilibrium per-capita conflict when $\alpha = 1$. Substituting this into (17) and dividing through by n_i, we obtain an expression for the correction factors:

$$\gamma_i = \frac{n_i \Delta_i}{\hat{\rho}^2 + n_i^2 \Delta_i} \tag{18}$$

for all i, where $\hat{\rho}^2$ must solve

$$\sum_{j=1}^{m} \frac{n_j^2 \Delta_j}{\hat{\rho}^2 + n_j^2 \Delta_j} = 1.$$

If we look at the case of purely private goods, we have $\lambda = 0$, so that (18) reduces to

$$\gamma_i = \frac{1}{\hat{\rho}^2 + n_i}.$$

Now smaller groups will have the higher value of γ (this is precisely the Pareto-Olson thesis). This means that terms with a low value of n_i and n_j in (16) will receive greater prominence than in the approximation, or equivalently, than in the fractionalization index. With purely public goods, we have $\lambda = 1$, and (18) reduces to

$$\gamma_i = \frac{n_i}{\hat{\rho}^2 + n_i^2}.$$

While a comparison with pure public goods is not immediate (in particular, the two values of $\hat{\rho}$ are *not* the same), there is less variation in γ across group sizes and the relationship between n_i and γ_i will generally be ambiguous.

How important are these deviations from the approximate specification in Proposition 2? Not very, as we shall now see.

5.2. **Numerical Analysis.**

We now examine the accuracy of our approximation by means of numerical analysis. To this end we run a series of simulations based on random draws for the parameter values describing group sizes and preferences. For each these randomly drawn societies we compute the exact level of conflict and compare it to our linear approximation. There are several cases that we consider.

5.2.1. *A Baseline Case.*

Our baseline exercise is the case of contests with quadratic costs. First consider large populations. Then, by the discussion in the previous section we may take $\alpha = 0.5$ without any loss of generality. We examine several degrees of publicness in the payoffs: $\lambda = 0, 0.2, 0.8$ and 1.0 (we report on $\lambda = 0.5$ in a later variation). In each of these cases, we take numerous random draws of a population distribution over five groups.

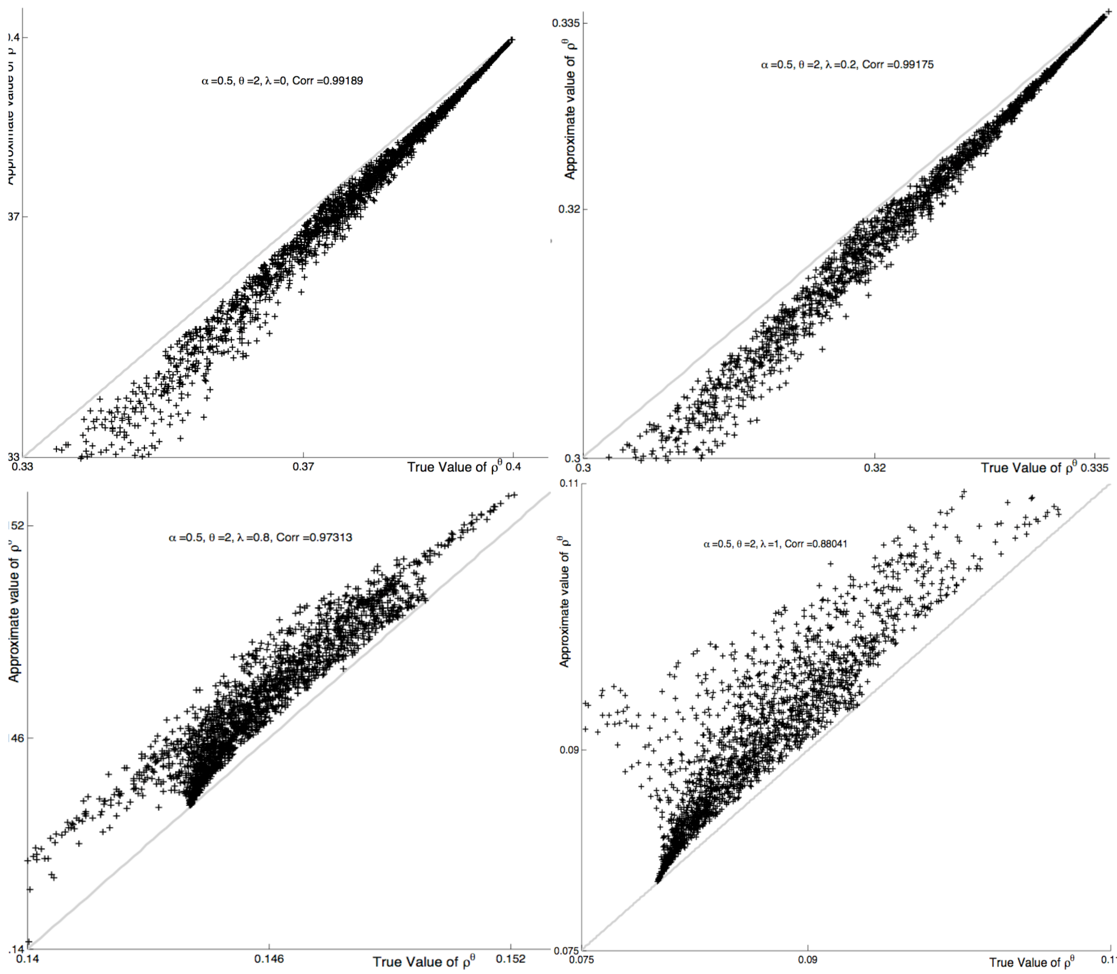

FIGURE 1. APPROXIMATE AND TRUE CONFLICT: BASELINE CASE

Figure 1 depicts the scatter plot of the approximate and true values of ρ^θ (ρ^2, in this case). In each situation (and in all successive figures as well), we plot the true value of conflict on the horizontal axis and the approximation on the vertical axis. We also use the same units for both values, so that the diagonal, shown in every figure, is interpretable as equality in the two values. The top two panels perform simulations when private goods are dominant ($\lambda = 0.0, 0.2$) and the bottom panels do the same when public goods are dominant ($\lambda = 0.8, 1.0$).

Remarkably, we obtain a very strong correlation between true and approximate values for equilibrium conflict suggesting that the "behavior correction factors" do not play a critical role in explaining conflict.

Notice how we underapproximate the true value of conflict when λ is close to zero, and the overall conflict is small. This is related to the Pareto-Olson argument discussed in the previous section. Low conflict will occur in non-polarized societies with one or more small groups. When the conflict is over private goods (which is the case with λ small), small groups will put in more resources per-capita. Because our approximation ignores this effect, it underestimates conflict, especially when the value of that conflict is relatively small. In a similar vein, we tend to overestimate conflict (for small values of conflict) when λ is close to 1. With public goods at stake, small groups put in less resources compared to their population share, and true conflict is smaller than the approximation predicts.

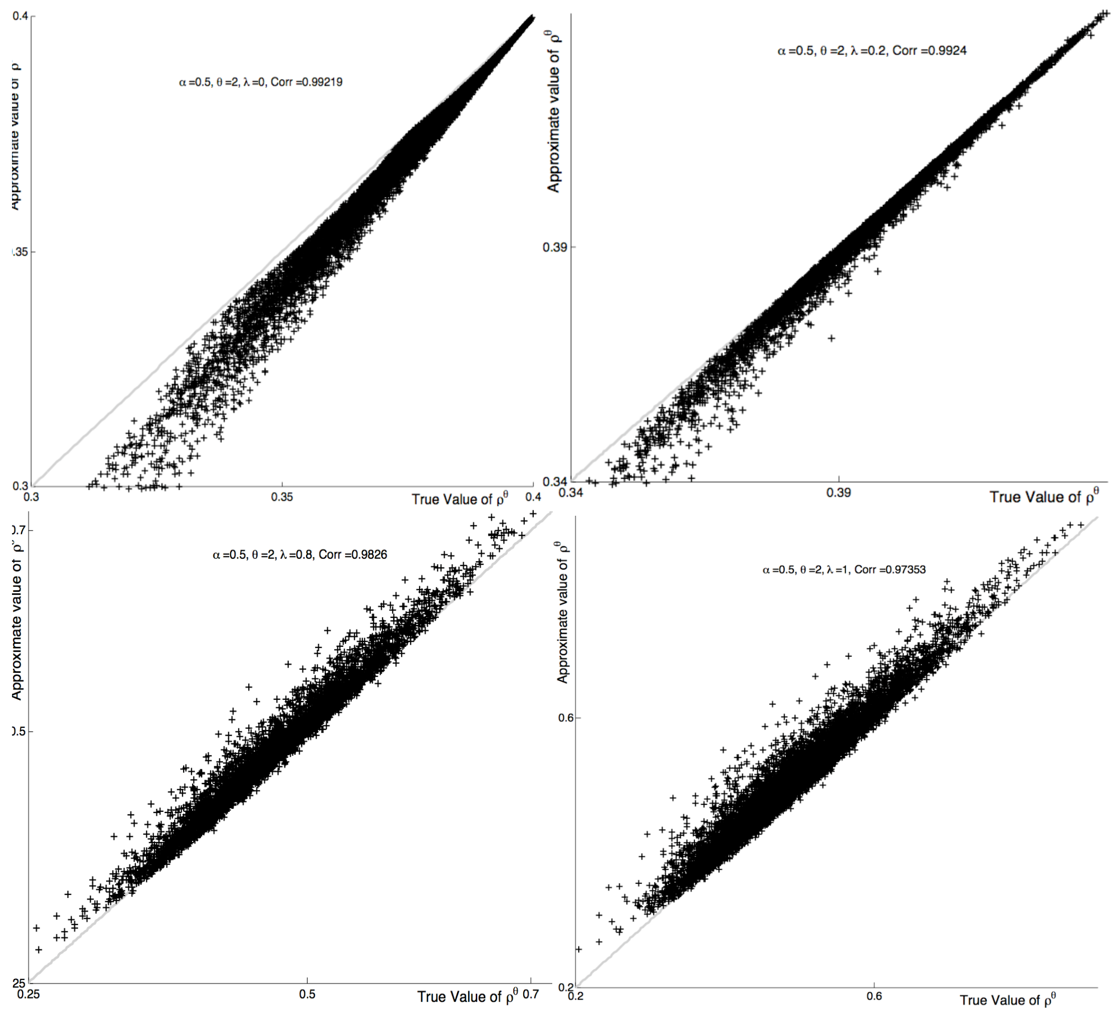

FIGURE 2. APPROXIMATE AND TRUE CONFLICT: VARYING UTILITY DISTANCES

That said, the correlation between the two variables is unaffected and the relationship appears broadly linear. What is remarkable is how close the approximation really is, and yet how difficult it appears to be to get a handle on this analytically. That there is no simple relationship between the two values is evident from the highly nontrivial (yet concentrated) scatter generated by the model.

However, note that in all situations, symmetric or near-symmetric population distributions over all groups with positive populations will have the property that correction factors are unimportant. This is why there are regions in every panel where the simulations take us precisely to the diagonal.

This high correlation is retained in all the reasonable variations that we have studied. Some examples follow.

5.2.2. *Inter-Group Distances.* The next set of simulations studies varying inter-group distances, instead of pure contests. Recall that distances are to be interpreted as losses from having the other public goods in place, instead of the group's favorite. We modify the previous simulation and now permit utility losses to vary across groups pairs (retaining the symmetry restriction that $u_{ij} = u_{ji}$). This is done by taking numerous independent draws of the matrix describing pairwise utility losses. We retain the baseline specification in all other ways. The results are reported in Figure 2, for various values of λ. As in the baseline case, the top panels perform simulations with private goods ($\lambda = 0.0, 0.2$) and the bottom panels do the same for public goods ($\lambda = 0.8, 1.0$). The correlations continue to be remarkably high and the general features of the baseline case are retained.

5.2.3. *Small Populations.* We return to the case of contests (with quadratic cost functions) and now study "small" populations. We suppose that there are 50 individuals in the economy, and consider numerous allocations of this population to the five groups. We report results for one case in which private goods are dominant ($\lambda = 0.2$) and another in which public goods are dominant ($\lambda = 0.8$). Notice that with small populations, the value of group cohesion α will generally matter. The top panel of Figure 3 reports our results for private goods under two values of α, 0.5 and 1.0. The bottom panel does the same for public goods. The correlations continue to be very high and the other features discussed for the baseline case are retained.

5.2.4. *Other Cost Elasticities.* Finally, we explore a set of variations in which we change the cost function from quadratic to other isoelastic specifications. We report four sets of results in Figure 4, all for the case with $\lambda = 0.5$ and large populations. One is for the baseline quadratic case. The remaining three are for progressively higher elasticities of the cost function: $\theta = 3, 4$ and 10.

Once again, the large correlations that we obtain remain undisturbed. Indeed, the simulations suggest that as the elasticity of the cost function goes up, our approximation improves even further. This is intuitive, as a highly curved cost function will lead to greater uniformity in the per-capita contribution of resources, thereby bringing the behavioral correction factors closer to unity.

It is certainly possible to try various combinations of these variations. We have done so, but do not report these results for the sake of brevity. In all the specifications we have tried, the approximation theorem we use appears to be more than satisfactory.

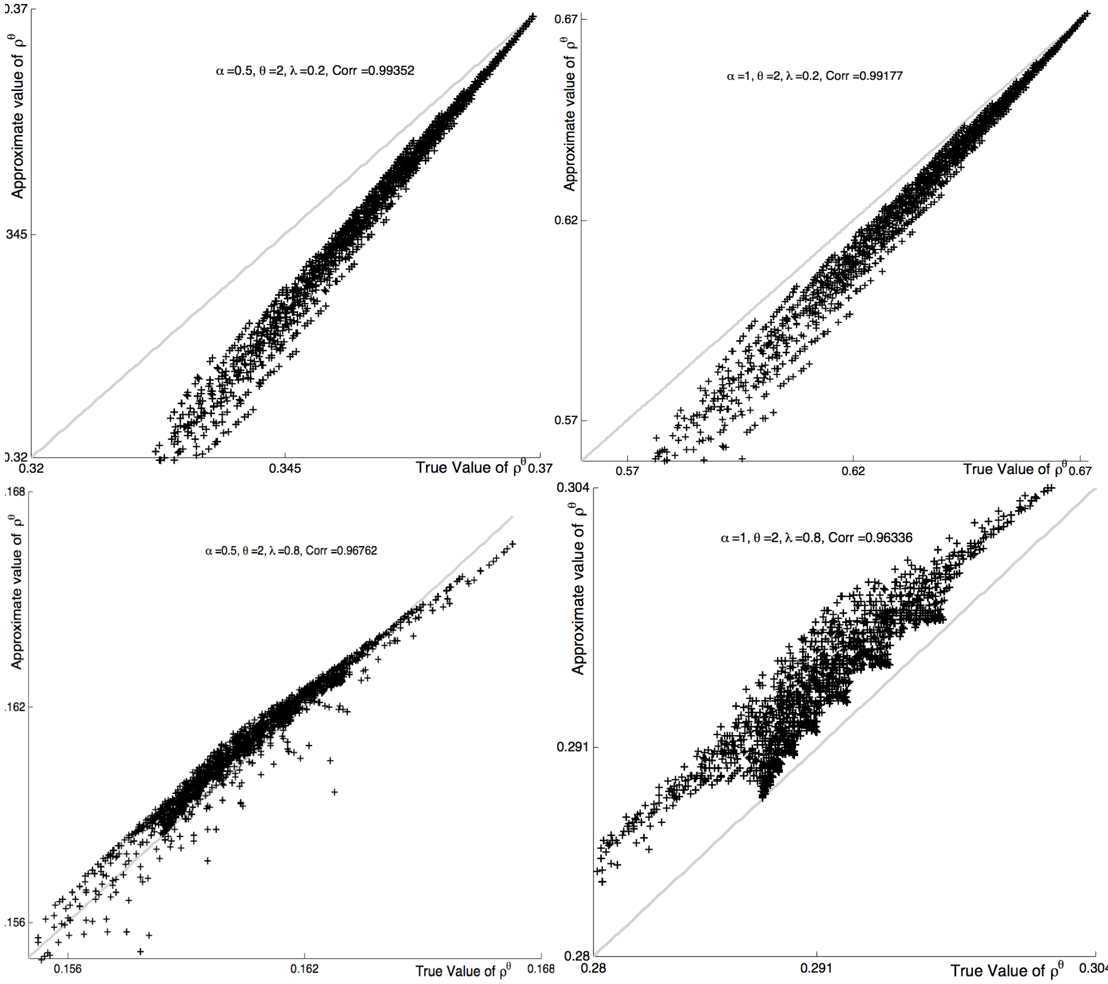

FIGURE 3. APPROXIMATE AND TRUE CONFLICT: SMALL POPULATION

6. SUMMARY

We have set up a behavioral model of conflict that provides a basis for the use of F, G and/or P as indicators for conflict.

[A] We have shown that the equilibrium level of resources expended in conflict can be approximated by a linear combination of the three indices, using the degree of altruism and of publicness as weights.

[B] The higher is the altruism the more pertinent fractionalization and polarization are in explaining conflict. The higher the degree of publicness the pertinent indices are inequality and polarization.

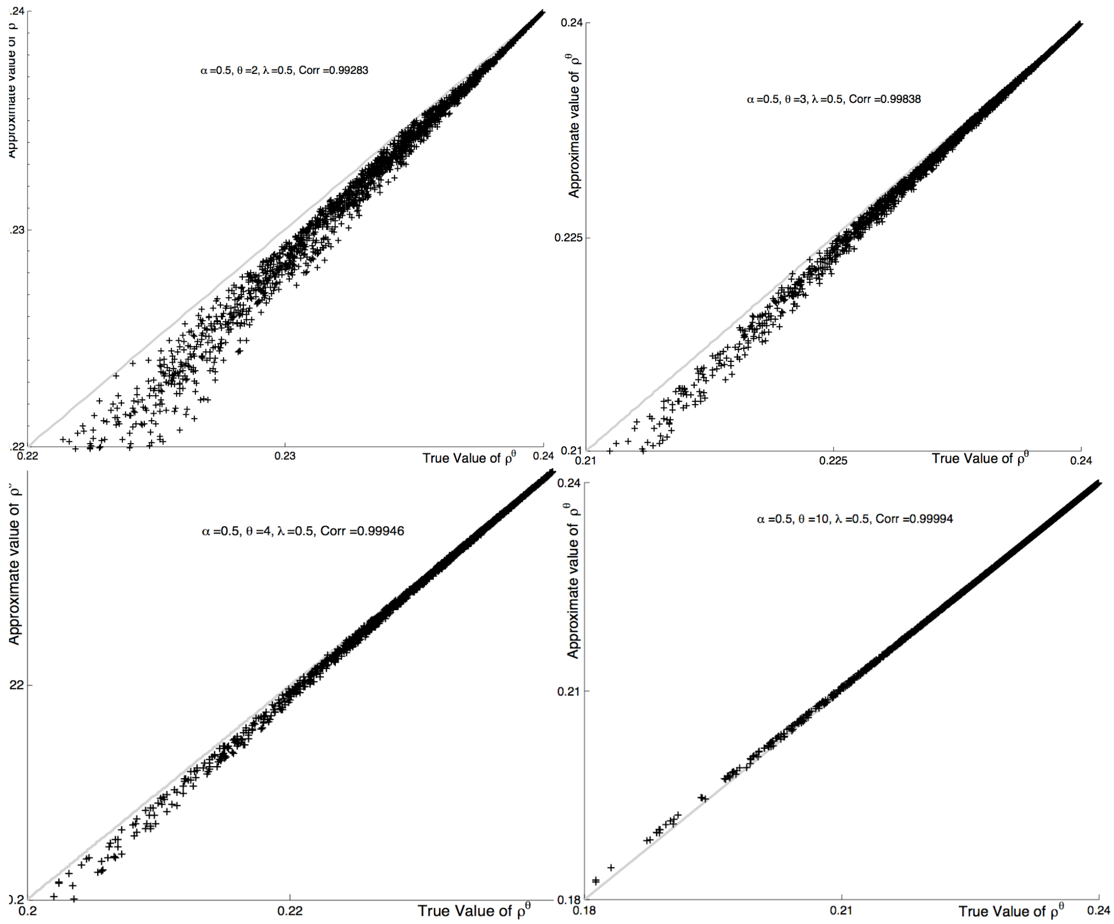

FIGURE 4. APPROXIMATE AND TRUE CONFLICT: NONQUADRATIC COSTS

[C] In simulations we find a very high correlation between our approximation and the true value of per capita conflict. This suggests that the behavior correction factors do not play a critical role.

Most importantly, this paper suggests new key features in explaining conflict: the degree of publicness in the payoff and the level of group mindedness in individual behavior. It seems plausible that the two dimensions are not independent of each other. One would expect high individualism in conflicts with a purely private payoff and higher group motivation when the payoff sought is essentially public in nature. The connection between these two dimensions is a matter of future research.

REFERENCES

BARRO, R.J. AND G.S. BECKER(1989) "Fertility Choice in a Model of Economic Growth," *Econometrica* **57**, 481-501.

BROCKETT, C.D. (1992) "Measuring Political Violence and Land Inequality in Central America" *American Political Science Review* **86** (1992), 169-176.

CHAKRAVARTY, S.R. AND A. MAJUMDER(2001), "Inequality, Polarization and Welfare: Theory and Applications," *Australian Economic Papers* **40**, 1–13.

COLLIER, P. AND A. HOEFFLER (2004), "Greed and Grievance in Civil War" *Oxford Economics Papers* 56, 563-595.

DUCLOS, J-Y., J. ESTEBAN, AND D. RAY (2004), "Polarization: Concepts, Measurement, Estimation" *Econometrica* **72**, 1737–1772.

EASTERLY W. AND R. LEVINE (1997), Africas Growth Tragedy: Policies and Ethnic Divisions, *Quarterly Journal of Economics* **111**, 1203-1250.

ESTEBAN, J., C. GRADÍN AND D. RAY (2007) An Extension of a Measure of Polarization, with an application to the income distribution of five OECD countries, *Journal of Economic Inequality* **5**, 1-19.

ESTEBAN, J. AND D. RAY (1994), "On the Measurement of Polarization" *Econometrica* **62**, 819-852.

ESTEBAN, J. AND D. RAY (1999), "Conflict and Distribution" *Journal of Economic Theory* **87**, 379-415.

ESTEBAN,J. AND D. RAY (2007), "A Model of Ethnic Conflict," mimeo., Department of Economics, new York University.

ESTEBAN, J. AND D. RAY (2008a), "Polarization, Fractionalization and Conflict," *Journal of Peace Research* **45**, 163-182.

ESTEBAN, J. AND D. RAY (2008b), "On the Salience of Ethnic Conflict," *American Economic Review* **98** forthcoming.

ESTEBAN, J. AND G. SCHNEIDER(2008) Polarization and Conflict: Theoretical and Empirical Issues (Introduction to the special issue), *Journal of Peace Research* **45**, 131-141.

FEARON, J. AND D. LAITIN (2003), "Ethnicity, Insurgency, and Civil War," *American Political Science Review* **97**, 75–90.

FOSTER, J. AND M.C. WOLFSON(1992), "Polarization and the Decline of the Middle Class: Canada and the US", Venderbilt University, mimeo.

LICHBACH, M. I. (1989), "An Evaluation of "Does Economic Inequality Breed Political Conflict?" Studies, *World Politics* **4**, 431470.

MIDLARSKI, M.I. (1988), "Rulers and the ruled: patterned inequality and the onset of mass political violence," *American Political Science Review* **82**, 491–509.

MIGUEL, E., SATYANATH, S. AND E. SERGENTI (2004), "Economic Shocks and Civil Conflict: An Instrumental Variables Approach," *Journal of Political Economy* **112**, 725–753.

MONTALVO, J. G. AND M. REYNAL-QUEROL (2005a), "Ethnic Polarization, Potential Conflict and Civil War," *American Economic Review* **95**, 796–816.

MONTALVO, J. G. AND M. REYNAL-QUEROL(2005b) “Ethnic diversity and economic development” *Journal of Development Economics* **76**, 293-323.

MULLER,E.N. AND M.A. SELIGSON (19987) “Inequality and Insurgency”, *American Political Science Review* **81**, 425-451.

MULLER, E. N., M.A. SELIGSON AND H. FU (1989), “Land inequality and political violence”, *American Political Science Review* **83**, 577–586.

NAGEL, J. (1974), “Inequality and discontent: a non-linear hypothesis,” *World Politics* **26**, 453–472.

PANDE, R. AND D. RAY (2003), “Preliminary Notes on Inequality and Conflict,” mimeo.

REYNAL-QUEROL, M. (2002), ”Ethnicity, Political Systems, and Civil Wars” *Journal of Conflict Resolution* 46, 29-54.

RODRÍGUEZ, J.G. AND R. SALAS(2002), “Extended Bi-Polarization and Inequality Measures,” *Research on Economic Inequality* **9** 69-84.

SAMBANIS, N. (2001), ”Do Ethnic and Non-Ethnic Civil Wars Have the Same Causes? A Theoretical and Empirical Enquiry (Part 1)” *Journal of Conflict Resolution* 45, 259-282.

SEN, A.(1966) “Sen A. (1966), “Labour Allocation in a Cooperative Enterprise” *Review of Economic Studies* **33**, 361-371.

SKAPERDAS, S. (1996), “Contest Success Functions,” *Economic Theory 7, 283-290.*

THON D.(1982) ”An axiomatization of the Gini coefficient”, *Mathematical Social Science* 2 131-143.

WANG, Y.Q. AND K.Y. TSUI(2000), “Polarization Orderings and New Classes of Polarization Indices,” *Journal of Public Economic Theory* **2**, 349–363.

WOLFSON, M. C. (1994), “When Inequalities Diverge”, *American Economic Review* Papers and Proceedings **84** 353-358.

WOLFSON, M.C.(1997), ”Divergent Inequalities: theory and empirical results”, *Review of Income and Wealth* **43** 401-421.

ZHANG, X. AND R. KANBUR(2001), “What Difference Do Polarisation Measures Make? An Application to China,” *Journal of Development Studies* **37**, 85–98.

APPENDIX

Esteban and Ray (1994) and Duclos, Esteban and Ray (2003) axiomatize the following class of polarization measures. Let population be distributed on $[0,\infty)$ with density $f(x)$. The class is given by

$$P_\beta = K\int\int f(x)^{1+\beta} f(y)|x-y|dxdy, \text{ for some constant } K>0 \text{ and } \beta\in[0.25,1].$$

Axiom 5. Suppose that a distribution consists of three equi-spaced uniform basic densities of sizes r, p and q, as shown in Figure 5, each of support 2ϵ. Assume that $p = q + r$. Then there is

$\eta > 0$ such that if $0 < r < \eta$ and $0 < \epsilon < \eta$, any uniform transfer of population mass from r to q cannot decrease polarization.

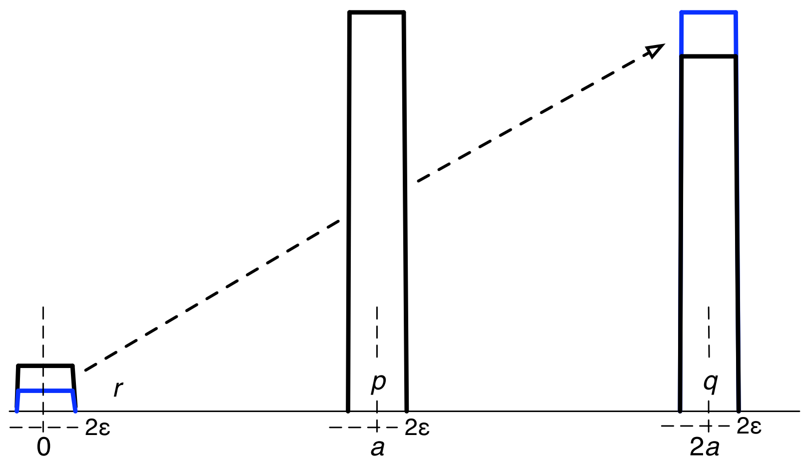

FIGURE 5. Figure for Axiom 5.

Intuitively, this axiom asserts that if the group of size r is extremely small, it cannot be contributing much on its own to social tension. If instead the population is transferred from that group to another group which is "equally opposed" to the largest group of size p (and of mass slightly smaller than p), then polarization cannot come down.

THEOREM **1.** *Under the additional Axiom 5, it must be that $\beta = 1$, so the unique polarization measure that satisfies the five axioms is proportional to*

$$\int\int f(x)^2 f(y)|y-x|dydx.$$

Proof. Consider a distribution generated from three copies of a uniform basic density as in Axiom 5, exactly as shown in Figure 5. The bases are centered at locations 0, a and $2a$. Each has width 2ϵ. The heights are r, p and q.

First we show necessity. Suppose that the axiom is true. Take parameters $z \equiv (p, q, r, \epsilon)$ to satisfy the conditions of the axiom, and transfer a small amount δ uniformly from the r-mass to the q-mass. Then polarization (viewed as a function of δ and the other parameters z) is given by the three "internal" polarizations of each basic density as well as the pairwise effective antagonisms across each pair of basic densities, which makes for nine terms in all:

$$\begin{aligned} P(\delta, z) &= I(\epsilon)\left[(r-\delta)^{2+\beta} + p^{2+\beta} + (q+\delta)^{2+\beta}\right] + C^2(\epsilon)\left[(r-\delta)^{1+\beta}(q+\delta) + (q+\delta)^{1+\beta}(r-\delta)\right] \\ &\quad + C(\epsilon)\left[(r-\delta)^{1+\beta}p + p^{1+\beta}(q+\delta) + (q+\delta)^{1+\beta}p + p^{1+\beta}(r-\delta)\right], \end{aligned}$$

where $I(\epsilon)$ is the "total internal distance" within each rectangle:

$$I(\epsilon) \equiv \int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon} |x-y|dxdy = \frac{8\epsilon^3}{3},$$

$C(\epsilon)$ is the "total distance" across neighboring rectangles:

$$C(\epsilon) \equiv \int_{-\epsilon}^{\epsilon} \int_{a-\epsilon}^{a+\epsilon} (x-y)dxdy = 4a\epsilon^2,$$

and $C^2(\epsilon)$ is the "total distance" between the side rectangles:

$$C^2(\epsilon) \equiv \int_{-\epsilon}^{\epsilon} \int_{2a-\epsilon}^{2a+\epsilon} (x-y)dxdy = 8a\epsilon^2.$$

Differentiating $P(\delta, z)$ with respect to δ (write this partial derivative as $P'(\delta, z)$) and evaluating the result at $\delta = 0$, we see that

$$\begin{aligned} P'(0,z) &= (2+\beta)I(\epsilon)\left[q^{1+\beta} - r^{1+\beta}\right] + (1+\beta)C(\epsilon)\left[q^{\beta}p - r^{\beta}p\right] \\ &- C^2(\epsilon)\left[q^{1+\beta} - r^{1+\beta} + (1+\beta)(r^{\beta}q - q^{\beta}r)\right]. \end{aligned}$$

Substituting the values for $I(\epsilon)$, $C(\epsilon)$ and $C^2(\epsilon)$, we see that

$$\begin{aligned} \frac{1}{4}\epsilon^{-2}P'(0,z) &= (2+\beta)\frac{2\epsilon}{3}\left[q^{1+\beta} - r^{1+\beta}\right] + (1+\beta)ap\left[q^{\beta} - r^{\beta}\right] \\ &- 2a\left[q^{1+\beta} - r^{1+\beta} + (1+\beta)(r^{\beta}q - q^{\beta}r)\right]. \end{aligned} \tag{19}$$

The axiom insists that $P'(0,z)$ must be nonnegative for all values of z such that $p = q + r$ and r sufficiently small. Fixing p and a, take a sequence of z's such that $r \to 0$, $q \to p$ and $\epsilon \to 0$. Noting that $P'(0,z) \geq 0$ throughout this sequence, we can pass to the limit in (19) to conclude that

$$(1+\beta) - 2 \geq 0,$$

which, given that $\beta \leq 1$, proves that $\beta = 1$.

To establish the converse, put $\beta = 1$ and consider (19) again. We see that for any configuration with $q > r$,

$$\begin{aligned} \frac{1}{4}\epsilon^{-2}P'(0,z) &= 2\epsilon\left[q^2 - r^2\right] + 2ap\left[q - r\right] - 2a\left[q^2 - r^2\right] \\ &> 2ap\left[q - r\right] - 2a(q-r)(q+r) \\ &= 2ap\left[q - r\right] - 2ap\left[q - r\right] = 0, \end{aligned}$$

where the penultimate equality uses the restriction that $p = q + r$. ■