

# BUDGET CONSTRAINED EXPENDITURE MULTIPLIERS

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**Abstract:** We show that standard expenditure multipliers capture economy-wide effects of new government projects only when financing constraints are not binding. In actual policy making, however, new projects usually need financing. Under liquidity constraints, new projects are subject to two opposite effects: an income effect and a set of spending substitution effects. The former is the traditional, unrestricted, multiplier effect; the latter is the result of expenditure reallocation to uphold effective financing constraints. Unrestricted multipliers will therefore be, as a general rule, upward biased and policy designs based upon them should be reassessed in the light of the countervailing substitution effects.

**Keywords:** Government multipliers, fiscal stimulus, expenditures substitution effects.

**JEL classification:** H61, C67

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Ferran Sancho is also a member of the MOVE consortium at UAB. Support from project SEC2006-0702 is gratefully acknowledged. Errors and shortcomings are the exclusive responsibility of the authors.

## 1. Introduction

In the wake of the current global recession little discussion has been undertaken for the need and the effectiveness of the so-called ‘fiscal stimulus packages’. These expenditure packages have been aimed at speeding up the economic recovery or, at the very least, slowing down its detrimental effects. Governments have come aboard the wagon hoping their fiscal actions will have results that are effective, quick and visible to the public. A variety of economic models, both macro and micro, seem to lend conceptual support to these initiatives although genuine doubts remain in the macro arena. See, for instance, Barro’s recent letter (2009) questioning the USA’s stimulus bill. In the micro field the support for expansionary policies comes usually from models that have an economy-wide perspective, like inter-industry or social accounting matrix based models (Miller and Blair, 1985, McGregor et al., 1996; Cardenete and Sancho, 2006). These micro models are demand-driven models with a, nonetheless, strong Keynesian flavour. They produce ‘multiplier effects’ taking advantage of some type of general equilibrium interactions that reflect, usually in quite good detail, the productive facets of an economy. Any new injections into the economy, regardless of their public or private origin, get ‘multiplied’ producing ripple effects in many sectors that in the aggregate seem to go, in fact, beyond and above the value of those injections.

Several considerations are however in order. Firstly, the origin of any injection is relevant but, usually, it is conveniently forgotten or omitted. New private injections can only come from consumers (for final consumption), firms (for investment), or external agents (for exports). But both consumers and firms are always subjected to some kind of budget constraints in the domestic economy and so one must wonder where those injections come so easily from. Exports, on the contrary, can be a source

of unrestricted new injections. Secondly, public injections in the form of new expenditure from the government need to be financed. There are three options here. One is financing by increasing taxes, but then this will negatively affect private agents' budget constraints and their spending decisions for consumption and investment, effectively 'crowding out' in some degree the expansionary effect of the public injection. A second option is debt financing borrowing from the savings of private agents, and again this may 'crowd out' private investment demand. Finally, the government may decide to finance a new policy by way of reshuffling its own budget constraint, i.e. more butter and fewer guns.

There are therefore two polar cases to be considered as far as the government expenditure is concerned: the standard one whereby any new expenditure is somehow materialised without regard to its financing ('free lunch' scenario) and a more realistic one that incorporates the fact that new expenditure may actually need a reallocation of current patterns ('down-to-earth' scenario). In the first case only output effects are considered whereas in the second case both output and expenditure substitution effects are incorporated. The results can be strikingly different as we will see in the next Section where we use recent Spanish data to illustrate.

## **2. Analysis and Results**

Let us consider an  $N$  sectors economy for which an  $N \times N$  multiplier matrix  $\mathbf{M}$  can be computed. This matrix can be obtained from a simple Leontief model, an extended SAM model or even from the Jacobian of a more sophisticated general equilibrium model, but we do not need the underlying details to be made explicit here. The only requirement is for matrix  $\mathbf{M}$  to have constant coefficients or, more simply, coefficients that are not affected by government policy decisions. The usual

interpretation for  $\mathbf{M}$  applies. A new policy injection of 1 unit (of euro worth) in sector  $j$  by the government will produce an overall increase of  $m_{ij}$  units (or euros' worth) of new output in sector  $i$ . Because matrix  $\mathbf{M}$  incorporates both direct and indirect effects, and possibly induced effects too, the initial unitary direct inflow 'multiplies' itself into a value greater than 1, i.e.  $\sum_{i=1}^N m_{ij} > 1$ . The composite multiplier value is therefore  $\mu_j = \sum_{i=1}^N m_{ij} - 1$ . This is the usual story if new government injections are fully unrestricted ('free lunch' scenario) and no countervailing effects are into play.

Let us consider now a scenario where any new expenditure is constrained by the aggregate current level of government outlays. Therefore a new injection targeted in sector  $j$  needs a reduction of expenditure in the remaining sectors  $i \neq j$  so as to maintain the aggregate budget constraint of the government. Let  $\langle \delta \rangle$  be a redistribution scheme that guarantees the upholding of the budget constraint, i.e.  $\delta_j + \sum_{i \neq j}^N \delta_i = 0$ , for an injection of type  $j$ . There are in fact many such redistribution schemes but we will illustrate here considering a simple homothetic pattern that assumes a reduction which is proportionate to initial outlays (we refer to it as the 'down-to-earth' scenario). In this case for  $\delta_j > 0$  we will have appropriate values  $\delta_i < 0$  for  $i \neq j$ . The initial, unrestricted, output effects  $m_{ij}$  are now conditioned by the countervailing spending substitution effects induced by  $\langle \delta \rangle$ . Under the constancy assumption for  $\mathbf{M}$ , if we denote by  $x_{ij}$  the change in output  $i$  caused by an injection in sector  $j$  restricted by scheme  $\langle \delta \rangle$  we will have:

$$x_{ij} = \delta_j \cdot m_{ij} + \sum_{k \neq j} \delta_k \cdot m_{ik}$$

Notice that  $x_{ij}$  is a measure of the opportunity costs, in terms of output, associated to implementation of the budget scheme  $\langle \delta \rangle$ . Also notice that  $x_{ij} = m_{ij}$  for

the usual, unrestricted ‘free lunch’ scenario. The new budget constrained, composite multiplier value under the reshuffling rule  $\langle \delta \rangle$  is therefore given by:

$$\hat{\mu}_j = \sum_{i=1}^N x_{ij} - 1 = \sum_{i=1}^N \delta_j \cdot m_{ij} + \sum_{i=1}^N \sum_{k \neq j} \delta_k \cdot m_{ik} - 1 \quad (1)$$

Unlike the always positive unrestricted multiplier value  $\mu_j$ , now the new multiplier value  $\hat{\mu}_j$  can be seen to be of any sign, positive or negative, depending on the relative strength of the overall positive output effects ( $\sum_{i=1}^N \delta_j \cdot m_{ij}$ ) versus the negative substitution effects ( $\sum_{i=1}^N \sum_{k \neq j} \delta_k \cdot m_{ik}$ ). The positive output effect would coincide, for unitary injections, with the standard multiplier value in the unrestricted scenario. Additionally, expression (1) can be further transformed and decomposed into two distinct components: the ‘within’ sector budget constrained multiplier effect ( $\sum_{i=1}^N \delta_i \cdot m_{ji}$ ) and the ‘external’ to the recipient sector effects ( $\sum_{k \neq j} \sum_{i=1}^N \delta_i \cdot m_{ki}$ ). In other words:

$$\hat{\mu}_j = \sum_{i=1}^N \delta_i \cdot m_{ji} + \sum_{k \neq j} \sum_{i=1}^N \delta_i \cdot m_{ki} - 1 \quad (2)$$

It can also be seen that both components, the ‘within’ sector effect and the derived ‘external’ output effects in the other production units contain positive and negative substitution effects.

Table 1 illustrates the empirical computations of the budget constrained multipliers with recent 2004 data for the Spanish economy for unitary new injections, i.e.  $\delta_j = 1$ , with the necessary adjustments in the rest of outlays to keep the size of the government programs constant. The sectoral break-down corresponds to an aggregation of the symmetric Spanish input-output table for 2004 describing seventeen production units. Down the first column of the Table, we show the gross multiplier value when sectors sequentially receive new unitary injections subject to

the budget restrictions. The remaining four columns of this table describe the multiplier components under the two alternative decompositions presented in expressions (1-2). As can be asserted from these results, and in sharp contrast with the standard approach, multiplier values are systematically smaller than one and, furthermore, can be positive or negative. In fact, for the current database a majority of the multipliers turn out to be negative. The second column picks up the positive output effects—which coincide with the standard, unrestricted multiplier values. They are all greater than unity, as predicted from the unrestricted multiplier model. Substitution effects, however, counteract the positive output effects and the overall sign of the restricted multiplier will depend on the relative strengths of the positive and negative effects. Lastly, the decomposition in the last two columns provides information on the relative strength of internal versus external effects. This could be useful as a categorization of sectors in terms of the transferral of multiplier effects, once the restrictions are accounted for and internalized, through circuits of influence.

### **3. Concluding remarks**

There is of course a whole range of possibilities between the unrestricted values for government multipliers and their restricted counterparts. Both cases correspond to polar situations but in terms of economic ‘realism’ the restricted ones should at least receive as much attention as the unrestricted ones. They describe a set of circumstances that correspond to tight budget situations that had better be put into the picture when assessing expenditure policies. In actual policy practice, governments do have some leeway to implement expenditure policies that are not fully constrained. The truth is probably ‘convex’ in terms of the effects of government spending and has to be found somewhere in-between the restricted and unrestricted multipliers. They

give us bounds for the effective effects of expenditure policies that are in need of a much more careful, detailed and systematic evaluation if we want to provide good and sound policy advice.

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**Table 1:** Unrestricted and Budget Constrained Multipliers and Decompositions.

Database: the Spanish Symmetric Input-Output Table for 2004.

<b>PRODUCTION UNITS</b>	$\hat{\mu}_j + 1$	Positive Output and Negative Substitution Effects Decomposition		Within and Out-sector Effects Decomposition	
		$\sum_{i=1}^N \delta_j \cdot m_{ij}^*$	$\sum_{i=1}^N \sum_{k \neq j} \delta_k \cdot m_{ik}$	$\sum_{i=1}^N \delta_i \cdot m_{ji}$	$\sum_{k \neq j} \sum_{i=1}^N \delta_i \cdot m_{ik}$
Primary sector	-0,118	1,705	-1,823	1,061	-1,180
Extraction of Anthracite, Coal, Lignite and Peat	-0,446	1,376	-1,822	1,002	-1,448
Extraction of Crude, Natural Gas, Uranium and Thorium	-0,802	1,019	-1,822	0,987	-1,789
other extraction industries	-0,078	1,743	-1,822	1,005	-1,084
Coke, Refinery and Nuclear Fuels	-0,104	1,718	-1,823	1,055	-1,160
Production and Distribution of Electricity	0,278	2,099	-1,820	1,163	-0,885
Production and Distribution of Gas	-0,057	1,764	-1,823	0,997	-1,054
Water Sector	0,036	1,858	-1,822	1,003	-0,964
Food, beverage, tobacco, textile and leather products	0,395	2,190	-1,795	1,248	-0,853
Other industries	0,276	2,097	-1,821	1,272	-0,995
Chemistry Industry, rubber and plastic industry	-0,045	1,777	-1,823	1,206	-1,252
Manufacturing industry	0,067	1,882	-1,814	1,267	-1,200
Construction sector	0,622	2,346	-1,724	1,440	-0,817
Commercial and Transport Activities	-0,029	1,796	-1,825	1,117	-1,147
Market services	-0,198	1,676	-1,875	1,077	-1,275
Market R&D	-0,052	1,770	-1,822	1,004	-1,056
Public Sectors	-0,403	1,515	-1,919	1,039	-1,442

\*The positive output effects coincide with the standard multiplier value under the unrestricted scenario:  $\mu_j + 1$ .