Sectoral Composition and Macroeconomic Dynamics

Jaime Alonso-Carrera
Jordi Caballé
Xavier Raurich

April 2011

Barcelona GSE Working Paper Series

Working Paper nº 545

Sectoral composition and macroeconomic dynamics*

Jaime Alonso-Carrera
Departamento de Fundamentos del Análisis Económico and RGEA
Universidade de Vigo

Jordi Caballé
Unitat de Fonaments de l'Anàlisi Economica
Universitat Autònoma de Barcelona, MOVE, and Barcelona GSE

Xavier Raurich
Departament de Teoria Econòmica and CREB
Universitat de Barcelona

April 5, 2011

Abstract

We analyze the transitional dynamics of a model with heterogeneous consumption goods. In this model, convergence is driven by two different forces: the typical diminishing returns to capital and the sectoral change inducing the variation in relative prices. We show that this second force affects the growth rate if the two consumption goods are not Edgeworth independent and if these two goods are produced with technologies exhibiting different capital intensities. Because the aforementioned dynamic sectoral change arises only under heterogeneous consumption goods, the transitional dynamics of this model exhibits striking differences with the growth model with a single consumption good. We also show that these differences in the transitional dynamics can give raise to large discrepancies in the welfare cost of shocks between the economy with a unique consumption good and the economy with multiple consumption goods.

JEL classification codes: O41, O47.

Keywords: multi-sector growth models, transitional dynamics, consumption growth.

*Financial support from the Government of Spain through grants ECO2009-09847, ECO2009-06953 and ECO2008-02752; PR2009-0162 and SM2009-0001; the Generalitat of Catalonia through the Barcelona GSE Research Network and grants SGR2009-00350 and SGR2009-1051; and the Xunta de Galicia through grant 10PXIB300177PR is gratefully acknowledged. Alonso-Carrera also thanks the Research School of Economics (Australian National University) for its hospitality. Caballé also thanks the financial support from the ICREA Academia program. The paper has benefited from comments by participants in the World Congres of the Econometric Society (Shanghai), DEGIT (Los Angeles), ESEM (Milan), SAEe (Granada), ASSET (Padova), Australasian Workshop in Macroeconomic Dynamics and seminars in UPV (Bilbao), IAE-CSIC (Barcelona), Australian National University, University of Melbourne, Monash University, Macquarie University, University of Wollongong, National University of Ireland (Maynooth), Geary Institute (UCD), Universidad de Murcia, Universidad de las Islas Baleares, Universitat Rovira i Virgili and Universidade do Minho.

Corresponding Author: Jordi Caballé. Universitat Autònoma de Barcelona. Departament d'Economia i d'Història Econòmica. Edifici B. 08193 Bellaterra (Barcelona). Spain. E-mail: jordi.caballe@uab.es

1. Introduction

The literature on economic growth has generally taken the standard model of capital accumulation with a single final consumption good as the canonical framework to study the growth pattern of an economy. In particular, this model has been widely used for the analysis of the dynamic effects of shocks in fundaments and for the normative and positive characterization of macroeconomic policy. The main feature of this model is that the economic dynamics is fully driven by the evolution of the return to capital. As the seminal contribution of Ramsey (1928) stated, the optimal intertemporal allocation of consumption and investment leads the growth of consumption expenditure to depend on the net interest rate only. In this paper, we claim that this result does not apply for models that allow for several heterogeneous consumption goods. More precisely, the aforementioned benchmark model may be unsuitable to study the dynamic effects of those shocks featuring a permanent effect on the sectoral composition of consumption. To illustrate this point, we characterize the properties of the transitional dynamics of a growth model where individuals derive utility from consumption of two heterogeneous goods.

The recent growing interest for the analysis of structural change and international trade has made popular the use of multi-sector growth models with heterogeneous consumption goods.[1] A typical by-product of this literature is that the dynamics of the aggregate variables are identical to those predicted by the model with a single consumption good: the growth rate of consumption expenditure only depends on the marginal product of capital. According to this result, the process of convergence would be only determined by the return to capital with independence of the number of consumption goods. We argue instead that this isomorphism between the two types of models is a consequence of some restrictive assumptions imposed on these multi-sector models, namely, either the utility function is additively separable in the amounts of consumption of the different goods or these consumption goods are produced by means of technologies with identical capital intensities. By relaxing these assumptions, we first prove that the rate of growth of expenditure depends not only on the interest rate, but also on the growth rate of relative prices of goods. Therefore, the process of convergence in a general multi-sector growth model is driven by two forces: the return to capital and the dynamic adjustment of relative prices arising from the change in the sectoral composition. Our main purpose in this paper is to analyze how the presence of the later force modifies the dynamic behavior of the economy.

The effect of the interest rate on consumption growth is measured by the intertemporal elasticity of substitution (IES, henceforth). On the contrary, the growth effect of the variation in the relative price of goods is jointly determined by the IES and the Edgeworth elasticity between goods.[2] Therefore, the relative importance of these two forces in determining the intertemporal allocation of consumption expenditure crucially depends on this Edgeworth elasticity. In fact, we show that the growth rate of relative prices increases (decreases) the rate of growth of expenditure when the

[1] Examples include, among many others, Echevarria (1997), Konsamung et al. (2001), Ngai and Pissarides (2008), or Perez and Guillo (2010).

[2] The Edgeworth elasticity between two goods is defined as the elasticity of the marginal utility of one good with respect the consumption level of the other good.

two consumption goods are Edgeworth substitute (complementary). The intuition of this result is that the increase in the relative price of one good reduces the demand of this good, which increases (decreases) the demand of the corresponding substitute (complementary) goods.

As was mentioned before, previous multi-sector growth models found in the literature impose assumptions that prevent the relative prices of consumption goods from displaying the aforementioned growth effects. Some authors assume that the consumption goods are Edgeworth independent (see, e.g., Echevarria, 1997; Laitner, 2000; or Perez and Guillo, 2010) or use a technology yielding a constant relative price between goods (Kongsamunt et al., 2001; or Steger, 2006). Two exceptions are the multi-sector growth models considered in Rebelo (2001) and Ngai and Pissarides (2007). In the later model, the growth of prices affects the rate of growth of expenditure. However, since the capital intensities are identical across sectors, the variation in prices arises only from exogenous, unbiased technological changes in sectoral productivities. On the contrary, in our model the dynamics of prices is endogenous as we consider different capital intensities across sectors. In this way, the dynamic adjustment of prices directly determines the response of the economy to changes in fundamentals. While Rebelo (2001) does also consider a model where prices are endogenous and the goods are not Edgeworth independent, he does not analyze the corresponding transitional dynamics. Therefore, to the best of our knowledge, the present paper is the first analyzing the transitional dynamics of a growth model with heterogeneous consumption goods when the two aforementioned forces driving the transition are operative.

In order to study the transitional dynamics when the variation of prices displays the aforementioned growth effects, we analyze a three sector growth model with a homothetic utility function whose argument is a composite good combining two different consumption goods. These goods are produced by means of constant returns to scale technologies that use physical and human capital as inputs. Furthermore, technologies exhibit different capital intensities across sectors. As was explained before, the last assumption makes the relative price between the two consumption goods not constant along the transition. To gain some intuition about this result, suppose that human capital becomes relatively scarcer than physical capital. Then, the consumption good produced in the physical capital intensive sector becomes less costly and the relative price of this consumption good decreases. Note that if the consumption goods were produced with technologies with the same capital intensity then the imbalances between the two capital stocks would not modify the relative price between these consumption goods. Finally, we assume in our analysis that the two consumption goods are not Edgeworth independent so that this dynamic adjustment of the relative price results in a modification of the growth rate of consumption expenditure.

As occurs in multi-sector growth models with two types of capital, the transitional dynamics will be governed by the imbalances between the two stocks of capital. However, the existence of two different forces governing the transition yields two interesting differences with respect to the transitional dynamics obtained in the standard growth model with a unique consumption good. First, in growth models with a unique consumption good, convergence in the consumption growth rate occurs from below (above) if the initial value of the ratio of physical to human capital is larger (smaller) than its stationary value. We will show that this behavior may be reversed by introducing heterogeneous consumption goods. In particular, we provide a condition that implies that convergence is from above when the initial value of the capital ratio is larger than its stationary value and from below otherwise. It should be noticed that, when this condition is satisfied, the initial effect on consumption growth of a shock in one of the capital stocks will be the opposite of the one obtained in a model with a single consumption good. As an example, consider an economy suffering a negative shock in human capital. Then, if there is a unique consumption good, this economy will experience a decrease in the growth rate of consumption. In contrast, in our model with heterogeneous consumption goods, the economy will display an increase in the growth rate of consumption expenditure.

Second, while the growth rate of consumption expenditure exhibits a monotonic behavior when the diminishing returns to capital is the only force governing the transition, it may exhibit instead a non-monotonic behavior in our model. Alvarez-Cuadrado et al. (2004) mention evidence of non-monotonic behavior of the consumption growth rate. Steger (2000), among others, has accounted for this non-monotonic behavior by means of the introduction of a minimum consumption level that makes preferences non-homothetic. In contrast, in our model the non-monotonic behavior is explained by the presence of the aforementioned two different forces acting on the transitional dynamics. In fact, the growth rate exhibits a non-monotonic behavior when these two forces exhibit opposite growth effects.

The two differences we have just mentioned imply that the patterns of growth along the transition crucially depend on the parameters values of our model. More precisely, we show that the capital intensity ranking across sectors and the value of the Edgeworth elasticity determine the nature of the transition. We will simulate the economy in order to analyze the transitional dynamics and show that the two forces governing the rate of growth of expenditure have opposite growth effects. As a consequence, in the simulated economy this growth rate exhibits a non-monotonic convergence towards the steady-state and, moreover, the sign of the growth effects of a shock in one of the capital stocks depends on the value of the Edgeworth elasticity. We also use the simulated model to study the growth and welfare effects of technological shocks. This analysis allows us to compare the effects of these shocks in the economy with a single consumption good with the effects in the economy with heterogeneous consumption goods. Regarding the welfare cost of shocks, we show that they will strongly depend on the sectoral composition of the composite consumption good when these shocks cause large effects on the unitary cost of this composite good. These large effects occur when we consider shocks that modify the long-run value of relative prices. In this case, the shocks result in a large distortion in the intratemporal decision concerning the sectoral composition of consumption, which translates in turn into sizeable additional welfare effects. We then conclude that the existing literature, by considering specific models where the force linked to the dynamics of the relative prices between goods is not operative, obtain biased results about the effects of those shocks.

The paper is organized as follows. Section 2 presents the ingredients of the model. Sections 3 and 4 characterize the equilibrium dynamics of relative prices and of the growth rate of expenditure, respectively. Section 5 develops the numerical analysis concerning the transitional dynamics and the effects of technological shocks. Section 6 presents some concluding remarks, while the Appendix contains the proofs of all the results of the paper.

2. The economy

Let us consider a three-sector growth model in which the output in each sector is obtained from combining amounts of two types of capital, k and h, which we dub physical and human capital, respectively. The first sector produces an amount y_1 of commodity using the following production function:

$$y_1 = A_1 (s_1 k)^{\alpha} (u_1 h)^{1-\alpha} = A_1 u_1 h z_1^{\alpha},$$

where s_1 and u_1 are the shares of physical and human capital allocated to this sector, $z_1 = s_1 k / u_1 h$ is the physical to human capital ratio, $A_1 > 0$ is the sectoral total factor productivity (TFP), and $\alpha \in (0,1)$ measures the intensity of physical capital in this sector. We interpret this sector as the one producing manufactures and assume that the commodity y_1 can be either consumed or added to the stock of physical capital. The law of motion of the physical capital stock is thus given by

$$\dot{k} = A_1 u_1 h z_1^{\alpha} - c_1 - \delta k, \tag{2.1}$$

where c_1 is the amount of good y_1 devoted to consumption, and $\delta \in [0,1]$ is the depreciation rate of the physical capital stock. To ease the notation we omit the time argument of all the variables. The second sector produces a consumption good y_2 by means of the production function

$$y_2 = A_2 (s_2 k)^{\beta} (u_2 h)^{1-\beta} = A_2 u_2 h z_2^{\beta}, \tag{2.2}$$

where s_2 and u_2 are the shares of physical and human capital allocated to this sector, respectively, $z_2 = s_2 k / u_2 h$ is the physical to human capital ratio, $A_2 > 0$ is the sectoral TFP, and $\beta \in (0,1)$ measures the intensity of physical capital in this sector. We interpret this sector as the one producing food and services devoted to consumption, such as cultural or entertainment goods. Thus, the output of this sector can only be devoted to consumption, which we denote by c_2, so that $y_2 = c_2$ in equilibrium. Finally, the third sector produces a commodity y_3 by means of the production function

$$y_3 = A_3 \left[(1 - s_1 - s_2) k \right]^{\pi} \left[(1 - u_1 - u_2) h \right]^{1-\pi} = A_3 (1 - u_1 - u_2) h z_3^{\pi},$$

where $z_3 = (1 - s_1 - s_2) k / (1 - u_1 - u_2) h$ is the physical to human capital ratio, $A_3 > 0$ is the sectoral TFP, and $\pi \in (0,1)$ measures the intensity of physical capital in this sector. This commodity is devoted exclusively to increase the stock of human capital and, therefore, we identify this sector with the education sector. The accumulation of the human capital stock is thus given by

$$\dot{h} = A_3 (1 - u_1 - u_2) h z_3^{\pi} - \eta h, \tag{2.3}$$

where $\eta \in [0,1]$ is the depreciation rate of human capital.

The economy is populated by an infinitely lived representative agent characterized by the instantaneous utility function

$$U(c_1, c_2) = \frac{\left(c_1^{\theta} c_2^{1-\theta} \right)^{1-\sigma}}{1 - \sigma}, \tag{2.4}$$

where the parameter $\theta \in [0,1]$ measures the share of good c_1 in the composite consumption good, $m = c_1^{\theta} c_2^{1-\theta}$, and $\sigma > 0$ is the (constant) elasticity of the marginal utility of this composite consumption good. Note that this utility function is homothetic, strictly concave, and increasing. The representative agent is endowed with k units of physical capital and h units of human capital. Let w be the rate of return on human capital (i.e., the real wage per unit of human capital) and r the rate of return on physical capital (i.e., the real interest rate). We assume perfect sectoral mobility so that the wage and interest rate are independent of the sector where the representative agent allocates the units of physical and human capital. Therefore, the budget constraint of the consumer is given by

$$wh + rk = (c_1 + pc_2) + (I_k + p_h I_h)\,, \tag{2.5}$$

where p is the relative price of good c_2 measured in units of good c_1, p_h is the relative price of human capital measured in units of physical capital (or consumption good c_1). Finally, I_h and I_k are the gross investment in human and physical capital, respectively,

$$I_k = \dot{k} + \delta k, \tag{2.6}$$

and

$$I_h = \dot{h} + \eta h. \tag{2.7}$$

3. Dynamics of relative prices

In this section we first solve the problems of consumers and firms and then we derive the system of differential equations characterizing the competitive equilibrium. We use these equations to find the long-run equilibrium and to study how the introduction of a second consumption good modifies the equilibrium dynamics of relative prices.

The representative agent maximizes

$$\int_0^{\infty} e^{-\rho t} U(c_1, c_2) dt, \tag{3.1}$$

subject to (2.5), (2.6), and (2.7), where $\rho > 0$ is the subjective discount rate. The solution to this optimization problem is given by the following equations derived in the Appendix:

$$p = \left(\frac{1-\theta}{\theta}\right)\left(\frac{c_1}{c_2}\right), \tag{3.2}$$

$$\frac{\dot{p}_h}{p_h} = r - \frac{w}{p_h} + \eta - \delta, \tag{3.3}$$

$$\frac{\dot{c}_1}{c_1} = \frac{r-\rho-\delta}{\sigma} - \left[\frac{(1-\sigma)(1-\theta)}{\sigma}\right]\left(\frac{\dot{p}}{p}\right), \tag{3.4}$$

and the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t} p^{(1-\theta)(\sigma-1)} c^{-\sigma} k = 0, \tag{3.5}$$

and

$$\lim_{t\to\infty} e^{-\rho t} p^{(1-\theta)(\sigma-1)} c^{-\sigma} h = 0. \tag{3.6}$$

Equation (3.2) tells us that the price ratio p is equal to the marginal rate of substitution between the two consumption goods. Equation (3.3) shows that the growth of the price p_h is determined by the standard non-arbitrage condition between the investments in physical and human capital. Finally, equation (3.4) characterizes the growth rate of consumption good c_1. From this equation we can easily obtain the growth rate of total consumption expenditure, which is defined as $c = c_1 + pc_2$. Note that equation (3.2) implies that

$$c = \frac{c_1}{\theta} = \frac{pc_2}{1-\theta}. \tag{3.7}$$

Hence, the growth rate of consumption expenditure c coincides with the growth rate of c_1 (the consumption expenditure in the good y_1, which is the numeraire). We then obtain from (3.4) that

$$\frac{\dot{c}}{c} = \frac{r-\rho-\delta}{\sigma} - \left[\frac{(1-\sigma)(1-\theta)}{\sigma}\right]\left(\frac{\dot{p}}{p}\right). \tag{3.8}$$

Equation (3.8) tells us that the growth rate of consumption expenditure is driven by both the interest rate and by the change in the relative price of the two consumption goods. The effect of a rise in the interest rate on the rate of growth of c is summarized by the intertemporal elasticity of substitution $IES = 1/\sigma$. On the contrary, the growth effect of a rise in the growth rate of the relative price is jointly determined by the IES and Edgeworth elasticity (i.e., the elasticity of the marginal utility of the consumption good c_1 with respect to the consumption good c_2) which is given by

$$\varepsilon \equiv -c_2\left(\frac{\partial^2 U/\partial c_1 \partial c_2}{\partial U/\partial c_1}\right) = -(1-\sigma)(1-\theta).$$

By using (3.8), we see that the growth rate of the relative price p directly affects the growth rate of consumption expenditure c when $\varepsilon \neq 0$, i.e., when the two consumption goods are not Edgeworth independent. Under the instantaneous utility function (2.4), the Edgeworth elasticity ε is determined by the parameters σ and θ. In particular, the two consumption goods are Edgeworth independent when $\sigma = 1$ because in this case the utility function is additively separable in the two goods c_1 and c_2. The previous literature on multisectoral growth models commonly uses a logarithmic specification for preferences and this explains why it does not obtain the growth effect of the variation in relative prices.

The intuition on the aforementioned growth effect of the dynamic adjustment of relative prices is as follows. Equation (3.8) is the Euler equation equating the market return from investing one unit of the numeraire y_1 and the growth of the marginal utility arising from consuming one additional unit of this commodity. When the two consumption goods are Edgeworth independent, then the marginal utility of one consumption good does not depend on the other consumption good. In this case, the growth rate of total consumption expenditure only depends on the interest rate. In contrast, when the two consumption goods are not Edgeworth independent a change in the consumption of good c_2 alters the marginal utility of consumption good c_1. Thus, in this case, the growth of the marginal utility of one good will depend on the growth of both consumption goods. As follows from equation (3.2), the consumption of these goods depends on the relative price. Actually, the concavity of the utility function implies that an increase in the relative price p reduces the amount consumed of good c_2. This reduction implies an increase (reduction) in the marginal utility of consumption good c_1 and in the amount of good c_1 consumed when the two goods are Edgeworth substitute (complementary).[3]

After having presented the equilibrium conditions on the demand side of our economy, we will now move to the supply side and we will characterize how the dynamics of relative prices is determined. This dynamics depends on the technologies used by the different sectors and on the market structure. In particular, firms maximize profits in each sector and, thus, the competitive factors payment must satisfy simultaneously the following equations:

$$r = \alpha A_1 z_1^{\alpha-1}, \tag{3.9}$$

$$r = p\beta A_2 z_2^{\beta-1}, \tag{3.10}$$

$$r = p_h \pi A_3 z_3^{\pi-1}, \tag{3.11}$$

$$w = (1-\alpha) A_1 z_1^{\alpha}, \tag{3.12}$$

$$w = p(1-\beta) A_2 z_2^{\beta}, \tag{3.13}$$

and

$$w = p_h (1-\pi) A_3 z_3^{\pi}. \tag{3.14}$$

Combining the system of equations (3.9) to (3.14) when $\alpha \neq \pi$, we obtain

$$z_i = \psi_i p^{\frac{1}{\alpha-\beta}}, \quad \text{for } i = 1, 2, 3, \tag{3.15}$$

where

$$\psi_1 = \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{1-\beta}{\alpha-\beta}} \left(\frac{A_2}{A_1}\right)^{\frac{1}{\alpha-\beta}},$$

$$\psi_2 = \left(\frac{\beta}{1-\beta}\right)\left(\frac{1-\alpha}{\alpha}\right)\psi_1, \tag{3.16}$$

and

$$\psi_3 = \left(\frac{\pi}{1-\pi}\right)\left(\frac{1-\alpha}{\alpha}\right)\psi_1. \tag{3.17}$$

From the previous set of equilibrium conditions we obtain the following well-known result, which has important consequences for the equilibrium dynamics of our economy.

Proposition 3.1. *The relative price p of consumption goods is constant over time for all initial values of the capital ratio $z = k/h$ if and only if at least one of the following conditions holds: (i) $\alpha = \beta$, (ii) $\alpha = \pi$.*

[3] Note that the effect of relative prices on expenditure growth appears because only the good c_1 can be used as physical capital. If the equilibrium mix of the two consumptions goods could be devoted to investment in physical capital, then the relative price would not affect the growth rate of consumption expenditure c (see Acemoglu and Guerrieri, 2008).

Let us first consider the condition $\alpha = \beta$, which means that the two consumption goods c_1 and c_2 are produced by means of technologies with the same capital intensity. We see that under this condition, equation (3.16) implies that $\psi_2 = \psi_1$ when $\alpha \neq \pi$ and then, from equation (3.15), we get $z_1 = z_2$. Therefore, by combining equations (3.9) and (3.10), it follows that the relative price between the two consumption goods remains constant and equal to $p = \frac{A_1}{A_2}$. This obviously means that the growth rate of consumption expenditure only depends on the interest rate (see equation (3.8)). Therefore, the transitional dynamics of our model when $\alpha = \beta$ coincides with the transitional dynamics of the two-sector growth model with a unique consumption good, which was analyzed by Uzawa (1965) and Lucas (1988).

Let us now consider the condition $\alpha = \pi$. Under this condition the two capital goods k and h are produced by means of technologies with the same capital intensity. Observe that in this case conditions (3.9), (3.11), (3.12) and (3.14) imply that $z_1 = z_3$ and, thus, the relative price between the two capital stocks is constant and given by $p_h = \frac{A_1}{A_3}$. Equation (3.3) implies that the wage to interest rate ratio w/r remains constant when p_h is constant. Then, from combining (3.9) and (3.12) we immediately see that z_1 is constant when p_h is constant. Therefore, both the interest rate r and z_2 are constant as follows from (3.9) and (3.11). Finally, equation (3.10) shows that in this case the relative price p between the two consumption goods remains constant. In fact, it is easy to see that the three sectors are using Ak technologies when $\alpha = \pi$.[4] Therefore, the transition dynamics in this case coincides with the transition in Ak growth models with several consumption goods (see, e.g., Rebelo, 1991).

We have just established the conditions under which the growth rate of consumption expenditure depends not only on the interest rate, but also on the growth rate of the relative price p. This new dependence requires that the consumption goods be not Edgeworth independent and to be produced by means of technologies with different capital intensities. These arguments then explain why the previous multi-sector growth models do not find a direct effect of relative prices on consumption growth. Some of these models consider logarithmic preferences so that they implicitly assume that consumption goods are Edgeworth independent. Other models assume that consumption goods are produced with technologies that share the same capital intensity. Obviously, in this case the variation of relative prices could still affect directly the growth rate of consumption expenditure under exogenous and biased technological change, that is, when the sectoral TFPs grow at exogenous growth rates that are different across sectors (see, e.g., Ngai and Pissarides, 2007). However, if technologies exhibit different capital intensities, the relative price between consumption goods appear as an endogenous channel for the propagation of shocks in fundamentals. In the rest of the paper, we will illustrate the consequences of this endogenous mechanism and, hence, we will assume that $\alpha \neq \pi$ and $\alpha \neq \beta$.

Note that relative prices would also affect the growth rate of consumption expenditure when $\beta = \pi$, that is, when services and human capital are produced with

[4] Note that the technology that produces commodity y_1 can be rewritten as follows $y_1 = \widehat{A}_1 u_1 h$, where $\widehat{A}_1 = A_1 (z_1^*)^\alpha$ is constant. The technology that produces commodity y_2 can be rewritten as $y_2 = \widehat{A}_2 u_2 h$, where $\widehat{A}_2 = A_2 z_2^\beta$ is constant and, finally, the technology that produces commodity y_3 can be rewritten as $y_3 = \widehat{A}_3 (1 - u_1 - u_2) h$, where $\widehat{A}_3 = A_3 (z_1^*)^\alpha$ is constant. Since goods y_1 and y_2 are produced with linear technologies, their relative prices are constant and given by $p = \frac{\widehat{A}_1}{\widehat{A}_2}$.

the same technology.[5] Moreover, this growth effect of prices would also hold if we had assumed a unique capital stock. In this latter case, the dynamics of prices would be driven by the accumulation of the capital stock, whereas in our two-capital model they are driven by the relative accumulation of these two capital stocks.

The dynamics of the relative price p can be easily derived. To this end, we use equations (3.10), (3.11), and (3.15), to obtain

$$p_h = (\varphi p)^{\frac{\alpha-\pi}{\alpha-\beta}}, \tag{3.18}$$

where

$$\varphi = \frac{\beta A_2 (\psi_2)^{\beta-1}}{\pi A_3 (\psi_3)^{\pi-1}}.$$

This previous relationship between the relative prices implies that

$$\frac{\dot{p}}{p} = \left(\frac{\alpha-\beta}{\alpha-\pi}\right)\left(\frac{\dot{p}_h}{p_h}\right). \tag{3.19}$$

Equation (3.19) shows that the relationship between the growth rate of the relative prices p and p_h only depends on the capital intensity ranking among sectors. Therefore, in our economy the dynamics of both prices p and p_h are fully determined by the non-arbitrage condition (3.3) and equation (3.19).

We next characterize the shares of physical and human capital in each sector. To this end, we consider the aggregate ratios $z = k/h$ and $q = c/k$. Then, we combine (2.2) with (3.2) and (3.7) to get

$$u_2 = (1-\theta)\left(\frac{qz}{pA_2 z_2^{\beta}}\right), \tag{3.20}$$

and we use the definition of z_2 to obtain

$$s_2 = (1-\theta)\left(\frac{q z_2^{1-\beta}}{pA_2}\right). \tag{3.21}$$

Next, we combine the definitions of z_1 and z_3 to get

$$u_1 = \frac{(1-u_2)\, z_3 - (1-s_2)\, z}{z_3 - z_1}, \tag{3.22}$$

and

$$s_1 = \left(\frac{z_1}{z}\right)\left(\frac{(1-u_2)\, z_3 - (1-s_2)\, z}{z_3 - z_1}\right). \tag{3.23}$$

We proceed to characterize the growth rate of the two capital stocks. For that purpose, we use (2.1) to obtain

$$\frac{\dot{k}}{k} = \frac{A_1 u_1 z_1^{\alpha}}{z} - \theta q - \delta, \tag{3.24}$$

[5] Note that if $\beta = \pi$ then the consumption good c_2 and human capital are produced by using technologies with the same capital intensity. In this case, the two relative prices satisfy $p = \frac{A_3}{A_2} p_h$.

and from (2.3) we get

$$\frac{\dot{h}}{h} = A_3\left(1 - u_1 - u_2\right) z_3^{\pi} - \eta. \tag{3.25}$$

Finally, we obtain the equations that characterize the equilibrium path. First, we combine (3.3), (3.9), (3.12), (3.15), (3.18) and (3.19) to obtain

$$\frac{\dot{p}}{p} = \left(\frac{\alpha-\beta}{\alpha-\pi}\right)\left[\alpha A_1 \psi_1^{\alpha-1} p^{\frac{\alpha-1}{\alpha-\beta}} - (1-\alpha)\left(\frac{A_1\psi_1^{\alpha}}{\varphi^{\frac{\alpha-\pi}{\alpha-\beta}}}\right) p^{\frac{\pi}{\alpha-\beta}} + \eta - \delta\right] \equiv \kappa\left(p\right). \tag{3.26}$$

Note that the right hand side of the previous equation can be written as a function $\kappa\left(\cdot\right)$ of the relative price p.

We combine (3.8) with (3.9), (3.15) and (3.19) to obtain

$$\frac{\dot{c}}{c} = \nu\left(p\right) + \left(\frac{\varepsilon}{\sigma}\right)\kappa\left(p\right) \equiv \gamma\left(p\right) \tag{3.27}$$

where

$$\nu\left(p\right) \equiv \frac{\alpha A_1 z_1^{\alpha-1} - \rho - \delta}{\sigma}. \tag{3.28}$$

Note that the function $\nu\left(\cdot\right)$ defined in (3.28) only depends on relative price p as follows from (3.15). Equation (3.27) shows the two forces governing the transition and the parameters measuring the intensity of these two forces. In particular, the net balance between the two forces depends crucially on the elasticity ε of the marginal utility of consumption good c_1 with respect to the consumption good c_2, which determines in turn the nature of the transitional dynamics of the economy.

Combining (3.24) and (3.25), we get

$$\frac{\dot{z}}{z} = \frac{A_1 u_1 z_1^{\alpha}}{z} - \theta q + \eta - \delta - A_3\left(1 - u_1 - u_2\right) z_3^{\pi}, \tag{3.29}$$

and combining (3.24) and (3.27) we obtain

$$\frac{\dot{q}}{q} = \nu\left(p\right) + \left(\frac{\varepsilon}{\sigma}\right)\kappa\left(p\right) - \frac{A_1 u_1 z_1^{\alpha}}{z} + \theta q + \delta. \tag{3.30}$$

The dynamic equilibrium is thus characterized by a set of paths $\{p, z, q\}$ such that, given the initial value z_0 of the physical to human capital ratio, solves the equations (3.26), (3.29), and (3.30), and satisfies (3.15), (3.20), (3.21), (3.22) together with the transversality conditions (3.5) and (3.6). As in the standard two-sector growth model, there is a unique state variable z and the transition will be governed by the imbalances between the two capital stocks.

We define a steady-state or balanced growth path (BGP, henceforth) equilibrium as an equilibrium path along which the ratios z and q and the relative prices p and p_h remain constant. The following result characterizes the steady-state equilibrium:

Proposition 3.2. *The unique steady-state value p^* of the relative price solves $\kappa\left(p^*\right) = 0$ and the two capital stocks and consumption expenditure grow at the same constant growth rate $g^* \equiv \nu\left(p^*\right)$. Moreover, the steady-state value z^* of the physical to human capital ratio and the steady-state value q^* of the consumption expenditure to capital ratio are unique.*

Note that neither the steady-state price level p^* nor the growth rate g^* depend on the parameter θ measuring the relative weight of the consumption goods in the utility function. As in the standard endogenous growth model with a single consumption good, the steady-state values of these two variables only depend on the technology. In contrast, the steady-state value of the ratios z^* and q^* depend on the utility parameter θ.[6] On the one hand, as θ increases, the weight of consumption good c_1 in the utility function increases. Since the relative price of goods does not depend on θ, any variation in this parameter will affect the ratios $\frac{c_1}{k}$ and $\frac{pc_2}{k}$ in the opposite direction and, therefore, the final effect on q^* is ambiguous. On the other hand, the change in the patterns of consumption due to an increase in θ also affects the steady-state value z^* of the physical to human capital ratio. In particular, when the sector producing the consumption good c_1 is relatively more (less) intensive in physical than the sector producing the consumption good c_2, a rise in θ increases (decreases) the demand of physical capital relative to the demand of human capital and, hence, the ratio z^* increases (decreases) with θ.[7]

4. Dynamics of consumption expenditure

Let us now analyze how the behavior of the growth rate of consumption expenditure during the transition is affected by the existence of two heterogeneous consumption goods.

Proposition 4.1. *The steady-state equilibrium is locally saddle-path stable.*

This result implies that the dynamic equilibrium is unique, which allows us to make comparisons between growth patterns and to analyze the asymptotic speed of convergence, i.e., the speed of convergence around the steady-state (or long-run) equilibrium. Concerning the asymptotic speed of convergence, in the proof of Proposition 4.1 it is shown that if $\alpha > \pi$ then the asymptotic speed of convergence is equal to $p^* \kappa' (p^*)$ and is independent of the utility parameter θ. In contrast, if $\alpha < \pi$ then the asymptotic speed of convergence depends on θ. In this case, the equilibrium value p of the relative price of good c_2 is always equal to its steady state value so that it is constant along the transition towards the steady state. This implies that the growth rate of consumption expenditure is constant and equal to $v(p^*)$ along the transition when $\alpha < \pi$. Therefore, there is no transition in terms of the growth rate of consumption expenditure in this case. Following Perli and Sakellaris (1998), we will impose from now on the standard assumption that the production of consumption good c_1 (or of physical capital k) is more intensive in physical capital than the production of human capital, $\alpha > \pi$, so that the rate of growth of consumption expenditure will exhibit transitional dynamics.[8]

Assumption A. $\alpha > \pi$.

[6] The exact expressions for z^* and q^* are given in the Appendix.

[7] The proof of these results is available upon request.

[8] The role of the factor intensity ranking in the transitional dynamics of multi-sector growth models is extensively discussed in Bond et al. (1996).

We proceed with the analysis of the two aforementioned forces governing the transition in this economy. It is important to note that this dynamic analysis is global in the sense that the conclusions obtained from this analysis hold even when the equilibrium path is far from the steady state. As shown in equation (3.27), those two forces are summarized by the terms $v(p)$ and $\kappa(p)$, which are functions of the relative price of goods. The function $v(p)$ collects the growth effect of an increase in the interest rate and $\kappa(p)$ is a measure of the growth effect of a variation in the relative price.[9] As the two forces only depend on the relative price, the properties of the transition will depend on the slope of the stable manifold relating the price p with the state variable z as this manifold determines the dynamic adjustment of relative prices along the transition. We proceed to characterize this dynamic adjustment. To this end, we denote the stable manifold relating p and z by $p = P(z)$. Note that the function $P(\cdot)$ is defined on the domain $(0, \infty)$.

Lemma 4.2. *If $\alpha > (<)\beta$ then $P'(z) > (<)0$. Moreover, the range of the function $P(\cdot)$ is $(0, \infty)$.*

The intuition behind this lemma is straightforward. Let us assume that $z_0 < z^*$. In this case, h_0 is large in comparison to k_0 and then the relative price of human capital p_h will be lower than its long-run value and, therefore, this price increases along the transition. This implies that the relative cost of producing the good relatively more intensive in physical capital will decrease along the transition. As firms behave competitively, this means that the relative price of consumption goods p dynamically evolves in such a way that $\kappa(p) > (<)\, 0$ when $\alpha > (<)\beta$. Obviously, the converse is true when $z_0 > z^*$. In any case, we finally conclude that the slope of the stable manifold relating relative price p and capital ratio z is strictly positive (negative) if $\alpha > (<)\beta$. In addition, by using identical arguments, we can directly see that the range of equilibrium values of p is the interval $(0, \infty)$. If the value of the physical to human capital ratio z tends to zero, then human capital becomes an abundant resource whose price tends to zero. Symmetrically, when the value of the capital ratio z tends to infinity physical capital becomes so abundant that its price tends to zero, that is, the relative price p_h of human capital in terms of physical capital tends to infinity.

Proposition 4.3. *The physical to human capital ratio z exhibits a globally monotonic transition.*

The result in Proposition 4.3 allows us to characterize analytically the global transitional dynamics of the growth rate of consumption expenditure $\gamma = \dot{c}/c$. We should first mention that the coexistence of two forces determining the transition implies that the dynamic path of this variable may be non-monotonic when these two forces have opposite growth effects. To show these non-monotonic dynamics, we use (3.27), (3.15) and (3.26) to obtain the following derivative of the rate of growth of consumption expenditure with respect to the capital ratio z:

$$\frac{\partial \gamma}{\partial z} = \left[\frac{(1-\alpha)\, A_1 \psi_1^{\alpha-1} P(z)^{\frac{\beta-1}{\alpha-\beta}}}{\alpha - \beta} \right] \Omega(z)\, P'(z), \tag{4.1}$$

[9] In the proof of Proposition 3.2 we have shown that $\kappa(p)$ is decreasing when $\alpha > \pi$, whereas it is immediate to see from (3.15) that $v(p)$ is a decreasing (increasing) function when $\alpha > (<)\, \beta$.

where

$$\Omega(z) = \left(\frac{\pi \chi \psi_1}{\varphi^{\frac{\alpha - \pi}{\alpha - \beta}}} \right) P(z)^{\frac{1-\alpha+\pi}{\alpha-\beta}} - \alpha \left(\frac{1}{\sigma} - \chi \right), \tag{4.2}$$

and

$$\chi \equiv - \left(\frac{\varepsilon}{\sigma} \right) \left(\frac{\alpha - \beta}{\alpha - \pi} \right). \tag{4.3}$$

According to Lemma 4.2, the function $\Omega(\cdot)$ is strictly increasing in z. Note that if $\chi \in (0, 1/\sigma)$ then there exists a unique value $\overline{z}$ of z, such that $\Omega(z) > (<) 0$ when $z > (<) \overline{z}$. The following result uses these arguments and Proposition 4.3 to provide conditions for the existence of non-monotonic behavior and to characterize the global transition dynamics of the growth rate $\dot{c}/c$ of consumption expenditure:

Proposition 4.4.

(a) *If* $\chi \leq 0$, *the time path of the growth rate of consumption expenditure is strictly decreasing (increasing) when* $z_0 < z^*$ $(z_0 > z^*)$.

(b) *If* $\chi \in (0, 1/\sigma)$ *and* $\overline{z} < z^*$, *the time path of the growth rate of consumption expenditure strictly decreases when* $z_0 > z^*$, *monotonically increases when* $z_0 \in (\overline{z}, z^*)$, *and exhibits a non-monotonic behavior when* $z_0 < \overline{z}$.

(c) *If* $\chi \in (0, 1/\sigma)$ *and* $\overline{z} \geq z^*$, *the time path of the growth rate of consumption expenditure strictly decreases when* $z_0 < z^*$, *strictly increases when* $z_0 \in (z^*, \overline{z})$, *and exhibits a non-monotonic behavior when* $z_0 > \overline{z}$.

(d) *If* $\chi > 1/\sigma$, *the time path of the growth rate of consumption expenditure is strictly increasing (decreasing) when* $z_0 < z^*$ $(z_0 > z^*)$.

The results in Proposition 4.4 imply that we can distinguish four types of transition in this economy depending on the value of χ, which is jointly determined by the IES, the Edgeworth elasticity, and the capital intensity ranking across sectors. These different types of transition are represented in Figure 1, where the growth rate $\gamma = \dot{c}/c$ of consumption expenditure is displayed as a function of the capitals ratio z. In particular, Panel (i) shows the growth rate of consumption expenditure when $\chi = 0$, i.e., when this rate is not affected by the growth of the relative price p. In this case, as in the Uzawa-Lucas model, the growth rate of consumption expenditure is a monotonic function that decreases when $z_0 < z^*$ and increases when $z_0 > z^*$ (see Mulligan and Sala-i-Martín (1993) and Caballe and Santos (1993) for a complete analysis of the transitional dynamics of the Uzawa-Lucas model). In fact, the condition $\chi = 0$ holds when the production structure of the economy coincides with the one in the Uzawa-Lucas model $(\alpha = \beta)$, there is a unique consumption good $(\theta = 1)$, or the two consumption goods are Edgeworth independent $(\sigma = 1)$. Moreover, the same type of convergence holds when $\chi < 0$. However, when $\chi \in (0, 1/\sigma)$ the two forces governing the transition have opposite growth effects and the patterns of growth are different from the ones in the Uzawa-Lucas model. On the one hand, the growth rate of consumption expenditure exhibits a non-monotonic behavior when the initial value of the capital ratio is sufficiently far from its stationary value. On the other hand, as shown in Panels (ii) and (iii), we must distinguish two types of transition, depending on the relationship between $\overline{z}$ and z^*. Interestingly, if $\overline{z} < z^*$ the convergence is from below when $z_0 < z^*$ and from above otherwise. Therefore, in this case, the conclusions from convergence are reversed due to the effect of the growth of prices. As shown in Panel (iv), this reversed transition also arises when $\chi > 1/\sigma$. To see the implications of this reversed transition, suppose that the economy suffers a decrease in the stock of physical capital so that the ratio z of physical to human capital goes down. This reduction implies an initial increase in the growth rate of consumption expenditure in a model with a single consumption good, whereas it could result in an initial reduction in the growth rate $\dot{c}/c$ in our general model.

[Insert Figure 1]

5. Numerical Analysis

The results in Proposition 4.4 imply that the transition crucially depends on both the value of the parameters and the initial conditions. We next discuss which is the most plausible type of transition, as well as how quantitatively important are the differences in the transitional dynamics across different parametric scenarios. We address these two issues by following some numerical simulations. In order to fit our model with data, we will consider that the commodity y_1 corresponds to manufactures, the consumption good c_2 is composed of primary goods and services, and h is human capital. We use the labor income shares in the primary, manufacturing and service sectors, and the sectoral composition of GDP reported by Echevarria (1997) for the US economy to set $\alpha = 0.34$ and $\beta = 0.49$.[10] We should mention here the long-standing debate about the capital intensity ranking among sectors producing consumption goods. A crucial point in this discussion is whether housing is considered as a service. If this is the case, since the stock of physical capital embeds residential capital, the service sector will be relatively more physical capital intensive than the manufacturing sector. This is the view that we adopt in our numerical analysis. We take the average share of physical capital in the final education output estimated by Perli and Sakellaris (1998) and we set $\pi = 0.18$. We assume $\delta = 0.056$ to replicate the fact that the investment in physical capital amounts to 7.6% of its stock. Moreover, Perli and Sakellaris (1998) pointed out that the estimates of the depreciation rate η vary widely. We choose $\eta = 0.025$, which corresponds with the low end of the range. We set arbitrarily $A_1 = A_2 = 1$, and set $A_3 = 0.0851$ to generate a long-run interest rate net of depreciation equal to 5.6%. The parameter θ measures the fraction of total consumption expenditures devoted to consumption goods produced in the manufacturing sector. According to Kongsamunt et al. (2001), this fraction was roughly constant during the last century and equal to 0.3. We then select this value for the parameter θ. Finally, we use the long-run growth rate g^* and the Edgeworth elasticity ε as a target to pin down the value of the other two preference parameters σ and ρ. As the Proposition 4.4 shows, the Edgeworth elasticity crucially determines the nature of the transition since it governs the relationship between the

[10] The value of β is a weighted average of the capital income shares in the agriculture (0.71%) and service (0.49%) sectors in the US and the weights are the fraction of GDP in agriculture (1.7%) and in services (72.2%). These weights are obtained from NIPA.

two dynamic forces for a given capital intensity ranking across sectors and expenditure share θ (see the expression of χ in equation (4.3)). We then consider three different values for ε : 0.7, 0.95 and 1.2. We set the values of σ and ρ that jointly replicate those values for ε and a long-run growth rate equal to 2%. In the low elasticity economy we obtain $\sigma = 2$ and $\rho = 0.016$, whereas we get $\sigma = 2.357$ and $\rho = 0.0089$ for the economy with $\varepsilon = 0.95$, and finally we get $\sigma = 2.7143$ and $\rho = 0.0017$ for the high elasticity economy. Observe that this calibration implies reasonable values for the IES: 0.5, 0.4243 and 0.3684.

We next simulate the response of each of the three parameterized economies to imbalances in the capital ratio, i.e., when $z_0 \neq z^*$. In order to show how important is the growth effect of price variation, we compare the response of these baseline economies with the response of the corresponding economy with a unique consumption good. In order words, we compare the dynamic behaviors of the economy with $\theta = 0.3$ and the economy with $\theta = 1$.

5.1. Transitional dynamics

The expression of χ in equation (4.3) implies that it takes positive values when $\alpha < \beta$ and $\varepsilon > 0$. Thus, the value of χ is positive under our empirically plausible values of the fundamental parameters. In this case, the two aforementioned forces governing the transition display opposite growth effects. In our numerical examples, we show that, if the force associated with the variation of prices is the dominating then the transition is going to be different from that of models with a single consumption good. Figures 2, 3 and 4 show that this is the case when the Edgeworth elasticity is high (i.e., when the value of σ is high). These figures show the dynamic response of some relevant variables to imbalances in the capital ratio. In particular, each of these figures contains six panels. Panels (i), (iv), (v) and (vi) display, respectively, the growth rate of consumption expenditure, the growth rate of GDP, the relative price of consumption goods and the speed of convergence of the state variable z as a function of the deviations of the capital ratio with respect to its stationary value. Note that, following Reiss (2000),we define the non-asymptotic speed of convergence of the ratio of capitals as $\dot{z}\,/(z - z^*)$. Panels (ii) and (iii) display, respectively, the time path of the growth rate of consumption expenditure when the state variable is initially below its long-run value and when it is initially above. Furthermore, all panels compare the transitional dynamics of the baseline economy with heterogeneous consumption goods (continuous line) with the transition in an equivalent economy with a unique consumption good, i.e., with $\theta = 1$ (dashed line). We parametrize the counterfactual economy with $\theta = 1$ so that it replicates the same empirical facts used to calibrate our benchmark economy with two heterogenous consumption goods. We observe that the differences between the two economies under consideration are quite significant in the three parametric scenarios. Hence, the direct effect of the price adjustment on the intertemporal allocation of consumption expenditure also has important quantitative consequences for macroeconomic dynamics.

[Insert Figures 2, 3 and 4]

The first three panels of Figures 2, 3 and 4 illustrate numerically the results in

Proposition 4.4. We observe that the dynamic adjustment of consumption expenditure is non monotonic under the higher values of σ in the economy with two consumption goods ($\theta = 0.3$). Moreover, when σ is high, the introduction of heterogeneous consumption goods reverses the transition. This occurs because σ determines the value of the Edgeworth elasticity ε provided a value θ for the consumption share. When the Edgeworth elasticity ε is high, the growth effect of changes in the interest rate is low in comparison with the growth effects of changes in the growth of the relative price. In this case, even if the initial values of the economy are close to the corresponding steady-state values, the transition is different from the one arising in an economy where the transition is governed only by the diminishing returns to capital.

The significant effects of the price variation on the intertemporal allocation of consumption expenditure and savings have important quantitative consequences for the dynamic behavior of the other macroeconomic variables. As an illustration, Figures 2, 3 and 4 shows that the paths of the GDP growth rate, the relative price of goods and the speed of convergence also depend on the value of the parameter θ. This parameter measures the weight of the human capital intensive good in the composite consumption good. Thus, a reduction in θ makes the composite good more intensive in physical capital, which explains the results displayed in these three figures. Intuitively, there are two non-competing ways of increasing in relative terms the stock of the scarce capital and, thus, of adjusting the imbalances in the capital ratio: (i) To decrease the accumulation of the relatively abundant capital; and (ii) to decrease the consumption expenditure. The more intensive in physical capital is the composite consumption good, the larger is the relative importance of the second way when $z < z^*$. The growth rate of GDP is then a decreasing function of θ if $z < z^*$. On the contrary, the more intensive in physical capital is the composite good, the larger is the relative importance of the first procedure when $z > z^*$. This implies that the growth rate of GDP is an increasing function of θ if $z > z^*$. Therefore, the dynamic adjustment of any imbalance in the capital ratio is faster when the composite consumption good is more physical intensive. This fact explains why the non-asymptotic speed of convergence always decreases with θ (see Panel (vi)).

We finally illustrate the implications of the differences in the transitional dynamics across the alternative parametric scenarios by computing the welfare effects of the initial imbalances in the capital ratio.[11] Table 1 reports the time-invariant increase (decrease) in consumption required to compensate the welfare costs (gains) of having an initial capital ratio smaller (larger) than the stationary ratio. We again show the results for our baseline economy with $\theta = 0.3$ and for the economy with a single consumption good (i.e., $\theta = 1$). The last column of this table compares the differences in welfare costs between these two economies and shows that they are large. In particular, the welfare cost is approximately 20% larger in the economy with two consumption goods, whereas the welfare gain is 17% larger. These results follow again from the fact that the composite consumption good in the economies with a low value of θ is more intensive in physical capital. Obviously, in these economies the unitary cost of the composite good is more sensitive to the relative endowment of physical capital.

[11] As in Lucas (1987), we measure the welfare cost of the imbalances in the capital ratio by the percentage increase in composite consumption good m necessary to obtain the same discounted sum of utility as in the situation where the capital ratio is initially equal to its stationary value.

[Insert Table 1]

By repeating the previous numerical exercises we obtain that the reported differences in welfare between the two economies are extremely robust to both the size of shocks and the value of σ. The insignificant effect of σ is explained by analyzing the dynamic behavior of the composite good $m = c_1^{\theta} c_2^{1-\theta}$, which is the fundamental variable for welfare analysis. By using conditions (3.2), (3.7) and (3.27), we obtain

$$\frac{\dot{m}}{m} = \left(\frac{1}{\sigma}\right) \left[\alpha A_1 z_1^{\alpha-1} - \rho - \delta - (1-\theta)\,\kappa\,(p)\right]. \tag{5.1}$$

Obviously, the growth rate of m also depends on the forces driving the intertemporal allocation of consumption expenditure c: the diminishing returns to capital and the growth rate of prices. However, observe that the net effect of these two forces does not depend in this case on the value of σ. This occurs because the direct effect of the variation in the relative price on the growth rate of m does not depend on the Edgeworth elasticity ε. This then explains the insignificant effect of σ on the welfare comparison between the economy with $\theta = 0.3$ and the economy with $\theta = 1$.

Next, we complement the analysis in this subsection by studying how the response of the economy to shocks in fundamentals depends on the value of θ. Given the previous conclusion about the independence of welfare effects on σ, we will only present the results for the case of $\sigma = 2$, which is associated with the value $\varepsilon = 0.7$ for the Edgeworth elasticity.

5.2. Comparative dynamics and welfare

We now proceed to study the dynamic adjustments and the welfare costs from two different shocks: a sectoral biased technological shock and a sectoral unbiased technological shock. For that purpose, we assume that the economy is initially in a BGP and, unexpectedly, one of these shocks is introduced in a permanent basis. The aim of this analysis is to compare the effects of these shocks in the baseline economy with two consumption goods ($\theta = 0.3$) with the effects in the economy with a unique consumption good ($\theta = 1$).

We first analyze the effects of a biased technological shock that consists of reducing the TFP of the manufacturing sector A_1 by a 15%. We explain these effects by using Figure 5, which summarizes how the economy responds to the shock; and Table 2, which provides the welfare cost of this shock. Observe that the rate of growth of expenditure initially suffers a strong decline and then it increases until it converges to its new long-run, which is smaller than the one before the shock. In the economy with a single consumption good, the growth rate only depends on the interest rate, which instantaneously falls due to the technological shock. This reduces investment and, as a consequence, the stock of physical capital declines during the transition. The reduction in the stock of physical capital implies that the interest rate increases during the transition. Note that the behavior of the interest rate fully explains the initial strong reduction in the rate of growth of expenditure and also its posterior increase during the transition. On the contrary, in the economy with two consumption goods, the rate of growth of expenditure also depends on the growth of the relative price p of consumption goods. This price decreases instantaneously because the shock directly affects the sector producing manufactures, whereas it increases during the transition because the continuous reduction in the stock of physical capital rises the cost of producing services, which is relatively intensive in this capital. This behavior of the relative price p has a positive effect on the rate of growth of expenditure as the Edgeworth elasticity in the benchmark economy satisfies $\varepsilon > 0$. The presence of this positive growth effect in the economy with two consumption goods explains both the smaller initial reduction in the rate of growth of expenditure and its larger values along the transition.

[Insert Figure 5 and Table 2]

The first row of Table 2 reports the welfare cost of the considered permanent reduction in the TFP of the manufacturing sector. The main result is that the welfare cost is a 45.6% larger in the economy with a unique consumption good. This large difference arises from the fact that the response of the composite good m to the shock is larger, the larger is the share θ of manufactures in the composite good. Figure 5 illustrates the dynamic adjustment of that good. Panel (iii) reports deviations of the composite good to physical capital ratio m/k from its initial stationary value. From this panel we conclude that the initial reduction in the value of m is smaller in the economy with $\theta = 0.3$. The intratemporal substitution between goods in this economy reduces the impact of the shock in the level of the composite good. On the contrary, as Panel (iv) shows, the growth rate of composite good increases during the transition and, what is more interesting, it is smaller in the economy with $\theta = 0.3$ due to the negative effect of the increase in the relative price p (see equation (5.1)). However, the larger recovery of the amount of the composite good in the economy with $\theta = 1$ is not enough to outweigh its larger instantaneous reduction. In other words, the initial difference in the response of the composite good in the two economies explains the larger welfare cost in the economy with $\theta = 1$.

Figure 6 displays the dynamic effects of an unbiased technological shock consisting of a 5% decrease in the TFP in each sector. We observe that the dynamic adjustment in this case is qualitatively similar to the one accruing from a biased technological shock when $\theta = 1$. Moreover, the differences between the two economies are now quantitatively insignificant because of the smaller incidence of the price adjustment on the rate of growth of expenditure. Since each sectoral TFP falls in the same proportion, the responses of the relative price p and of consumption composition are both smaller when the shock is unbiased. This explains the small discrepancies between the two economies under consideration concerning the dynamic response of the rate of growth of expenditure and the level of composite consumption. Finally, this implies that the welfare cost associated with the unbiased shock is very similar in the two economies. As the second row of Table 2 shows, the welfare cost in the economy with $\theta = 0.3$ is less than 2% larger than in the economy with $\theta = 1$.

At this point, we should also mention that the differences in the effects of the unbiased shock between the two economies only arise because the depreciation rates of both capital stocks are different, which makes the shock distort the optimal allocation of capital among sectors. If $\delta = \eta$, then the stationary value of p is not affected by the unbiased shock as it can be derived from (3.15) and (3.26). Moreover, in this case we obtain that the welfare cost in the two economies would coincide. As can be seen from Figure 6, even if some differences arise in the dynamic adjustment of both the growth rate of expenditure and the amount of composite good between the two economies, the larger recovery of the composite good in the economy with $\theta = 1$ will fully offset its larger instantaneous reduction. Therefore, in spite of displaying identical welfare costs, the time-path of the welfare cost associated with a shock is different across the two economies even if the technological shock is unbiased. We can thus conclude that the discrepancy in the welfare cost of shocks between the two economies under consideration only arises when these shocks have permanent effects on the relative prices and on the sectoral composition of consumption in the economy with two goods.

[Insert Figure 6]

6. Concluding remarks

We have analyzed the transitional dynamics of an endogenous growth model with two consumption goods. We have shown that the growth rate of expenditure not only depends on the interest rate, but also on the growth rate of the relative price of consumption goods. Convergence in this case may be determined by two different forces: the diminishing returns to capital and the growth of prices. In particular, this result arises when the two consumption goods are not Edgeworth independent and the technologies producing the two consumption goods have different capital intensities. These growth effects of relative prices yield interesting differences with respect to the transitional dynamics obtained in the standard growth model with a unique consumption good. We illustrate these differences using a growth model with two capital stocks that we identify with human and physical capital. First, we show that in contrast with the standard growth model, convergence in the growth rate may occur from above if the initial value of the ratio of physical to human capital is larger than its stationary value and may occur from below otherwise. Second, we show that the growth rate of consumption expenditure may exhibit a non-monotonic behavior when the two aforementioned dynamic forces have opposite growth effects. These differences in the transition have other noteworthy implications.

First, economies with the same interest rate may exhibit different growth rates of consumption along the transition. Therefore, our model provides an additional explanation to the cross-country differences in the growth rates. Rebelo (1992) shows that the introduction of a minimum consumption requirement also implies that the growth rates do not equalize. This occurs because the minimum consumption makes preferences non-homothetic so that the *IES* is no longer constant along the transition. In this framework, convergence is driven by the interest rate and by the time-varying *IES*. More recently, Steger (2006) shows that, if there are heterogeneous consumption goods and a unique capital stock, then the *IES* is not constant and the growth rates do not equalize. Obviously, he derives this result when preferences are non-homothetic. In contrast, we show that, when there are heterogeneous consumption goods, the growth rates are different even with a constant *IES* because of the effect of the growth of the relative prices along the transition.

The previous remark can be illustrated in a different way. By combining (3.8), (3.3), (3.19) and (4.3) we obtain that the rate of growth of consumption expenditure satisfies

$$\frac{\dot{c}}{c} = \gamma\left(p_h\right) = \left(\frac{1}{\sigma} - \chi\right) r + \chi\left(\frac{w}{p_h}\right) - \left(\frac{\rho + \delta}{\sigma}\right) - \chi\left(\eta - \delta\right).$$

This equation shows that the rate of growth of total expenditure depends both on the interest rate and on the wage rate when $\chi \neq 0$. This implies that cross-country differences in the growth rates will also be explained by wage differentials when $\chi \neq 0$ (i.e., when there are several consumption goods that are Edgeworth dependent and produced by technologies with different capital intensity). Moreover, for values of χ close to the *IES*, interest rate differentials will not explain cross country differences in the growth rates.

According to our results, the welfare cost of shocks will also depend on the sectoral composition of the composite consumption good. The relationship between the welfare cost of shocks and the sectoral composition of consumption expenditure will be particularly strong when the shocks permanently modify the value of relative prices. In this case, the effect of these shocks on the cost of the composite consumption good will depend on its sectoral composition. We have shown that biased technological shocks that increase the gap between the return on physical and human capital cause large and permanent effects on prices. We have also shown that the welfare cost of these shocks depends on the intensity of the direct growth effect of dynamic price adjustment. Therefore, this growth effect of relative price is an unexplored channel affecting the persistence and propagation of shocks.

We summarize our analysis by saying that the results obtained in aggregate growth models with a single consumption good cannot be generalized to more disaggregated models with heterogeneous consumption goods. In these disaggregated models, the welfare costs of shocks depend on the value of the parameters measuring the sectoral composition of consumption and on the physical capital intensities of the sectors producing these consumption goods. Therefore, the empirical estimation of the sectoral composition parameters should be an important concern for future research on the assessment of the welfare cost of macroeconomic shocks.

A natural extension of our paper is to introduce a minimum consumption requirement in one of the consumption goods. The price of this good will be high in the initial stages of development since the minimum consumption requirement will induce a high marginal utility of this good. Then, as the economy develops, the price will fall sharply until convergence is attained. Therefore, it seems that the introduction of a minimum consumption may accelerate the change of prices and, hence, the introduction of this consumption requirement may increase the effect of the growth of the relative price on both the growth rate of consumption expenditures and on the welfare cost of shocks.

References

[1] Acemoglu, D. and Guerrieri V. (2008). "Capital Deepening and Nonbalanced Economic Growth," *Journal of Political Economy* 116, 467-498.

[2] Alvarez-Cuadrado, F., Monterio, G. and Turnovsky, S. (2004). "Habit Formation, Catching-up with the Joneses, and Economic Growth," *Journal of Economic Growth* 9, 47-80.

[3] Bond E., Wang P. and Yip C. (1996). "A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics," *Journal of Economic Theory* 68, 149-173.

[4] Caballé J. and Santos M. (1993). "On Endogenous Growth with Physical and Human Capital," *Journal of Political Economy* 101, 1042-1067.

[5] Echevarria, C. (1997). "Changes in Sectoral Composition Associated with Economic Growth," *International Economic Review* 38, 431-452.

[6] Kongsamunt, P., Rebelo, S. and Xie, D. (2001). "Beyond Balanced Growth," *Review of Economic Studies* 68, 869-882.

[7] Lucas, R. E. (1987). "Models of Business Cycles," Basil Blackwell.

[8] Lucas, R. (1988). "On the Mechanics of Economic Development," *Journal of Monetary Economics* 22, 3-42.

[9] Mulligan C. and Sala-i-Martín X. (1993). "Transitional Dynamics in Two-Sector Models of Endogenous Growth," *Quarterly Journal of Economics* 108, 737-773.

[10] Ngai, R. and Pissarides, C. (2007). "Structural Change in a Multi-sector Model of Growth," *American Economic Review* 97, 429-443.

[11] Perez, F. and Guillo, D. (2010). "Reexamining the Role of Land in Economic Growth," Manuscript.

[12] Perli R. and Sakellaris P. (1998). "Human Capital Formation and Business Cycle Persistence," *Journal of Monetary Economics* 42, 67-92.

[13] Ramsey, F.P. (1928). "A Mathematical Theory of Saving," *Economic Journal* 38, 543–559.

[14] Rebelo, S. (1991). "Long-run Policy Analysis and Long-run Growth," *Journal of Political Economy* 99, 500-521.

[15] Rebelo, S. (1992). "Growth in Open Economies," *Carnegie-Rochester Conference Series on Public Policy* 36, 5-46.

[16] Reiss, J. P. (2000). "On the Convergence Speed in Growth Models," FEMM Working. Paper 22/2000.

[17] Steger, T.M., (2000). "Economic Growth with Subsistence Consumption," *Journal of Development Economics* 62, 343-361.

[18] Steger, T.M., (2006). "Heterogeneous Consumption Goods, Sectoral Change and Economic Growth," *Studies in Nonlinear Dynamics and Econometrics* 10, No. 1, Article 2.

[19] Uzawa, H. (1965). "Optimum Technical Change in an Aggregative Model of Economic Growth," *International Economic Review* 60, 12-31.

A. Appendix

Solution to the consumer's optimization problem.

The Hamiltonian function associated with the maximization of (3.1) subject to (2.5), (2.6) and (2.7) is

$$\begin{aligned} H &= e^{-\rho t} U(c_1, c_2) + \\ &\quad \lambda (wh + rk - c_1 - pc_2 - I_k - p_h I_h) + \mu_1 (I_k - \delta k) + \mu_2 (I_h - \eta h), \end{aligned}$$

where λ, μ_1, and μ_2 are the co-state variables corresponding to the constraints (2.5), (2.6) and (2.7), respectively. The first order conditions are

$$e^{-\rho t} \left[\frac{\theta \left(c_1^{\theta} c_2^{1-\theta} \right)^{1-\sigma}}{c_1} \right] - \lambda = 0, \tag{A.1}$$

$$e^{-\rho t} \left[\frac{(1-\theta) \left(c_1^{\theta} c_2^{1-\theta} \right)^{1-\sigma}}{c_2} \right] - \lambda p = 0, \tag{A.2}$$

$$\lambda = \mu_1, \tag{A.3}$$

$$p_h \lambda = \mu_2, \tag{A.4}$$

$$\lambda r - \delta \mu_1 = -\dot{\mu}_1, \tag{A.5}$$

$$\lambda w - \eta \mu_2 = -\dot{\mu}_2. \tag{A.6}$$

Combining (A.1) and (A.2), we obtain (3.2) and

$$\frac{\dot{c}_2}{c_2} = \frac{\dot{c}_1}{c_1} - \frac{\dot{p}}{p}. \tag{A.7}$$

Using (A.3) and (A.4), we obtain

$$p_h \mu_1 = \mu_2,$$

which implies that

$$\frac{\dot{p}_h}{p_h} + \frac{\dot{\mu}_1}{\mu_1} = \frac{\dot{\mu}_2}{\mu_2},$$

and (3.3) follows from using (A.5) and (A.6). Combining (A.1), (A.3) and (A.5), we obtain

$$-r + \delta = -\rho + [(1-\sigma)\theta - 1] \left(\frac{\dot{c}_1}{c_1} \right) + (1-\sigma)(1-\theta) \left(\frac{\dot{c}_2}{c_2} \right),$$

and (3.4) follows from using (A.7). Finally, the transversality conditions (3.5) and (3.6) follow from combining (A.1) and (3.2).

Proof of Proposition 3.2. The uniqueness of p^* follows from the monotonicity of $\kappa(p)$, which can be shown using (3.26),

$$\kappa'(p) = \left[\frac{(1-\alpha) A_1 \psi_1^{\alpha-1} p^{\frac{\beta-1}{\alpha-\beta}}}{\pi - \alpha} \right] \left[\alpha + \left(\frac{\pi \psi_1}{\varphi^{\frac{\alpha-\pi}{\alpha-\beta}}} \right) p^{\frac{1-\alpha+\pi}{\alpha-\beta}} \right] > (<)\, 0 \text{ if } \alpha < (>)\, \pi,$$

and the fact that $\lim_{p\to 0}\kappa(p) = -\infty(\infty)$ and $\lim_{p\to\infty}\kappa(p) = \infty(-\infty)$ when $\alpha < (>)\,\pi$.

Combining (3.20), (3.21) and (3.22), we obtain

$$u_1 = \frac{z_3 - z}{z_3 - z_1} + \left(\frac{1-\theta}{\theta p A_2 z_2^{\beta}}\right)\left(\frac{z_2 - z_3}{z_3 - z_1}\right)\theta q z \tag{A.8}$$

and

$$1 - u_1 - u_2 = \frac{z - z_1}{z_3 - z_1} + \left(\frac{1-\theta}{\theta p A_2 z_2^{\beta}}\right)\left(\frac{z_1 - z_2}{z_3 - z_1}\right)\theta q z. \tag{A.9}$$

In a steady state, equations (3.25) and (3.24) simplify to

$$1 - u_1^* - u_2^* = \frac{g^* + \eta}{A_3\left(z_3^*\right)^{\pi}},$$

$$\frac{A_1 u_1^*\left(z_1^*\right)^{\alpha}}{z^*} - \theta q^* = g^* + \delta.$$

By using (A.8) and (A.9), the previous two equations can be rewritten as the following system of two equations:

$$z^* + \underbrace{\left(\frac{1-\theta}{\theta p^* A_2\left(z_2^*\right)^{\beta}}\right)}_{\phi_1}\left(z_1^* - z_2^*\right)\theta q^* z^* = \underbrace{\left(\frac{g^* + \eta}{A_3\left(z_3^*\right)^{\pi}}\right)\left(z_3^* - z_1^*\right) + z_1^*}_{\phi_2},$$

$$z_3^* + \left[\phi_1^*\left(z_2^* - z_3^*\right) - \frac{\left(z_3^* - z_1^*\right)}{A_1\left(z_1^*\right)^{\alpha}}\right]\theta q^* z^* = \underbrace{\left[\left(z_3^* - z_1^*\right)\left(\frac{g^* + \delta}{A_1\left(z_1^*\right)^{\alpha}}\right) + 1\right]}_{\phi_3} z^*.$$

The steady state values of $z^*$and q^* are the unique solution of this system of equations and they are equal to

$$z^* = \frac{\phi_1\phi_2\left(z_2^* - z_3^*\right) + \phi_1\left(z_1^* - z_2^*\right)z_3^* - \frac{\phi_2\left(z_3^* - z_1^*\right)}{A_1\left(z_1^*\right)^{\alpha}}}{\phi_1\left(z_2^* - z_3^*\right) + \phi_1\phi_3\left(z_1^* - z_2^*\right) - \frac{z_3^* - z_1^*}{A_1\left(z_1^*\right)^{\alpha}}},$$

and

$$q^* = \frac{\phi_2\phi_3 - z_3^*}{\theta\left[\phi_1\phi_2\left(z_2^* - z_3^*\right) - \frac{\phi_2\left(z_3^* - z_1^*\right)}{A_1\left(z_1^*\right)^{\alpha}} + \phi_1\left(z_1^* - z_2^*\right)z_3^*\right]},$$

where the steady-state values of z_i, $i = \{1, 2, 3\}$, satisfy $z_i^* = \psi_i\left(p^*\right)^{\frac{1}{\alpha-\pi}}$ as follows from (3.15).■

Proof of Proposition 4.1. Let J be the Jacobian matrix evaluated at the steady state of the system of differential equations formed by (3.26), (3.29) and (3.30),[12]

$$J = \begin{pmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial z} & \frac{\partial \dot{p}}{\partial q} \\ \frac{\partial \dot{z}}{\partial p} & \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial q} \\ \frac{\partial \dot{q}}{\partial p} & \frac{\partial \dot{q}}{\partial z} & \frac{\partial \dot{q}}{\partial q} \end{pmatrix},$$

where

$$\frac{\partial \dot{p}}{\partial p} = p\kappa'(p),$$

$$\frac{\partial \dot{p}}{\partial z} = 0,$$

$$\frac{\partial \dot{p}}{\partial q} = 0,$$

$$\frac{\partial \dot{z}}{\partial p} = z \left\{ \begin{array}{c} \underbrace{\left(\frac{A_1 z_1^{\alpha}}{z}\right)\left(\frac{\partial u_1}{\partial p}\right) + \left(\frac{A_1 u_1 \alpha z_1^{\alpha-1}}{z}\right)\left(\frac{\partial z_1}{\partial p}\right)}_{\epsilon_p} \\ -A_3 z_3^{\pi}\left[\frac{\partial(1-u_1-u_2)}{\partial p}\right] - A_3\left(1-u_1-u_2\right)\pi z_3^{\pi-1}\left(\frac{\partial z_3}{\partial p}\right) \end{array} \right\},$$

$$\frac{\partial \dot{z}}{\partial z} = z \left\{ \underbrace{-\frac{A_1 u_1 z_1^{\alpha}}{z^2} + \left(\frac{A_1 z_1^{\alpha}}{z}\right)\left(\frac{\partial u_1}{\partial z}\right)}_{\epsilon_z} - A_3 z_3^{\pi}\left[\frac{\partial\left(1-u_1-u_2\right)}{\partial z}\right] \right\},$$

$$\frac{\partial \dot{z}}{\partial q} = z \left\{ \underbrace{\left(\frac{A_1 z_1^{\alpha}}{z}\right)\left(\frac{\partial u_1}{\partial q}\right) - \theta}_{\epsilon_q} - A_3 z_3^{\pi}\left[\frac{\partial\left(1-u_1-u_2\right)}{\partial q}\right] \right\},$$

$$\frac{\partial \dot{q}}{\partial p} = q\left\{\left[\frac{\alpha\left(\alpha-1\right)A_1 z_1^{\alpha-2}}{\sigma}\right]\left(\frac{\partial z_1}{\partial p}\right) + \left(\frac{\varepsilon}{\sigma}\right)\kappa'(p) - \epsilon_p\right\},$$

$$\frac{\partial \dot{q}}{\partial z} = -q\epsilon_z,$$

and

$$\frac{\partial \dot{q}}{\partial q} = -q\epsilon_q.$$

The determinant of the Jacobian matrix is

$$Det\left(J\right) = \left(\frac{\partial \dot{p}}{\partial p}\right)\left[\left(\frac{\partial \dot{z}}{\partial z}\right)\left(\frac{\partial \dot{q}}{\partial q}\right) - \left(\frac{\partial \dot{z}}{\partial q}\right)\left(\frac{\partial \dot{q}}{\partial z}\right)\right] = zq\kappa'(p)\,pA_3 z_3^{\pi} M,$$

[12] In this proof all the variables are valued at the BGP equilibrium. To ease the notation, we omit the asterisk denoting the steady-state.

where

$$M=\epsilon_{q}\left[\frac{\partial\left(1-u_{1}-u_{2}\right)}{\partial z}\right]-\epsilon_{z}\left[\frac{\partial\left(1-u_{1}-u_{2}\right)}{\partial q}\right]=$$

$$=\left\{\begin{array}{c}\theta\left(\frac{\partial u_{1}}{\partial z}\right)-\left(\frac{A_{1}u_{1}z_{1}^{\alpha}}{z^{2}}\right)\left(\frac{\partial u_{1}}{\partial q}\right)\\ \\ -\left[\left(\frac{A_{1}z_{1}^{\alpha}}{z}\right)\left(\frac{\partial u_{1}}{\partial q}\right)-\theta\right]\left(\frac{\partial u_{2}}{\partial z}\right)+\left[-\frac{A_{1}u_{1}z_{1}^{\alpha}}{z^{2}}+\left(\frac{A_{1}z_{1}^{\alpha}}{z}\right)\left(\frac{\partial u_{1}}{\partial z}\right)\right]\left(\frac{\partial u_{2}}{\partial q}\right)\end{array}\right\}.$$

Using (3.20), (A.8) and (A.9), and after some algebra, M simplifies to

$$M=-\left(\frac{\theta}{z_{3}-z_{1}}\right)\left[\underbrace{1+\phi_{1}z_{2}\left(g^{*}+\delta\right)+\phi_{1}z_{1}\left(A_{1}z_{1}^{\alpha-1}-g^{*}-\delta\right)}_{N}\right].$$

Note that $N>0$ because

$$A_{1}z_{1}^{\alpha}-g^{*}-\delta=\frac{g^{*}\left(\sigma-\alpha\right)+\rho+\delta\left(1-\alpha\right)}{\alpha}>0,$$

where the inequality follows from the transversality condition, which implies that $\rho>\left(1-\sigma\right)g^{*}$. Thus, the determinant is given by

$$Det\left(J\right)=$$

$$-\left(\frac{\theta zq\kappa'\left(p\right)pA_{3}z_{3}^{\pi}}{z_{3}-z1}\right)\left\{1+\phi_{1}z_{2}\left(g^{*}+\delta\right)+\phi_{1}z_{1}\left[\frac{g^{*}\left(\sigma-\alpha\right)+\rho+\delta\left(1-\alpha\right)}{\alpha}\right]\right\}.$$

By using (3.15) and (3.17), we obtain that $z_{3}>(<)\,z_{1}$ when $\alpha<(>)\pi$ and, therefore, we derive from the proof of Proposition 3.2 that $\kappa'\left(p\right)>(<)\,0$ when $z_{3}>(<)\,z_{1}$. We then conclude that $Det\left(J\right)<0$. Next, we obtain the value of the trace,

$$Tr\left(J\right)=\frac{\partial\dot{p}_{h}}{\partial p_{h}}+\frac{\partial\dot{z}}{\partial z}+\frac{\partial\dot{q}}{\partial q}=$$

$$=\left\{\begin{array}{c}p\kappa'\left(p\right)+A_{1}z_{1}^{\alpha}\left(\frac{\partial u_{1}}{\partial z}\right)-\frac{A_{1}u_{1}z_{1}^{\alpha}}{z}\\ \\ -\left(\frac{A_{3}z_{3}^{\pi}}{z}\right)\left[\frac{\partial(1-u_{1}-u_{2})}{\partial z}\right]-q\left[\left(\frac{A_{1}z_{1}^{\alpha}}{z}\right)\left(\frac{\partial u_{1}}{\partial q}\right)-\theta\right]\end{array}\right\}.$$

Using (A.8) and (A.9), the trace simplifies, after some tedious algebra, to

$$Tr\left(J\right)=\alpha A_{1}\psi_{1}^{\alpha-1}p^{\frac{\alpha-1}{\alpha-\beta}}+\frac{\left(1-\alpha\right)A_{1}\psi_{1}^{\alpha}p^{\frac{\pi}{\alpha-\beta}}}{\varphi^{\frac{\alpha-\pi}{\alpha-\beta}}}-\left(g^{*}+\eta\right)-\left(g^{*}+\delta\right).$$

Making $\kappa\left(p\right)=0$, we obtain

$$Tr\left(J\right)=2\left(\alpha A_{1}\psi_{1}^{\alpha-1}p^{\frac{\alpha-1}{\alpha-\beta}}-g^{*}-\delta\right),$$

and, by using (3.27) at BGP, we derive

$$Tr\left(J\right)=2\left[\left(\sigma-1\right)g^{*}+\rho\right]>0,$$

as follows from the transversality condition.

Since the trace of J is positive and the determinant is negative, there exists a unique negative root and the equilibrium is saddle-path stable. When $\alpha > \pi$ the adjustment process of the relative price p is stable so that the negative root of the Jacobian J is $p\kappa'(p)$. Otherwise, the dynamic process of p is unstable. In this case, the relative price p instantaneously jumps to its stationary value, and the negative root of J is one of the roots obtained from the sub-system of differential equations formed by equations (3.29) and (3.30) with $p = p^*$ for all t.[13]■

Proof of Lemma 4.2. Equation (3.15) shows that all the physical to human capital ratios in the three sectors, z_1, z_2 and z_3, depend positively (negatively) on the relative price p when $\alpha > (<)\,\beta$. We can write the aggregate physical to human capital ratio $z = k/h$ as

$$z = \frac{k_1 + k_2 + k_3}{h_1 + h_2 + h_3}, \tag{A.10}$$

where k_i and h_i are the stocks of physical and human capital used in the production of good i, $i = \{1, 2, 3\}$. When all the ratios z_1, z_2 and z_3 vary in the same direction, the aggregate physical to human capital ratio z also varies in this direction. For instance, if all the ratios z_1, z_2 and z_3 rise, then the following relationship between the increments of the sectoral capital stocks must apply: $\Delta k_1 > \Delta h_1$, $\Delta k_2 > \Delta h_2$, and $\Delta k_3 > \Delta h_3$. Therefore,

$$\Delta k_1 + \Delta k_2 + \Delta k_3 > \Delta h_1 + \Delta h_2 + \Delta h_3.$$

Using the previous inequality in (A.10), and the dependence of the ratios z_1, z_2 and z_3 on relative price p, we obtain the monotonically increasing (decreasing) relationship between the aggregate physical to human capital ratio z and the relative price p of human capital along the stable manifold when $\alpha > (<)\,\beta$.

Note that equation (3.15) implies that $\lim_{p\to 0} z_i = 0\,(\infty)$ when $\alpha > (<)\,\beta$, with $z_i = k_i/h_i$, $i = \{1, 2, 3\}$. This means that either $\lim_{p\to 0} k_i = 0\,(\infty)$ or $\lim_{p\to 0} h_i = \infty\,(0)$ when $\alpha > (<)\,\beta$. In both cases, we will get that $\lim_{p\to 0} z = 0\,(\infty)$ if $\alpha > (<)\,\beta$. However, $\lim_{p\to\infty} z_i = \infty\,(0)$ when $\alpha > (<)\,\beta$, with $z_i = k_i/h_i$, $i = \{1, 2, 3\}$, which means that either $\lim_{p\to\infty} k_i = \infty\,(0)$ or $\lim_{p\to\infty} h_i = 0\,(\infty)$ when $\alpha > (<)\,\beta$. In both cases, we will get that $\lim_{p\to\infty} z = \infty\,(0)$ if $\alpha > (<)\,\beta$. Therefore, as the ratio z may take potentially any value in the interval $(0, \infty)$, the range of values of the price p along the stable manifold is also $(0, \infty)$.■

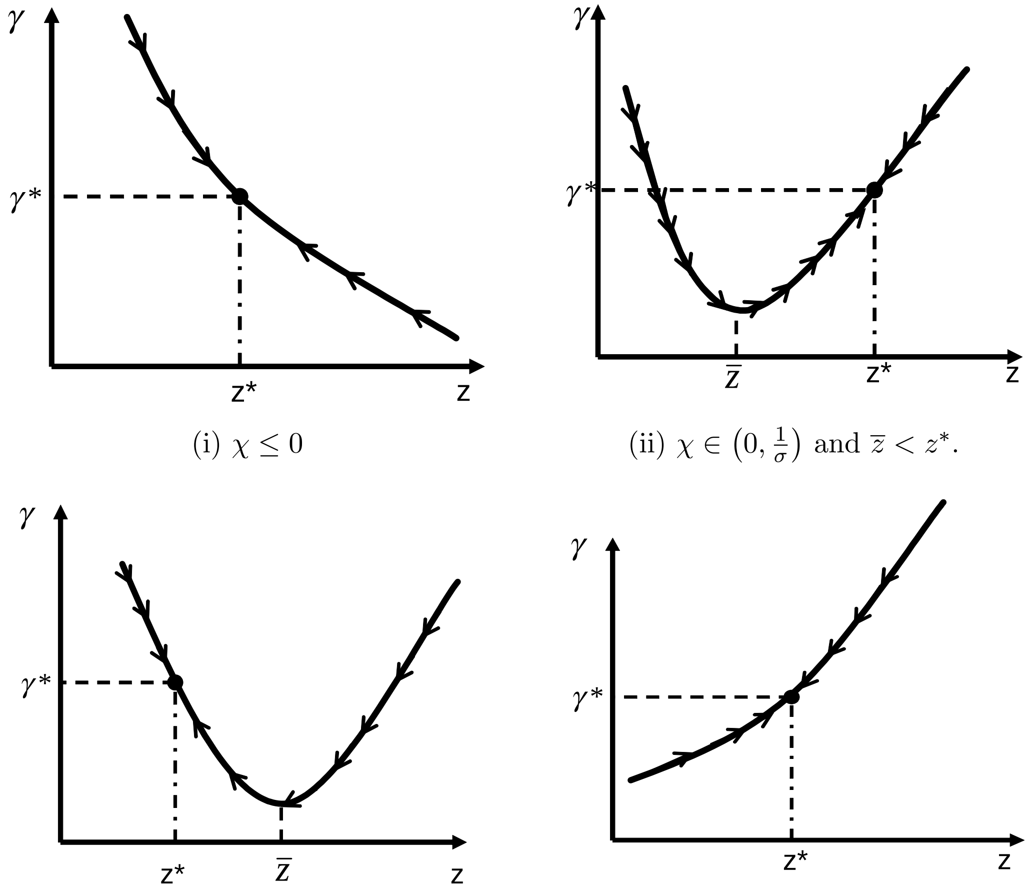
Proof of Proposition 4.3. In the proof of Proposition 4.1, we have shown that $\kappa'(p) < 0$ if $\alpha > \pi$. This means that relative prices exhibit a monotonic transition. In addition, Lemma 4.2 states that the stable manifold relating prices and the ratio of capitals is strictly monotone. This implies that the ratio z of capitals must also exhibit a monotonic behavior along the entire transition.■

Proof of Proposition 4.4. Given the sign of $P'(z)$ characterized by Lemma 4.2, we conclude from (4.1) that the growth rate of consumption expenditure γ is increasing

[13] Note that he dynamic system characterizing the equilibrium maintains the duality between quantities and prices that emerges in the Lucas-Uzawa-type growth models. More precisely, the dynamic adjustment of prices is determined independently of the quantities and is dictated by the capital intensity ranking across sectors.

(decreasing) when $\Omega(z) > (<) 0$. Therefore, the proposition directly follows from (4.2). Parts (a) and (d) follow since $\Omega(z) < 0$ when $\chi \leq 0$ and $\Omega(z) > 0$ when $\chi > \frac{1}{\sigma}$. For Part (b) note that we get $\Omega(z) > 0$ along the transition when $z_0 > z^*$ and $\Omega(z) < 0$ when $z_0 < \overline{z} < z^*$. In the first case, the rate of growth of consumption is monotonically decreasing, whereas it exhibits a non-monotonic behavior when $z_0 < \overline{z}$. In particular, if $z_0 < \overline{z}$ the growth rate of consumption expenditure initially decreases and ends up being increasing with time as the dynamic equilibrium approaches its steady state. In Part (c), we have that $\Omega(z) > 0$ along the transition when $z_0 < z^*$ and $\Omega(z) < 0$ when $z_0 > \overline{z} \geq z^*$. In the first case, the consumption growth rate is monotonically decreasing, whereas it exhibits a non-monotonic behavior when $z_0 > \overline{z}$. In particular, if $z_0 > \overline{z}$ the growth rate of consumption expenditure initially decreases and becomes eventually increasing as the equilibrium path approaches its steady state.■

Figure 1. Growth rate of consumption expenditure



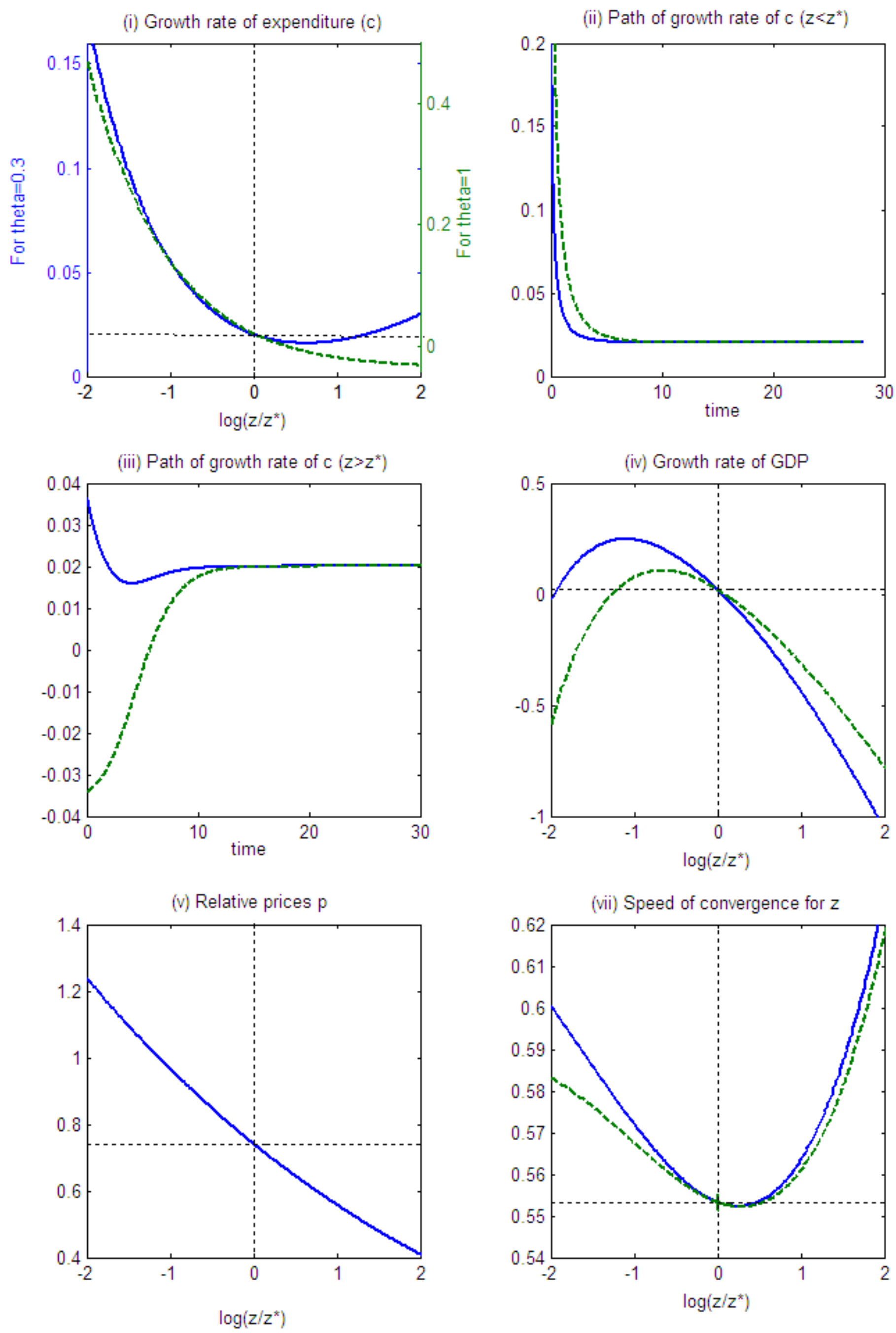
(i) $\chi \leq 0$

(ii) $\chi \in \left(0, \frac{1}{\sigma}\right)$ and $\overline{z} < z^*$.

(iii) $\chi \in \left(0, \frac{1}{\sigma}\right)$ and $\overline{z} > z^*$.

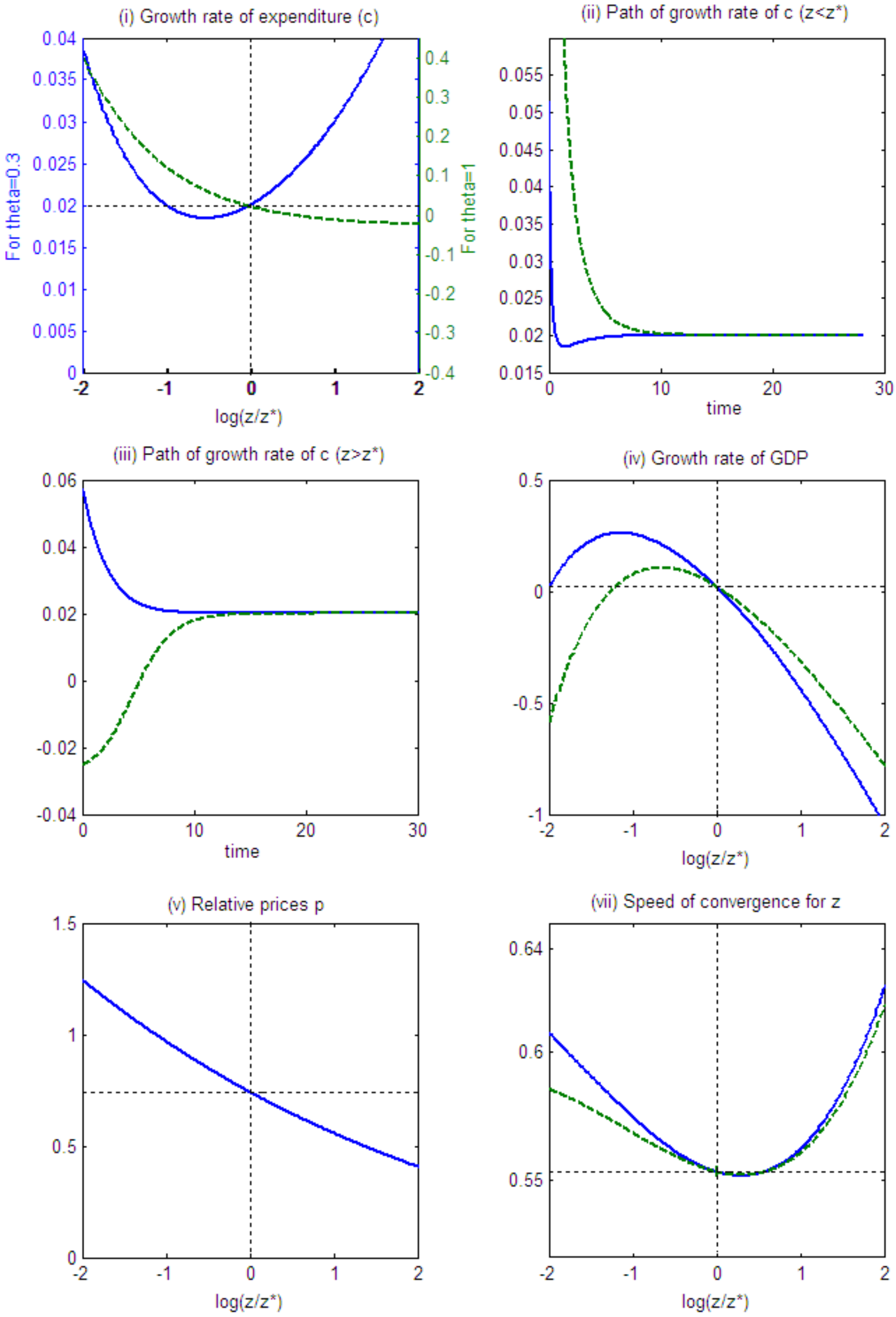
(iv) $\chi > \frac{1}{\sigma}$.

Figure 2. Transitional dynamics with $\sigma = 2$



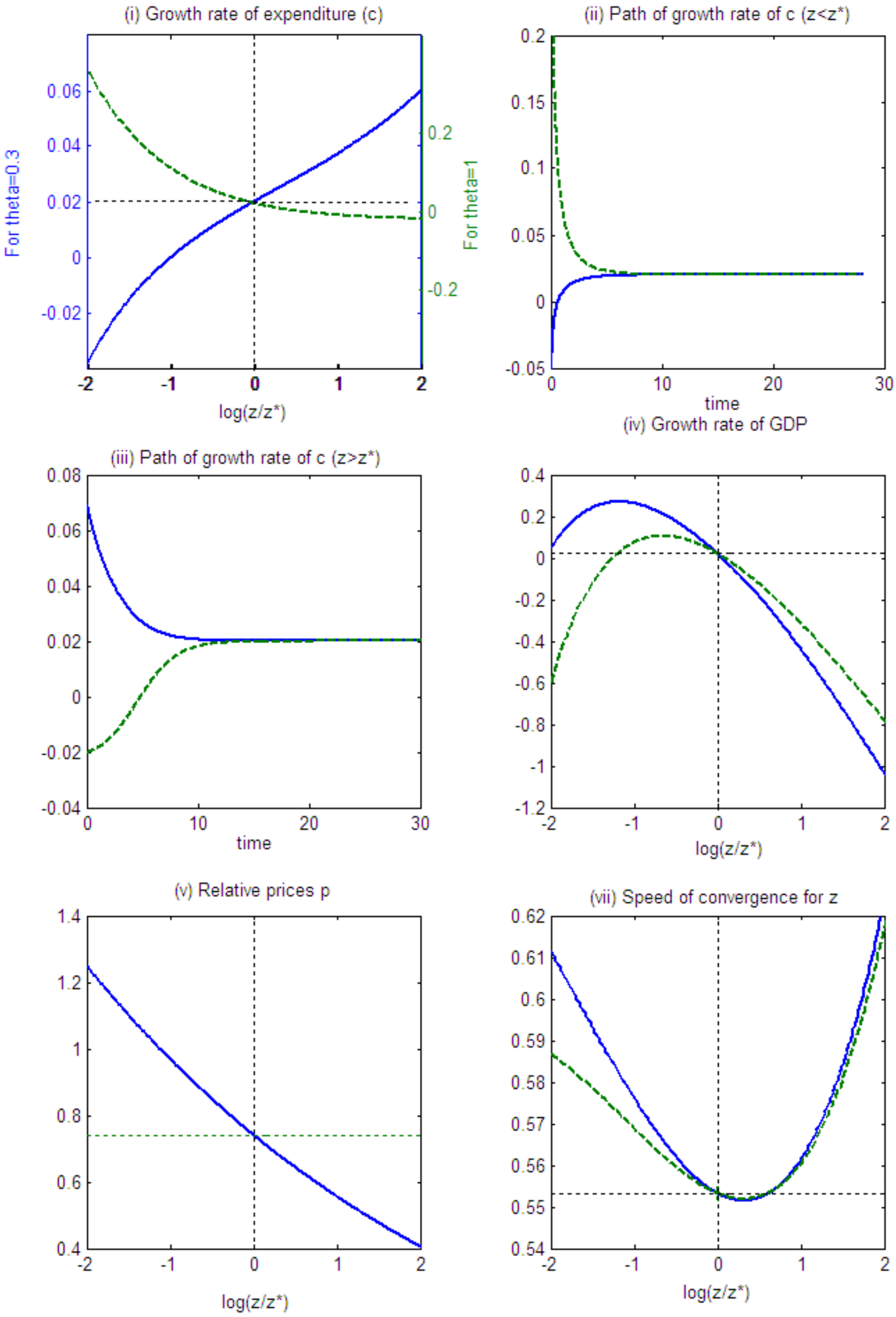
— Economy with $\theta = 0.3$ - - - Economy with $\theta = 1$

Figure 3. Transitional dynamics with $\sigma = 2.357$



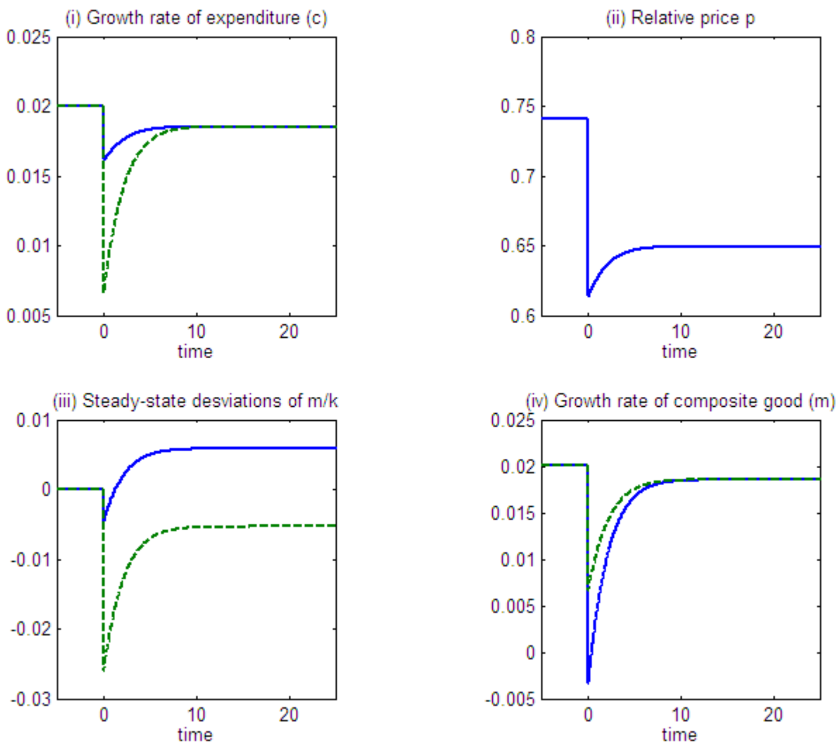
— Economy with $\theta = 0.3$ - - - Economy with $\theta = 1$

Figure 4. Transitional dynamics with $\sigma = 2.7143$



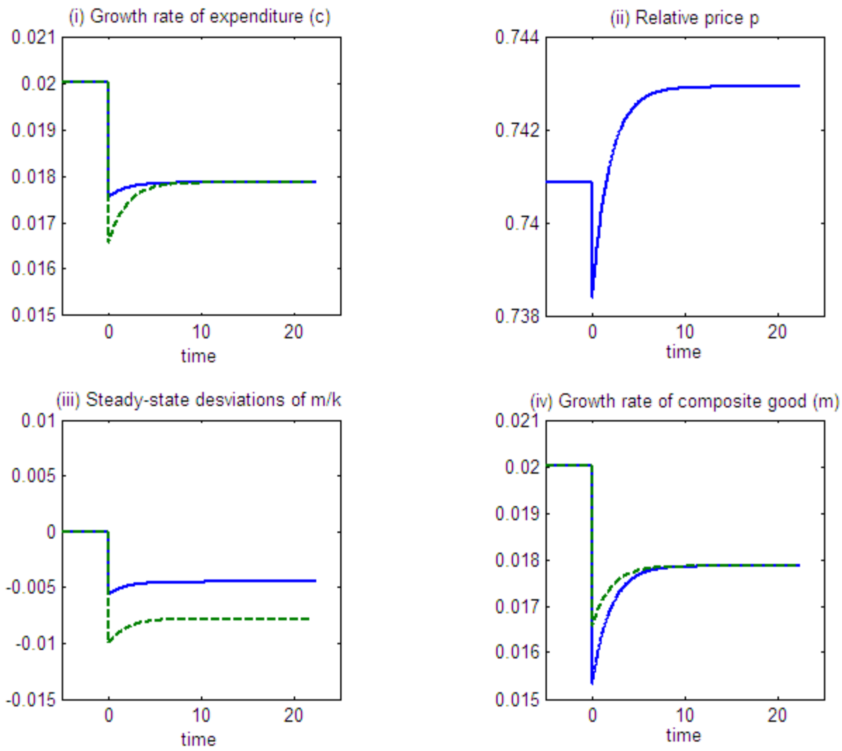
— Economy with $\theta = 0.3$ - - - Economy with $\theta = 1$

Figure 5. Dynamic effects of a biased technological shock when $\sigma = 2$



— Economy with $\theta = 0.3$ - - - Economy with $\theta = 1$

Figure 6. Dynamic effects of an unbiased technological shock when $\sigma = 2$



— Economy with $\theta = 0.3$ - - - Economy with $\theta = 1$

Table 1. Welfare cost of imbalances in the capital ratio

$z_0 = (0.75)\, z^*$			
σ	$\theta = 0.3$ (a)	$\theta = 1$ (b)	a/b
2	7.0608%	5.8959%	1.1976
2.357	7.0619%	5.8954%	1.1979
2.7143	7.0634%	5.8959%	1.1980
$z_0 = (1/0.75)\, z^*$			
σ	$\theta = 0.3$ (a)	$\theta = 1$ (b)	a/b
2	-7.6300%	-6.5049%	1.1730
2.357	-7.6284%	-6.5039%	1.1729
2.7143	-7.6275%	-6.5031%	1.1729

Table 2. Welfare cost of technological shocks ($\sigma = 2$)

Type of shock		$\theta = 0.3$ (a)	$\theta = 1$ (b)	a/b
Sectoral biased:	$\Delta A_1 = -0.15 A_1$	14.3821%	26.4386%	0.5440
Sectoral unbiased:	$\dfrac{\Delta A_1}{A_1} = \dfrac{\Delta A_2}{A_2} = \dfrac{\Delta A_3}{A_3} = -0.05$	13.5843%	13.3788%	1.0154