

Growth, Unemployment and Wage Inertia*

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Abstract

We introduce wage setting via efficiency wages in the neoclassical one-sector growth model to study the growth effects of wage inertia. We compare the dynamic equilibrium of an economy with wage inertia with the equilibrium of an economy without wage inertia. We show that wage inertia affects the long run employment rate and that the transitional dynamics of the main economic variables will be different because wages are a state variable when wage inertia is introduced. In particular, we show non-monotonic transitions in the economy with wage inertia that do not arise in the economy with flexible wages. We also study the growth effects of permanent technological and fiscal policy shocks in these two economies. During the transition, the growth effects of technological shocks obtained when wages exhibit inertia may be the opposite from the ones obtained when wages are flexible. In the long run, these technological shocks may have long run effects if there is wage inertia. We also show that the growth effects of fiscal policies will be delayed when there is wage inertia.

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1 Introduction

Wage inertia implies that current wages depend on past wages.¹ This relationship is a well-known empirical fact in labor economics (see Bell, 1996; Blanchard and Katz, 1997 and the survey by Montuenga-Gómez and Ramos-Parreño, 2005). Moreover, wage inertia has been justified in different wage settings. In models of wage bargaining between unions and firms, the wage is set as a markup over a reference wage that is interpreted as a fall back position. This reference wage is typically related to the unemployment benefit. In most OECD countries, this unemployment benefit depends on past wages. This introduces the relationship between current and past wages (Burkhard and Morgenstern, 2000 and Beissinger and Egger, 2004). Wage inertia has also been justified in efficiency wage models, where the wage is set by the firm in order to make workers exert the right amount of effort (de la Croix and Collard, 2000; Danthine and Donaldson, 1990, and Danthine and Kurmann, 2004). In this wage setting, the wage also depends on a reference wage if fairness is introduced in the disutility of effort of workers. This reference wage is frequently interpreted as a social norm that depends on past wages. In this way, wage inertia is introduced in the efficiency wage model.

Wage inertia introduces a process of wage adjustment that drives the transitional dynamics of wages and modifies the time path of the other variables in the economy, including the GDP growth rate. These differences in the transitional dynamics have been explored in New Keynesian models to explain facts of the business cycle and the effects of monetary shocks (Danthine and Kurmann, 2004; and Blanchard and Galí, 2007). In contrast, wage inertia has almost not been introduced in growth models, even when a non competitive labor market is assumed.² Therefore, to study how wage inertia modifies the time path of the GDP growth rate is an open question. The purpose of this paper is to address this question.

In order to study the growth effects of wage inertia, we analyze a version of the neoclassical one-sector exogenous growth model with efficiency wages. In the model, wages are set by the firms in order to make workers exert the right amount of effort. These non-walrasian wages cause non frictional unemployment. We assume that the workers' disutility of effort depends on the comparison between current wages and an external reference wage. Therefore, the amount of effort exerted by the workers will depend on

¹The literature also refers to wage inertia as persistence or sluggishness.

²Growth models with wage bargaining between firms and unions and non frictional unemployment have been studied by Benassy (1997), Daveri and Tabellini (2000), Brattinger (2000), Daveri and Mafezzoli (2000), Doménech and García (2008) and growth models with efficiency wages have been studied by Van Shaik and de Groot (1998), Mehl (2004), Nakajima (2006), Brecher, Chen and Choudhri (2002) and Pierpaolo (2010). In all these papers, wages do not exhibit inertia. Two exceptions that introduce wage inertia in a growth model are the papers by Raurich, Sala and Sorolla (2006) and Greiner and Flaschel (2010). However, these two papers only study the long run growth effects of the interaction between wage inertia and some particular fiscal policies.

this comparison and then wages will be set by the firms in relation to this reference wage. In this paper, this reference wage is interpreted as a social norm and it is defined as the weighted average of past average labor income. It follows that current wages depend on past wages, implying that wage inertia is introduced and that wages are a state variable. Moreover, the weights in the weighted average that defines the social norm will determine the intensity of wage inertia and, thus, the speed of wage adjustment. Therefore, in this version of the neoclassical growth model, two forces drive the transition: first, as in the neoclassical growth model with full-employment, the diminishing returns to capital; and second, the process of wage adjustment.

We distinguish two effects of wage inertia. On the one hand, the time path of the employment rate depends on both capital accumulation and the process of wage adjustment. Capital accumulation increases the labor demand and, thus, increases employment. Wage growth reduces employment. Thus, a fast (slow) accumulation of capital in comparison to the speed of wage adjustment will imply an increase (decrease) in the employment rate. Therefore, the interaction between capital accumulation and wage adjustment can explain periods of fast employment creation and also non-monotonic transitions of the employment rate that are not present when wages are flexible.³

On the other hand, the returns on capital and wages are related because we assume perfect competition and constant returns to scale. Then, wage inertia modifies the time path of capital accumulation because it changes the returns on capital. In particular, if wages are initially high then the interest rate will be initially low, implying low capital accumulation. The opposite holds when wages are initially low. Moreover, a process of fast wage adjustment also causes fast changes in capital accumulation.

During the transition, the GDP growth rate depends on the exogenous growth rate of technology, capital accumulation and employment growth. As wage inertia modifies both capital accumulation and employment growth, the introduction of wage inertia modifies the GDP growth rate in a non-obvious way and causes non-monotonic transitions in this variable. This non-monotonic transitions do not appear in the neoclassical growth model with flexible wages. In contrast, in the long run, the GDP growth rate coincides with the exogenous growth rate of technology and, thus, it is independent of the process of wage adjustment.

We use numerical simulations to study the effects of wage inertia during the transition. First, we show that in economies with initially high wage, the employment rate is initially low and increasing. As a consequence, in these economies, the growth rate of GDP will be initially large and decreasing during the transition. In contrast, initially low wages imply that the employment rate will be initially high and decreasing during the transition. This

³By flexible wages we mean that there is no wage inertia and, thus, current wages do not depend on past wages. However, it does not mean that this flexible wage clear the labor market and, hence, there is full employment.

implies that the growth rate of GDP will be initially low and it will increase during the transition. We conclude that economies with the same initial stock of capital may exhibit different time paths of the growth rate of GDP if initial wages are different. This suggests that initial wages should be taken into account as a relevant variable in the empirical analysis of convergence.

We also show that wage inertia can generate a process of fast employment creation that may cause a non-monotonic transition of the GDP growth rate. Before this process starts, GDP growth will be low. Then, during the process of employment growth, the GDP growth rate will be large due to the increasing employment rate. Finally, when the creation of employment ends, the GDP growth rate decreases until it converges to its long run value. This non-monotonic transition implies that the log of the GDP exhibits a S-shaped curve along the transition. This is a well-known fact of the development process that the neoclassical growth model with flexible wages fails to show.

We compare the growth effects of technological shocks in the model with flexible wages and in the model with wage inertia. We consider two different technological shocks: a permanent increase in the level of total factor productivity (TFP) and a permanent increase in the growth rate of TFP. The first shock only has transitional effects on the GDP growth rate. However, these effects will depend on the intensity of wage inertia. When wages are flexible and transitional dynamics are driven only by diminishing returns to capital, this shock initially increases the marginal product of capital and therefore the GDP growth rate increases. When there is wage inertia, this shock implies an initially decreasing path of the employment rate. This causes a negative effect on GDP growth that is not present when wages are flexible. This negative growth effect dominates the positive growth effect associated to capital accumulation during the first periods and, therefore, this shock initially reduces the GDP growth rate. Thus, a permanent increase in the level of TFP causes an opposite transition of the GDP growth rate when there is wage inertia.

The second shock, the permanent increase in the growth rate of TFP, implies a faster growth of the marginal product of labor. Flexible wages adjust immediately to this faster growth of the marginal product, implying that the employment rate does not change. In contrast, when there is wage inertia, wages do not adjust to this faster growth of the marginal product. As a consequence, when there is wage inertia, this technological shock causes an increase in the employment rate both during the transition and in the long run. Therefore, wage inertia introduces a positive relationship between the exogenous long run growth rate and the long run employment rate. Note that, by interpreting the productivity growth slowdown of the seventies as a reduction in the TFP growth rate, this positive relationship explains the persistently high unemployment rates in Europe.

Finally, we study the growth effects of the introduction of an unemployment benefit.

This unemployment benefit rises the reference wage and, therefore, wages increase. If wages are flexible, they immediately adjust, implying an immediate jump downwards on both employment and GDP. When there is wage inertia, wages do not immediately adjust to this fiscal policy and, therefore, the employment rate and the level of GDP will suffer a slow decline. Thus, the macroeconomic effects of this fiscal policy suffer a delay when wages exhibit inertia.

The paper is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium. Section 4 develops the numerical analysis that is used to study the transitional dynamics when there is wage inertia. Section 5 contains some concluding remarks.

2 The economy

In this section we first describe the problem of the consumers; second, we introduce the technology and the optimal decisions of the firms; and, finally, we specify a wage setting rule with wage inertia.

2.1 Consumers

Consider an economy populated by N_t ex ante identical infinitely lived consumers. Each consumer chooses consumption, C_t , and effort, e_t^f , in order to maximize the discounted sum of the utility

$$\int_{t=0}^{\infty} e^{-(\rho-n)t} \left[u(C_t) - d_t v_t(e_t^f) \right] dt,$$

where $\rho > 0$ is the subjective discount rate, $n \geq 0$ is the constant population growth rate, d_t is a dummy variable that equals 1 if the consumer is employed and equals 0 if he is unemployed. The utility function is separable in consumption and effort. We assume that $u(C_t)$ is a twice continuously differentiable utility function, increasing and concave that satisfies the Inada conditions. We follow Akerlof (1982) and we introduce fairness in the disutility of effort. In particular, we assume the following functional form:

$$v_t(e_t^f) = \left[e_t^f - \ln \left(\frac{(1-\tau)w_t}{\tilde{\gamma}w_t^s} \right) \right]^2,$$

where τ is the labor income tax rate, w_t is the wage, w_t^s is an external reference wage and $\tilde{\gamma} > 0$ is a preference parameter. Note that the consumers disutility of effort decreases if consumers feel that they are well paid, which happens when the wage net of taxes is larger than the reference wage. Obviously, the disutility increases when consumers do not consider that they are well paid.

There is not a consensus in the literature on the determinants of the reference wage. Layard et al. (1991, chapter 2) identifies the reference wage with the average labor income. Blanchard and Katz (1999) and Blanchard and Wolfers (2000) assume that the reference wage also depends on past wages. We follow these authors and we interpret the reference wage as a social norm that depends on the average past labor income in the economy. As a consequence, it is external to the individual. This assumption is crucial as it implies that consumers do not consider that their decisions today can modify the current or future value of the reference wage.

In each period t a fraction of the consumers will be unemployed. As a consequence, there is expost heterogeneity. To avoid the complexity associated with expost heterogeneity, we follow Collard and de la Croix (2000) and we introduce unemployment insurance contracts that are not affected by transaction costs and that are offered by risk neutral insurance companies. Thus, risk averse consumers choose complete insurance against the risk of being unemployed. This means that the income of employed and unemployed consumers will coincide, implying that consumers are expost identical.⁴ If consumers are employed, they obtain

$$I_t^e = (1 - \tau) w_t - m_t,$$

where m_t is the cost of an unemployment insurance. If consumers are unemployed obtain

$$I_t^u = b_t + \frac{m_t}{1 - l_t} - m_t$$

where b_t is the unemployment benefit, $\frac{m_t}{1 - l_t}$ is the unemployment insurance and l_t is the employment rate. The labor income of both employed and unemployed workers coincides when the cost of the insurance is

$$m_t = [(1 - \tau) w_t - b_t] (1 - l_t).$$

In this case, $I_t^e = I_t^u = I_t$, where I_t is the average labor income and it is defined as⁵

$$I_t = (1 - \tau) w_t l_t + b_t (1 - l_t).$$

⁴Collard and de la Croix (2000) show that this contract exists.

⁵Expost heterogeneity associated to unemployment can also be avoided by assuming that all members of the economy belong to the same big family. Daveri and Maffezzoli (2000), Doménech and García (2008), Eriksson (1997) follow this big family assumption. If instead of a big family we have heterogeneous agents, the solution would not change as long as we assume complete competitive insurance markets for unemployment or that a union pursues a redistributive goal, acting as a substitute for the insurance markets (Maffezzoli, 2001 and Benassy, 1997).

Therefore, the budget constraint of the representative consumer is

$$\dot{S}_t = r_t S_t + I_t + T_t - C_t, \quad (1)$$

where S_t is the stock of assets, I_t is the labor income and T_t is a lump-sum subsidy.

The consumer decides optimally on effort, consumption and investment by maximizing the discounted sum of utilities subject to the budget constraint. The solution of the maximization problem of the consumers is characterized by the effort function

$$e_t^f = \ln [(1 - \tau) w_t] - \ln (\tilde{\gamma} w_t^s), \quad (2)$$

the Euler condition

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\sigma(C_t)}, \quad (3)$$

where $\sigma(C_t) = \frac{u''(C_t)C_t}{u'(C_t)}$, the budget constraint and the following transversality condition:

$$\lim_{t \rightarrow \infty} e^{-\rho t} u'(C_t) S_t = 0. \quad (4)$$

2.2 Firms

There is a continuum of firms distributed in the interval $[0, 1]$. These firms produce using the following neoclassical production function:

$$Y_t = F(K_t, e_t^f A_t L_t),$$

where Y_t is gross domestic product (GDP), K_t is capital, e_t^f is effort, and $A_t L_t$ are efficiency units of employed labor. We assume that technology, A_t , exogenously increases at a constant growth rate $x \geq 0$, and the production function satisfies the following properties: constant returns to scale, twice continuously differentiable, $F_1 > 0$, $F_2 > 0$, $F_{11} < 0$, $F_{22} < 0$ and the Inada conditions.⁶ Note that effort increases the productivity of efficiency units of labor. This is taken into account by firms that set wage contracts as a partial gift exchange: workers in exchange of higher salaries will exert a higher amount of effort. Obviously, this non-walrasian wages will create unemployment in this model.

Firms take into account the effort function when they maximize profits, $F(K_t, e_t^f A_t L_t) - w_t L_t - (r_t + \delta)K_t$, where r_t is the interest rate and δ is the constant depreciation rate $\delta \in (0, 1)$. The first order conditions with respect to capital, labor and wages imply

$$r_t = F_1 - \delta, \quad (5)$$

⁶We define $F_1 = \frac{\partial F}{\partial K}$, and $F_2 = \frac{\partial F}{\partial e_t^f A_t L_t}$.

$$w_t = e_t^f A_t F_L, \quad (6)$$

and

$$\frac{\partial e_t^f}{\partial w_t} \frac{w_t}{e_t^f} = 1. \quad (7)$$

The third equation is the Solow condition. From combining the effort function and the Solow condition, we obtain that $e_t^f = 1$. Therefore, the optimal amount of effort of employed workers is constant through the transition. Using this constant effort level and the constant returns to scale assumption, (6) can be rewritten as

$$\widehat{w}_t = f(\widehat{k}_t) - \widehat{k}_t f'(\widehat{k}_t), \quad (8)$$

where $\widehat{w}_t = \frac{w_t}{A_t}$ is the wage per efficiency units of labor, $\widehat{k}_t = \frac{K}{A_t L_t}$ is capital per efficiency units of employed labor and the production function in intensive form satisfies the following properties: $f' > 0$ and $f'' < 0$. Using (8), we get

$$\widehat{k}_t = \tilde{k}(\widehat{w}_t), \quad (9)$$

where $\tilde{k}' > 0$ and from (9) we obtain the labor demand function

$$L_t^d = \frac{K_t}{A_t \tilde{k}(\widehat{w}_t)}. \quad (10)$$

Combining (5) and (9), we obtain the zero profit condition as a function relating the interest rate and wages

$$r_t = f'[\tilde{k}(\widehat{w}_t)] - \delta \equiv \tilde{r}(\widehat{w}_t), \quad (11)$$

where $\tilde{r}' < 0$. Thus, if the wage increases faster than efficiency units of labor, the interest rate decreases in order to have zero profits.

We assume that the labor supply is exogenous and equal to population, N_t . Population grows at a constant rate n .⁷ Using the labor supply, we can define GDP per efficiency units of labor (population) $y_t = \frac{Y_t}{A_t N_t}$, capital per efficiency units of labor (population) $k_t = \frac{K_t}{A_t N_t}$ and the employment rate $l_t = \frac{L_t}{N_t} \in [0, 1]$. Moreover, we use the constant returns to scale assumption to rewrite the production function as follows

$$y_t = g(k_t, l_t).$$

In models with wage setting, factor markets do not clear if the wage is different from the competitive walrasian wage. When the wage is smaller than the walrasian wage, there is excess supply in the capital market and excess demand in the labor market; and if the

⁷Conclusions regarding the growth effects of wage inertia would not be modified if an endogenous labor supply had been assumed and leisure had been introduced additively in the utility function.

wage is larger than the walrasian wage then there is excess demand in the capital market and excess supply in the labor market. In this paper, we will consider the later case, which implies that the equilibrium stock of capital is determined by the supply and the equilibrium level of employment is determined by the labor demand, that is

$$L_t = \frac{K_t}{A_t \tilde{k}(\hat{w}_t)}. \quad (12)$$

Using (12), we obtain the employment rate function

$$l_t = \tilde{l}(\hat{w}_t, k_t) = \frac{k_t}{\tilde{k}(\hat{w}_t)}. \quad (13)$$

Note that the employment rate increases with the capital stock and decreases with the wages per efficiency unit. Finally, we use (13) to rewrite the ratio of GDP to capital as the following decreasing function of wages:

$$\frac{y_t}{k_t} = g\left(1, \frac{1}{\tilde{k}(\hat{w}_t)}\right) = h\left[\tilde{k}(\hat{w}_t)\right], \quad h' < 0. \quad (14)$$

2.3 Wage setting rule with inertia

We first summarize the decisions of the consumers and firms to obtain two well-known equations that make explicit the crucial role of wages in this model. To this end, we define $c_t = \frac{C_t}{A_t}$ as consumption per efficiency units of labor, we use the equilibrium condition of the capital market, $S_t = A_t k_t$, and we assume that the government budget constraint is balanced in each period and equal to

$$\tau L_t w_t = N_t T_t + (N_t - L_t) b_t.$$

Note that tax revenues are returned to the consumers as either a lump-sum transfer or as an unemployment benefit. Therefore, labor income taxes do not cause wealth effects in this economy.

Using equations (1), (3), (8), (11) and (14), the equilibrium condition in the capital market and the government budget constraint, we obtain

$$\frac{\dot{c}_t}{c_t} = \frac{f'[\tilde{k}(\hat{w}_t)] - \delta - \rho - \sigma(c_t A_t) x}{\sigma(c_t A_t)}, \quad (15)$$

and

$$\frac{\dot{k}_t}{k_t} = h[\tilde{k}(\hat{w}_t)] - \frac{c_t}{k_t} - (n + x + \delta). \quad (16)$$

Equations (15) and (16) characterize the growth rates of consumption and capital

per efficiency units of labor. These equations depend on the time path of the wage per efficiency unit of labor. On the one hand, a higher wage reduces the interest rate and causes a substitution effect that deters future capital accumulation. On the other hand, higher wages reduce the employment rate and, therefore, cause a negative wealth effect that reduces the rate of growth of capital. We conclude that the transitional dynamics of the one sector growth model will depend on the particular assumptions regarding wage setting. We can distinguish three different wage settings.

First, if wages are flexible and they are set in order to clear the markets, there is full-employment implying that $l_t = 1$ and that $\tilde{k}(\hat{w}_t) = k_t$. In this case, equations (15) and (16) characterize the dynamic equilibrium of the standard neoclassical growth model with full-employment.

Second, if wages are flexible but they are set above the competitive wage, markets do not clear and $l_t < 1$. In this case, $\tilde{k}(\hat{w}_t) = k_t / l_t$, where $l_t = \tilde{l}(\hat{w}_t, k_t)$ is defined in (13). Obviously, the system of differential equations (15) and (16) alone does not characterize the dynamic equilibrium. The equilibrium will also depend on the wage equation obtained from the wage setting and that determines the wage. The growth literature that studies the joint dynamics of growth and unemployment has considered this framework of non-walrasian wages without inertia. In this framework, the wage equation is static implying that the equilibrium employment rate is either a function of capital or, under appropriate assumptions regarding the wage setting, constant along the transition. In the later case, the transitional dynamics are identical to the ones obtained in the standard version of the neoclassical growth model with full-employment.

Finally, if wages exhibit inertia, there is an additional dynamic equation that governs wage dynamics. In this paper we show that the transitional dynamics in this case are different from the ones obtained when wages do not exhibit inertia and we outline that these differences can explain some facts of the growth process. We proceed to obtain the wage equation.

In the efficiency wage model considered in this paper, wages are set by the firm so that workers exert the optimal amount of effort, $e_t^f = 1$. Using the effort function (2), it follows that wages set by the firm satisfy

$$w_t = \frac{\gamma w_t^s}{1 - \tau}, \quad (17)$$

where $\gamma = \tilde{\gamma}e \geq 1$. Note that a higher value of γ implies that firms must pay a higher wage in order to obtain the same amount of effort. Regarding the reference wage, in this paper we follow de la Croix and Collard (2000) and Raurich, et. al (2006) and assume that the reference wage is a social norm that is defined as the following weighted average

of past average labor income:

$$w_t^s = w_0^s e^{-\theta t} + \theta \int_0^t e^{-\theta(t-i)} I(i) di, \quad (18)$$

where w_0^s is the initial value of the reference wage, $\theta > 0$ provides a measure of the wage adjustment rate and I_t is the workers' average labor income

$$I_t = (1 - \tau) l_t w_t + (1 - l_t) b_t, \quad (19)$$

with the unemployment benefits being $b_t = \lambda(1 - \tau) w_t$ and $\lambda \in (0, 1)$.

Note that a larger value of θ reduces the weights given to the past and thus it implies a lower wage inertia. Therefore, θ provides a very convenient parametrization of the speed of wage adjustment. Moreover, if θ diverges to infinite, the reference wage coincides with the current average labor income and, in this limiting case, there is no wage inertia. Therefore, we must distinguish between the limiting case in which θ diverges to infinite and the case in which θ takes a finite value.

If θ diverges to infinite, (18) simplifies as follows: $w_t^s = I_t$. We solve the system of equations (17), (19) and $w_t^s = I_t$ to obtain that when wages do not exhibit inertia the employment rate is constant and equal to

$$l_t = 1 + \frac{1 - \gamma}{\gamma(1 - \lambda)} \equiv \bar{l}. \quad (20)$$

An increase in the parameter γ raises wages and thus reduces the employment rate. We will assume that $\gamma \in [1, \frac{1}{\lambda})$ in order that $\bar{l} \in (0, 1]$. Note that there is full employment when $\gamma = 1$, whereas there is unemployment when $\gamma > 1$. Note also that the employment rate does not depend on the growth rate of the economy. In other words, it does not depend on capital accumulation, nor technological progress. This result is a consequence of the fact that the increase in wages crowds out completely the positive effect on employment of capital accumulation or technological progress when wages are flexible. In the following section, we show that this complete crowding out does not arise when wages exhibit inertia.

If θ takes a finite value, wages exhibit inertia implying that there is a process of wage adjustment. To characterize this process, we first obtain the law of motion of the reference wage by differentiating (18) with respect to time

$$\dot{w}_t^s = \theta [I_t - w_t^s]. \quad (21)$$

Combining (17), (21) and (19), we obtain

$$\frac{\dot{w}_t^s}{w_t^s} = \theta [\gamma (1 - \lambda) l_t + \gamma \lambda - 1].$$

We log-differentiate (17) and $\hat{w}_t = \frac{w_t}{A_t}$ and substitute the expression of the growth rate of the reference wage to obtain the growth rate of wages per efficiency unit of labor

$$\frac{\dot{\hat{w}}_t}{\hat{w}_t} = \theta [\gamma (1 - \lambda) l_t + \gamma \lambda - 1] - x. \quad (22)$$

Equation (22) is the dynamic wage equation of this model and it drives convergence in the labor market. To see this, assume that the employment rate is initially large. As a consequence, the average labor income is initially large implying that the reference wage and wages will be larger in the future. As follows from (22), the growth of wages will be initially large in this case. The fast growth of wages causes a reduction in the employment rate. This reduction deters wage growth and makes wages per efficiency unit and employment converge to its long run value.

3 Equilibrium

We must distinguish between the equilibrium when θ diverges to infinite and wages are flexible and the equilibrium when wages exhibit inertia.

Definition 1 *Given the initial condition k_0 , an equilibrium with flexible wages is a path $\{c_t, k_t, \hat{w}_t, l_t\}$ that solves the system of differential equations (15) and (16), satisfies (20), (13) and the transversality condition (4).*

Definition 2 *Given the initial conditions k_0 and w_0 , an equilibrium with wage inertia is a path $\{c_t, k_t, \hat{w}_t, l_t\}$ that solves the system of differential equations (15), (16), and (22), satisfies (13), and the transversality condition (4).*

Note that the main difference between these two equilibrium definitions is the number of state variables.

In order to simplify the analysis and obtain results, we will assume the following isoelastic utility function:

$$u(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma}$$

and the following Cobb-Douglas production function:

$$Y_t = K_t^\alpha \left(e_t^f A_t L_t \right)^{1-\alpha}, \alpha \in (0, 1).$$

With these assumptions, we obtain that $\sigma(c_t A_t) = \sigma$, $\tilde{k}(\hat{w}_t) = \left(\frac{\hat{w}_t}{1-\alpha}\right)^{\frac{1}{\alpha}}$, $f'[\tilde{k}(\hat{w}_t)] = \alpha \left(\frac{\hat{w}_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}$, $l_t = k_t \left(\frac{\hat{w}_t}{1-\alpha}\right)^{-\frac{1}{\alpha}}$, and $h[\tilde{k}(\hat{w}_t)] = \left(\frac{\hat{w}_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}$. Therefore, the system of differential equations characterizing the equilibrium with wage inertia, (15), (16), and (22), simplifies as follows:

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha \left(\frac{\hat{w}_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \delta - \rho - \sigma x}{\sigma}, \quad (23)$$

$$\frac{\dot{k}_t}{k_t} = \left(\frac{\hat{w}_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \frac{c_t}{k_t} - (n + x + \delta), \quad (24)$$

and

$$\frac{\dot{\hat{w}}_t}{\hat{w}_t} = \theta \left(\gamma (1 - \lambda) k_t \left(\frac{\hat{w}_t}{1-\alpha}\right)^{-\frac{1}{\alpha}} + \gamma \lambda - 1 \right) - x. \quad (25)$$

If wages are flexible then $l_t = \bar{l}$. In this case, from (13), we obtain that the wage is the following function of capital:

$$\hat{w}_t = (1 - \alpha) \left(\frac{k_t}{\bar{l}}\right)^{\alpha}.$$

Note that this implies that capital is the only state variable and the system of differential equations characterizing the equilibrium, (15) and (16), simplify as follows:

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha \left(\frac{k_t}{\bar{l}}\right)^{\alpha-1} - \delta - \rho - \sigma x}{\sigma}, \quad (26)$$

and

$$\frac{\dot{k}_t}{k_t} = \left(\frac{k_t}{\bar{l}}\right)^{\alpha-1} - \frac{c_t}{k_t} - (n + x + \delta). \quad (27)$$

The equilibrium with flexible wages is characterized by one control variable, c_t , and one state variable, k_t . This equilibrium has a transition qualitatively equivalent to the transition in the neoclassical growth model with full-employment because the employment rate remains constant along the transition. In fact, the model is the neoclassical growth model with full-employment (the Ramsey-Cass-Koopmans model) when $\gamma = 1$.

We define a steady state equilibrium as an equilibrium path along which \hat{w}_t , c_t , and k_t remain constant.

Proposition 3 *There is a unique steady state equilibrium. The steady state values of the*

employment rate and of wages, capital, consumption and GDP per efficiency unit satisfy

$$\begin{aligned}
l^* &= 1 + \frac{1 - \gamma}{(1 - \lambda)\gamma} + \frac{x}{(1 - \lambda)\gamma\theta}, \\
\widehat{w}^* &= (1 - \alpha) \left(\frac{\delta + \rho + \sigma x}{\alpha} \right)^{\frac{\alpha}{\alpha-1}}, \\
k^* &= \left(\frac{\delta + \rho + \sigma x}{\alpha} \right)^{\frac{1}{\alpha-1}} l^*, \\
\frac{c^*}{k^*} &= \frac{\delta + \rho + \sigma x}{\alpha} - (n + x + \delta), \\
\frac{y^*}{k^*} &= \frac{\delta + \rho + \sigma x}{\alpha}.
\end{aligned}$$

Note that the long run values of the ratios $\frac{y}{k}$ and $\frac{c}{k}$ and the wage \widehat{w}^* do not depend on the parameters characterizing the labor market. Thus, the equilibrium of the one sector growth model converges to the same values of these variables regardless of the wage setting assumed. However, the long run values of the employment rate and the levels of capital, GDP and consumption per efficiency units of population depend on both γ and θ . On the one hand, a larger value of γ causes a reduction in the employment rate. This reduction implies a decrease in the levels of capital, consumption and GDP. On the other hand, the employment rate increases with x only if θ takes a finite value. The parameter x measures the long run growth rate of per capita GDP. Therefore, wage inertia introduces a positive relationship between the long run values of the growth rate and of the employment rate that is not present when wages are flexible. The intuition is as follows. Sustained growth implies that the labor demand increases at the growth rate x . Wage inertia prevents wages to increase with the labor demand, which explains the positive effect on the employment rate. The same intuition explains that the employment rate decreases with the speed of wage adjustment, θ , when there is sustained growth. This result contributes to the literature that analyzes the long run relationship between growth and employment in different wage settings and in different growth models. This literature has proposed different mechanisms explaining a positive (or negative) relationship between these two variables. In this paper, we show that wage inertia is another mechanism relating growth with employment in the long run.

Proposition 4 *The steady state equilibrium of the economy with wage inertia is locally saddle path stable.*

This result shows that there is a unique dynamic equilibrium. In the following section we show how wage inertia affects the transitional dynamics along this unique equilibrium.⁸

⁸Given that we have a two dimensional manifold, there are two roots with a negative real part. In the numerical example of the following section, the roots are complex numbers when θ is sufficiently low.

4 Numerical analysis

When wages are flexible, transitional dynamics are governed by the diminishing returns to capital and, thus, they only depend on the initial value of capital per efficiency units of labor. In contrast, when there is wage inertia, transitional dynamics are governed by both the diminishing returns to capital and wage dynamics. In this case, transitional dynamics will depend on the initial conditions on both capital and wages per efficiency units of labor. The existence of two different forces driving the transition implies that the dynamic equilibrium will exhibit relevant differences with respect to the equilibrium path of a model with flexible wages. In this section, we compare these different transitional dynamics by means of a numerical analysis. Therefore, the value of the parameters must be fixed.

We assume that $\sigma = 2$ which implies a value of the $IES = 0.5$ and $\alpha = 0.35$ which implies a constant value of the labor income share equal to 0.65. We fix $x = 2\%$ and $n = 1\%$, which are within the range of empirically plausible values of these two parameters. The depreciation rate $\delta = 6.46\%$ implies a long run net interest rate equal to 5.2% and the subjective discount rate $\rho = 0.012$ implies that the long run ratio of capital to GDP equals 3. We assume that there is no unemployment benefit in the benchmark economy, $\lambda = 0$, and the value of γ is set so that the long run employment rate equals $l^* = 0.9$. As γ is used to calibrate the long run value of the employment rate, this parameter will take a different value in the economy with flexible wages and in the economy with wage inertia. In particular, $\gamma = 1.22$ in the economy with wage inertia and $\gamma = 1.11$ in the economy with flexible wages. Finally, the parameter θ determines the speed of wage adjustment.⁹ We set the value of θ equal to 0.2, which implies an intensity of wage inertia that makes half distance in wages be satisfied in only three periods.¹⁰

In what follows, we show the results of four selected numerical exercises aimed to illustrate the transitional dynamics when there is wage inertia and compare the growth effects of shocks in an economy with flexible wages and in an economy with wage inertia. In the numerical examples we use the relaxation algorithm proposed by Trimborn, Koch and Steger (2008) to obtain the transition in a two-dimensional stable manifold.

This implies that for a sufficiently high intensity of wage inertia the equilibrium exhibits oscillations. In the examples of the following section, we assume a value of θ such that the roots are real numbers. Therefore, the non-monotonic behavior of the variables will be just a consequence of the existence of two different forces driving the transition.

⁹The value of θ could be calibrated using the autoregressive coefficient in the estimated wage equations. However, there is not a consensus on the value of this coefficient in the empirical labor literature (see Montuenga and Ramos-Parreño, 2005). The value of the parameter θ used in the simulation exercises would imply a conservative value of the autoregressive coefficient equal to 0.82.

¹⁰Half distance is defined as $\frac{\hat{w}_\tau - \hat{w}_0}{\hat{w}^* - \hat{w}_0} = \frac{1}{2}$, where τ is the number of periods needed to satisfy half distance.

4.1 Transitional dynamics

In a first numerical exercise, Figure 1 compares the transitional dynamics of two economies with the same initial capital stock, which is 5% larger than the long run capital stock, and a different initial wage. Therefore, this numerical exercise is aimed to show how the initial conditions on wages modify the transitional dynamics. The economy described by the continuous line has an initial wage that is 1% smaller than the long run wage and the economy illustrated by the dashed line has an initial wage that is 5% larger than its long run value. The first two panels show the time path of the two state variables: wages and capital per efficiency units. The third panel shows the time path of the interest rate. This time path of the interest rate follows from (11). This equation implies a negative relationship between wages per efficiency units of labor and the interest along the growth process. Panel (iv) displays the time path of the employment rate. Equation (13) shows that the path of the employment rate is completely determined by the path of capital and wages per efficiency units of labor. As capital accumulates, the labor demand increases and so does the employment rate. However, an increase in wages reduces the employment rate. Thus, the two forces driving the transition have opposite effects on employment, which may cause a non-monotonic behavior in the time path of this variable. To see this, consider the economy with initially low wages and high capital stock described by the continuous line. On the one hand, the low wages explain the initially large value of the interest rate. This explains the initial increase in the capital stock. On the other hand, the initially large capital stock and the low wages imply that the employment rate is initially high. Obviously, this high employment rate makes the current average income be large, which causes the increase in the reference wage and, thus, wages per efficiency unit raise. The increase in wages per efficiency unit explains both the reduction in the employment rate and also the reduction in the interest rate during the first periods. The later explains the reduction in the capital stock. Therefore, the interaction between the labor market and the interest rate explains the non-monotonic time path of the capital stock. Moreover, the reduction in the employment rate decreases the growth of wages per efficiency unit and, eventually, wages decrease. The reduction in wages implies that the employment rate will finally increase and, therefore, the time path of this variable also exhibits a non-monotonic transition (see panel iv).

Panels v) and vi) display, respectively, the time path of the growth rate of per capita GDP and the logarithm of per capita GDP. We use the production function to obtain the logarithm of per capita GDP as the following function:

$$\ln GDP = a + \alpha \ln k + (1 - \alpha) \ln l + xt,$$

where a is a constant. In the long run, the logarithm of GDP is a linear function of

time implying a constant long run growth rate equal to x . However, during the transition, both the accumulation of capital and the growth of the employment rate determine the time path of the GDP growth rate. In the economy illustrated by the continuous line, the employment rate is initially high and suffers a process of fast reduction. The high employment rate explains the initially large level of per capita GDP and the fast reduction in employment explains the initially low growth rate of GDP.

The dashed line in Figure 1 illustrates the transitional dynamics of an economy that has the same initial capital stock but an initially larger wage. The transitional dynamics of this economy are the opposite from the ones obtained in the economy with initially low wages. In particular, the growth rate of GDP decreases in the economy with initially high wages, whereas increases in the other economy. We then conclude that economies with the same initial capital stock but different initial wages exhibit different time paths of the GDP growth rate. Therefore, the initial cost of labor is a relevant variable explaining convergence and this variable should then be taken into account by the empirical growth literature when analyzing convergence.

In Figure 2, we show that differences in wage inertia also cause large differences in transitional dynamics. We compare two economies with the same initial conditions, but different assumptions on wage inertia. In the economy characterized by the continuous line we assume that $\theta = 0.2$, whereas the dashed line shows the time path of the variables when there is no wage inertia, i.e. $\theta \rightarrow \infty$. In both economies the initial stock of capital per efficiency units of labor is 50% smaller than its long run value and the initial wage in the economy with wage inertia is set so that wages per efficiency unit in the initial period coincide in both economies. The main differences are in the time paths of the employment rate (panel iv) and of the growth rate of GDP (panel v). In the economy with flexible wages, the employment rate is constant and the transition of the growth rate is driven only by the diminishing returns to capital. In the economy with wage inertia, capital increases faster than wages in the initial periods, implying an increase in the employment rate. As a consequence, the reference wage increases, which accelerates the process of wage growth. This implies that the employment rate eventually decreases until it converges to its long run value. While the employment rate increases, the growth rate of GDP is larger than that of the economy with flexible wages, whereas it is smaller when the employment rate decreases. This implies a fast and sharp transition of the GDP growth rate in the economy with wage inertia. Note also that we can explain large values of the GDP growth rates with plausible values of the interest rate and of the intertemporal elasticity of substitution that can not be explained when wages are flexible. Therefore, the introduction of wage inertia can explain the observed large GDP growth rate in emerging economies.

Figure 3 illustrates the transitional dynamics of two economies with different initial conditions and different wage inertia. The continuous line shows the time path of the

variables of an economy with wage inertia ($\theta = 0.08$), an initial capital stock per efficiency unit that is 25% smaller than its long run value and an initial wage per efficiency unit that is 10% larger than its long run value. The dashed line shows the time path of the variables of an economy with flexible wages and an initial capital stock per efficiency unit that is 25% smaller than its long run value. In the economy with flexible wages, the employment rate is constant and equal to its long run value. Given that the stock of capital per efficiency unit is initially low, the interest rate will be large. This initially large interest rate causes a fast capital accumulation, which explains the increasing time path of wages per efficiency unit and the decreasing time path of the interest rate. The time path of the interest rate explains the time path of the growth rate, which is initially large and then, during the transition, it decreases until it converges to its long run value. Thus, the growth rate exhibits a monotonic transition.

The economy with wage inertia exhibits a completely different transition. The initially large value of wages per efficiency unit implies that the initial value of both the employment rate and the interest rate are initially low. On the one hand, the low interest rate implies that capital per efficiency unit decreases in the initial periods. As capital decreases, the interest rate increases and, eventually, capital increases. On the other hand, wages decrease because of the low employment rate. The reduction of wages causes the growth of employment during the initial periods of the transition. In these initial periods, the growth of employment is partially compensated by the reduction in capital per efficiency unit and thus the growth rate of GDP is low. During the transition, when capital starts accumulating and still there is employment growth, the GDP growth rate increases. Finally, the process of employment and capital growth ends, which implies a decrease in the growth rate of GDP. Note that the time path of the growth rate of GDP exhibits a non-monotonic behavior along the growth process. This non-monotonic behavior implies that the time path of the log of GDP exhibits a S-shaped curve.

S-shaped curves are a well-known fact of the development process, implying that initially the GDP growth rate is low and when the development process starts it exhibits a period of fast growth that eventually stops. The neoclassical growth model with no wage inertia fails to show this type of transition and thus fails to explain the development process.¹¹ In this paper, we show that a version of the neoclassical growth model with wage inertia explains this non-monotonic transition as the result of labor market dynamics. Obviously, the model in this paper is too simple to be a serious theory of development. However, it suggests that labor market dynamics driven by wage inertia could be a relevant part of a development theory.

¹¹Recent growth literature has developed models aimed to explain this behavior. Examples are the endogenous preferences literature (Alonso-Carrera, et al., 2005 and Steger, 2006) and models of technological change (Parente and Prescott, 1999).

4.2 Technological and fiscal policy shocks

In this subsection we compare the transitional dynamics implied by two different technological shocks and a fiscal policy shock. In the figures, the continuous line illustrates the transitional dynamics in our benchmark economy with wage inertia and the dashed line shows the transitional dynamics of an economy with flexible wages. In the examples of this subsection we assume that the economies before the shock are initially in the steady state.

Figure 4 shows the transitional dynamics implied by a permanent technological shock that increases the level of TFP by a 5%. In the economy with flexible wages, this shock causes an initial increase in the interest rate. The increase in the interest rate implies that the capital stock per efficiency units of labor, after the initial reduction due to the increase in the efficiency units, increases during the transition. This explains both the increasing time path of wages per efficiency unit and the decreasing time path of the interest rate. Note that the time path of the interest rate explains both the initial increase in the GDP growth rate and also the decreasing time path of this variable along the transition. The effects of this shock are completely different in the economy with wage inertia. In this economy, the shock does not initially increase the wage. As a consequence, the wage per efficiency units of labor initially suffers a strong reduction. This has two effects. First, the increase in the interest rate will be larger when there is wage inertia, implying a faster accumulation of capital. Second, the employment rate will initially jump upwards, as wages initially remain constant and the marginal product of labor increases. As a consequence, the average labor income of the economy increases. This causes the increase in wages per efficiency unit and the reduction in the employment rate during the transition. The reduction of employment has a negative effect on the growth rate of GDP. Thus, the growth rate of GDP will be driven by two different and opposite effects: the reduction in the employment rate and the increase in the capital stock. Given our assumption of low wage inertia, the employment rate will experience a process of fast reduction, implying that this effect initially dominates the transition. This explains the reduction in the growth rate of GDP after the positive technological shock. Note that the growth effects of this technological shock are the opposite from the ones obtained when wages do not exhibit inertia.

Figure 5 shows the transitional dynamics implied by a permanent technological shock that increases the exogenous growth rate from 2% to 3%. In the economy with flexible wages, the employment rate is constant and the transitional dynamics will be governed only by the diminishing returns to capital. The growth rate increases along the transition until it converges to its new long run value. As explained in Proposition 3, this shock has permanent effects on the employment rate in the economy with wage inertia. This is a consequence of the positive relationship between growth and employment introduced

by wage inertia. Therefore, the employment rate increases until it converges to its new steady state. Obviously, this permanent effects on employment also imply permanent effects on the levels of GDP and capital that are not present when wages are flexible.

The purpose of presenting these two exercises is to show the relevant consequences of assuming wage inertia in understanding the short and long run effects of technological shocks. As an illustrative example of this, consider the productivity slowdown of the seventies that most western economies suffer. If we interpret this slowdown as a permanent reduction of the exogenous growth rate then, according to a model with flexible wages, it will not have long run effects on the employment rate, whereas it will cause a permanent reduction in the employment rate in an economy with wage inertia. This suggests that the different long run effects on the labor market that this shock had in the European and US economies could be explained by a different intensity of wage inertia.

Figure 6 shows the transitional dynamics implied by the introduction of an unemployment benefit, $\lambda = 1/3$. This fiscal rises the current average income and thus increases wages. In the economy with flexible wages, this causes an initial jump upwards in wages per efficiency unit that has two different effects. On the one hand, the employment rate decreases, which implies a reduction in GDP. On the other hand, the interest rate decreases which causes the reduction of capital per efficiency units of labor during the transition. The reduction in the capital stock explains the lower growth rate during the transition. In the economy with wage inertia, wages do not jump after the introduction of the unemployment benefit. As a consequence, the interest rate does not initially decrease, nor does the employment rate. This implies that the level of GDP remains constant after the introduction of the unemployment benefit. However, during the initial periods of the transition, wages will experience a process of rapid growth that reduces both the interest rate and the employment rate. As a consequence, the stock of capital decreases during the transition. The combined effect of the reduction in the stock of capital per efficiency units of labor and in the employment rate explains the larger reduction in the growth rate and the time path of the log of per capita GDP. This example shows that wage inertia delays the macroeconomics effects of this fiscal policy. We conjecture that this conclusion could be generalized to any fiscal policy that affects the economy by increasing wages.

5 Conclusions

In this paper we develop a version of the neoclassical growth model with wage inertia. We use this model to compare the equilibrium with wage inertia with the equilibrium of another economy with flexible wages. We show that these economies exhibit very different transitional dynamics.

We study the transitional dynamics using selected numerical exercises. In a first

exercise, we show that economies with the same initial capital stock may exhibit opposite transitions of the GDP growth rates because of different values of the initial wage. We conclude that wages should be considered in the empirical analysis of convergence.

In a second exercise, we compare the transitional dynamics in an economy with wage inertia and in an economy with flexible wages. In the economy with flexible wages, the time path of the macroeconomic variables exhibits a monotonic behavior, while we show that this time path can be non-monotonic when wage inertia is introduced. We argue that this non-monotonic transition is a consequence of the interaction between two forces: diminishing returns to capital and the process of wage adjustment. We show that this non-monotonic transition may imply that the time path of the logarithm of per capita GDP exhibits a S-shaped curve. This suggests that a closer analysis of labor market dynamics when wages exhibit inertia could explain some facts of the development process.

In a third exercise, we study the transitional dynamics implied by two different technological shocks: a permanent increase in the level of TFP and a permanent increase in the TFP growth rate. We show that the effects of the first shock on the growth rate of GDP will be the opposite if wage inertia is introduced. We also show that the second shock has permanent effects on employment and on the level of GDP only when wages exhibit inertia. We conclude that wage inertia crucially changes the effects of technological shocks.

Finally, in a last numerical exercise, we study the effects of a fiscal policy that consists of introducing an unemployment benefit. We show that wage inertia causes a delay on the effects of this fiscal policy.

The introduction of wage inertia does not modify the long run GDP growth rate because it is exogenous in this model. The aim of future research is to extend this analysis of wage inertia to endogenous growth models. In these models, the long run growth rate of GDP is endogenously determined by the fundamentals of the economy and, therefore, it may depend on the intensity of wage inertia.

References

- [1] Akerlof, G. (1982). Labor contracts as partial gift exchange, *Quarterly Journal of Economics* 97, 543-569.
- [2] Alonso-Carrera, J., Caballé, J., and Raurich X. (2005). Growth, Habit Formation, and Catching Up with the Joneses, *European Economic Review* 49, 1665-1691.
- [3] Beissinger, T. and Egger, H. (2004). Dynamic wage bargaining if benefits are tied to individual wages, *Oxford Economic Papers*, 56, 437-460.
- [4] Bell, B. (1996). Wage curve or Phillips curve? Mimeo, Nuffield College, University of Oxford.
- [5] Benassy, J.P. (1997). Imperfect Competition, Capital Shortages and Unemployment Persistence, *Scandinavian Journal of Economics* 99, 15-27.
- [6] Blanchard, O. and Katz, L. (1997). What do we know and we do not know about the natural rate of unemployment, *Journal of Economic Perspectives* 11, 51-73.
- [7] Blanchard, O. and Katz, L.F. (1999). Wage Dynamics: Reconciling Theory and Evidence, *NBER Working Paper* 6924.
- [8] Blanchard, O. and Galí, J. (2007). Real wage rigidities and the new keynesian model, *Journal of Money, Credit and Banking*, 39, 35-65.
- [9] Blanchard, O.J. and J. Wolfers (2000) The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence. *The Economic Journal* 110, C1-C33.
- [10] Bräuning, M. (2000). Wage Bargaining, Unemployment and Growth, *Journal of Institutional and Theoretical Economics*, 156/4, 646-660.
- [11] Brecher, R., Chen, Z. and Choudhri, E. (2002). Unemployment and growth in the long run: an efficiency-wage model with optimal savings, *International Economic Review*, 43, 875-894.
- [12] Burkhard, H. and Morgenstern, A. (2000). Indexation of unemployment benefits to previous earning, employment and wages, *Munich Discussion Paper, 2000-13*.
- [13] Collard, F. and de la Croix, D. (2000). Gift Exchange and the Business Cycle: The Fair Wage Strikes Back, *Review of Economic Dynamics* 3, 166-193.
- [14] Danthine, J.P. and Donaldson, J.B. (1990). Efficiency wages and the business cycle puzzle, *European Economic Review*, 34, 1275-1301.

- [15] Danthine, J.P. and Kurmann, A. (2004). Fair wages in a New Keynesian model of the Business cycle, *Review of Economic Dynamics* 7, 107-142.
- [16] Daveri, F. and Mafezzoli, M. (2000). A numerical approach to fiscal policy, unemployment and growth in Europe, unpublished manuscript.
- [17] Daveri, F. and Tabellini, G. (2000). "Unemployment and Taxes", *Economic Policy*, 5: 49-104.
- [18] Doménech, R. and García, J.R. (2008). Unemployment, taxation and public expenditure in OECD economies, *European Journal of Political Economy* 24, 202-217.
- [19] Greiner, A. and Flaschel, P. (2010). Public debt and public investment in an endogenous growth model with real wage rigidities, *Scottish Journal of Political Economy*, 57, 68-82.
- [20] Layard, P.R.J., S.J. Nickell and R. Jackman (1991) *Unemployment: Macroeconomic Performance and the Labor Market* Oxford: Oxford University Press.
- [21] Mafezzoli, M. (2001). Non-Walrasian Labor Markets and Real Business Cycles, *Review of Economic Dynamics* 4, 860-892.
- [22] Meckl, J. (2004). Accumulation of technological knowledge, wage differentials, and unemployment, *Journal of Macroeconomics*, 26, 65-82.
- [23] Montuenga-Gómez, V. and Ramos-Parreño, J.M. (2005). Reconciling the wage curve and the the Phillips curve, *Journal of Economic Surveys*, 19, 735-765.
- [24] Nakajima, T. (2006). Unemployment and indeterminacy, *Journal of Economic Theory*, 126, 314-327.
- [25] Parente, S., and Prescott. E. (2004). A unified theory of the evolution of international income levels, Federal Reserve Bank of Minneapolis, Staff Report, 333.
- [26] Pierpaolo, C. (2010). A Shumpeterian growth model with equilibrium unemployment, *Metroeconomica*, 61, 398-426.
- [27] Raurich X., Sala, H. and Sorolla, V. (2006). Unemployment, Growth and Fiscal Policy, New Insights on the Hysteresis Hypothesis, *Macroeconomic Dynamics*, 10, 285-316.
- [28] Steger, T.M. (2000). Economic growth with subsistence consumption, *Journal of Development Economic* 62, 343-361.

- [29] Trimborn, T., Koch, K.J. and Steger, T.M. (2008). Multidimensional transitional dynamics: a simple numerical procedure, *Macroeconomic Dynamics* 12, 301-319.
- [30] Van Schaik, B.T.M. and De Groot, H.L.F. (1998). Unemployment and endogenous growth, *Labour* 12, 189-219.

A Appendix

Proof of Proposition 4 The Jacobian matrix associated to (23), (24) and (25) is

$$J = \begin{pmatrix} \frac{\partial \dot{w}_t}{\partial w_t} & \frac{\partial \dot{w}_t}{\partial k_t} & \frac{\partial \dot{w}_t}{\partial c_t} \\ \frac{\partial \dot{k}_t}{\partial w_t} & \frac{\partial \dot{k}_t}{\partial k_t} & \frac{\partial \dot{k}_t}{\partial c_t} \\ \frac{\partial \dot{c}_t}{\partial w_t} & \frac{\partial \dot{c}_t}{\partial k_t} & \frac{\partial \dot{c}_t}{\partial c_t} \end{pmatrix} = \begin{pmatrix} -\theta(1-\lambda)\gamma\left(\frac{k^*}{\alpha}\right)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} & \gamma\theta(1-\lambda)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}w^* & 0 \\ -\left(\frac{1}{\alpha}\right)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}k^* & \frac{c^*}{k^*} & -1 \\ -\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}c^* & 0 & 0 \end{pmatrix}$$

The characteristic polynomial is

$$P(J) = -\mu^3 + \mu^2 Tr(J) + \mu H + Det(J),$$

where the determinant of the Jacobian Matrix is

$$Det(J) = \gamma\theta(1-\lambda)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{2}{\alpha}}w^*c^* > 0,$$

the trace of this matrix is

$$\begin{aligned} Tr(J) &= -\gamma\theta(1-\lambda)\left(\frac{k^*}{\alpha}\right)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} + \frac{c^*}{k^*} = \\ &= -\frac{\gamma}{\alpha}\left[\left(\frac{x+\theta}{\gamma}\right) - \lambda\theta\right] + \frac{\delta + \rho + \sigma x}{\alpha} - (n + x + \delta), \end{aligned}$$

and

$$\begin{aligned} H &= \frac{\partial \dot{k}_t}{\partial w_t} \frac{\partial \dot{w}_t}{\partial k_t} - \frac{\partial \dot{w}_t}{\partial w_t} \frac{\partial \dot{k}_t}{\partial k_t} = \\ &= -\left(\frac{1}{\alpha}\right)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}k^*\gamma\theta(1-\lambda)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}w^* + \gamma\theta(1-\lambda)\left(\frac{k^*}{\alpha}\right)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}\frac{c^*}{k^*} \\ &= \gamma\theta(1-\lambda)\left(\frac{k^*}{\alpha}\right)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}\left[-\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}w^* + \frac{c^*}{k^*}\right] \\ &= \gamma\theta(1-\lambda)\left(\frac{k^*}{\alpha}\right)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}\left[\alpha\left(\frac{\delta + \rho + \sigma x}{\alpha}\right) - (n + x + \delta)\right] \\ &= \theta(1-\lambda)\gamma\left(\frac{k^*}{\alpha}\right)\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}}[\rho + (\sigma - 1)x - n] > 0, \end{aligned}$$

where the inequality follows from the bounded utility condition. Given that $H > 0$ and $Det(J) > 0$, the equilibrium is saddle path stable regardless of the sign of the trace.

Figures

Figure 1. Transitional Dynamics: different initial conditions

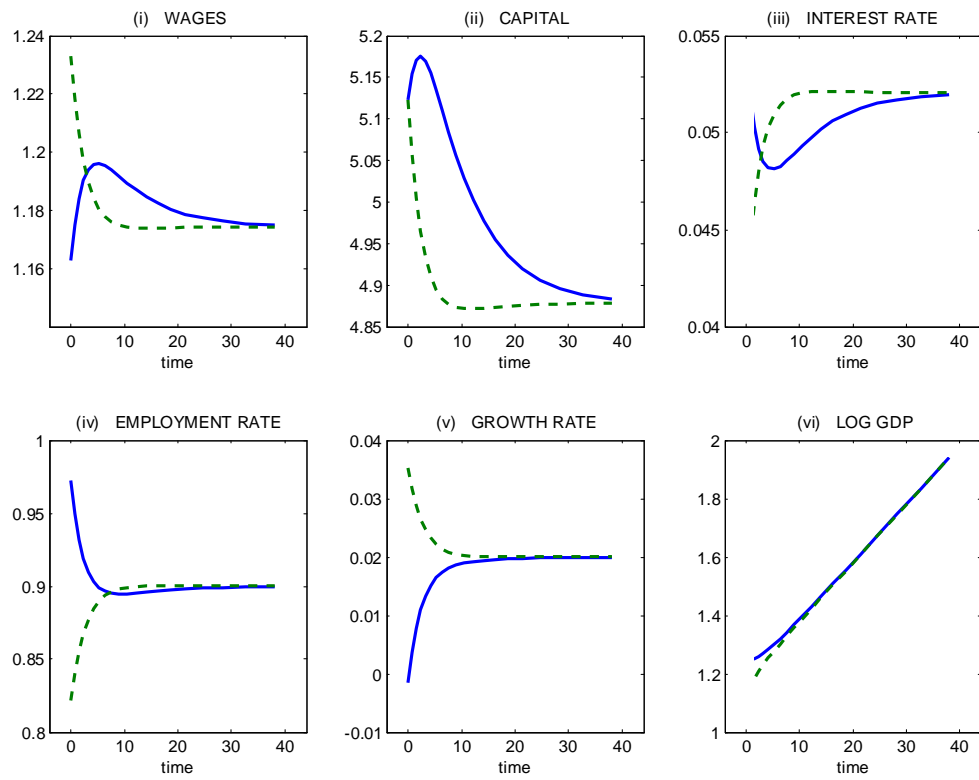


Figure 2. Transitional Dynamics: different wage inertia

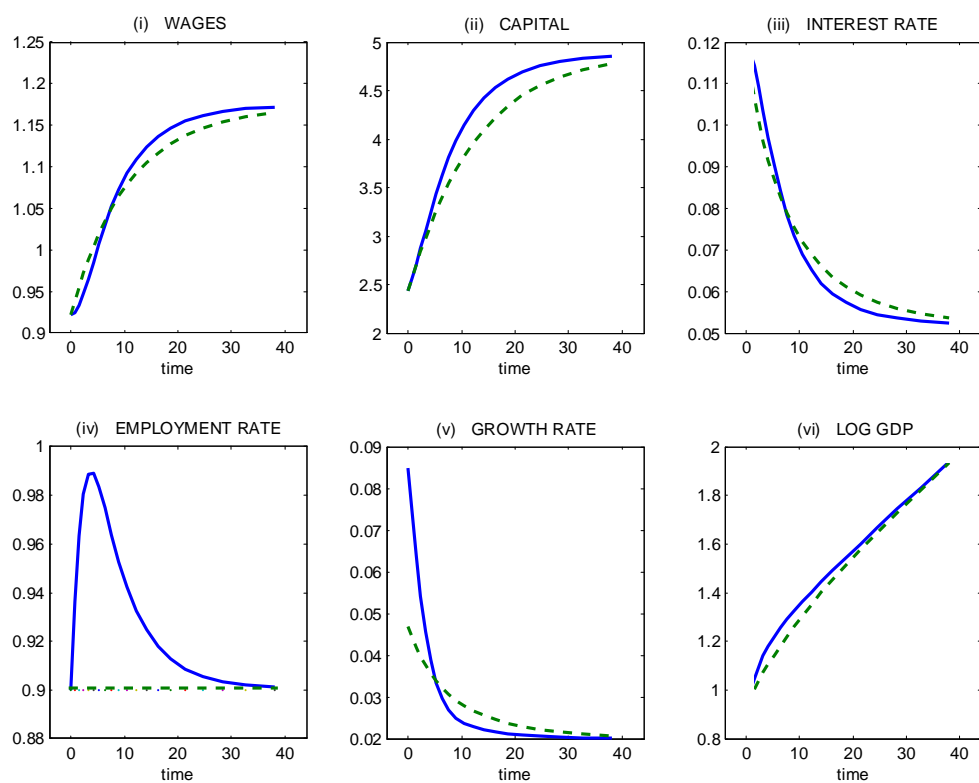


Figure 3. Transitional Dynamics: different wage inertia and initial conditions

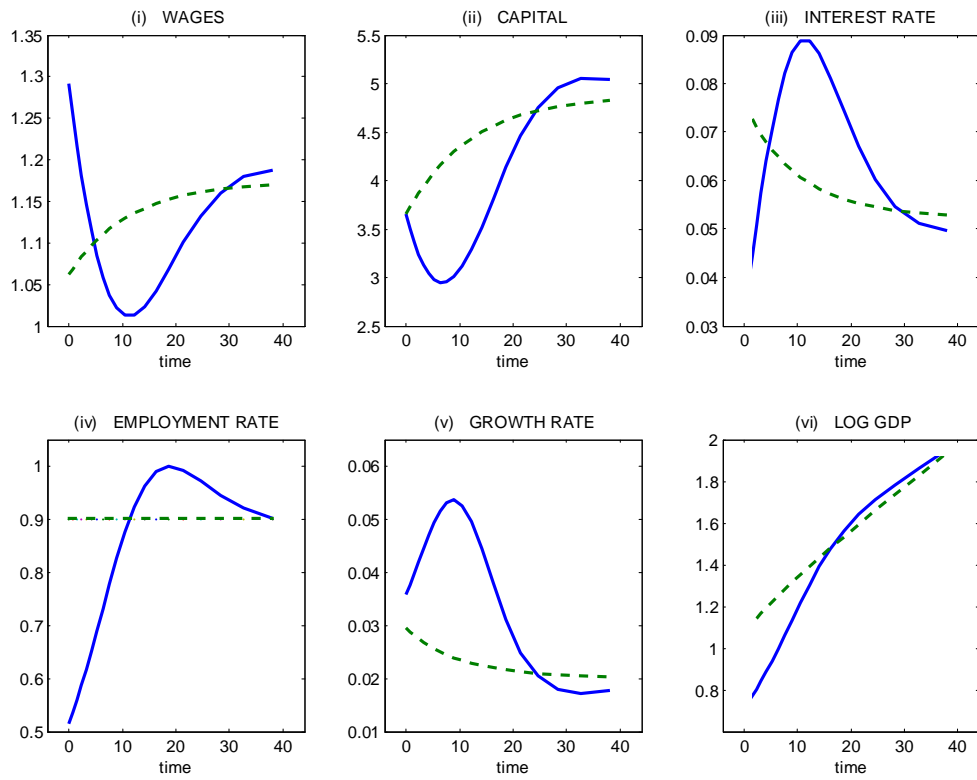


Figure 4. Permanent increase in the level of TFP

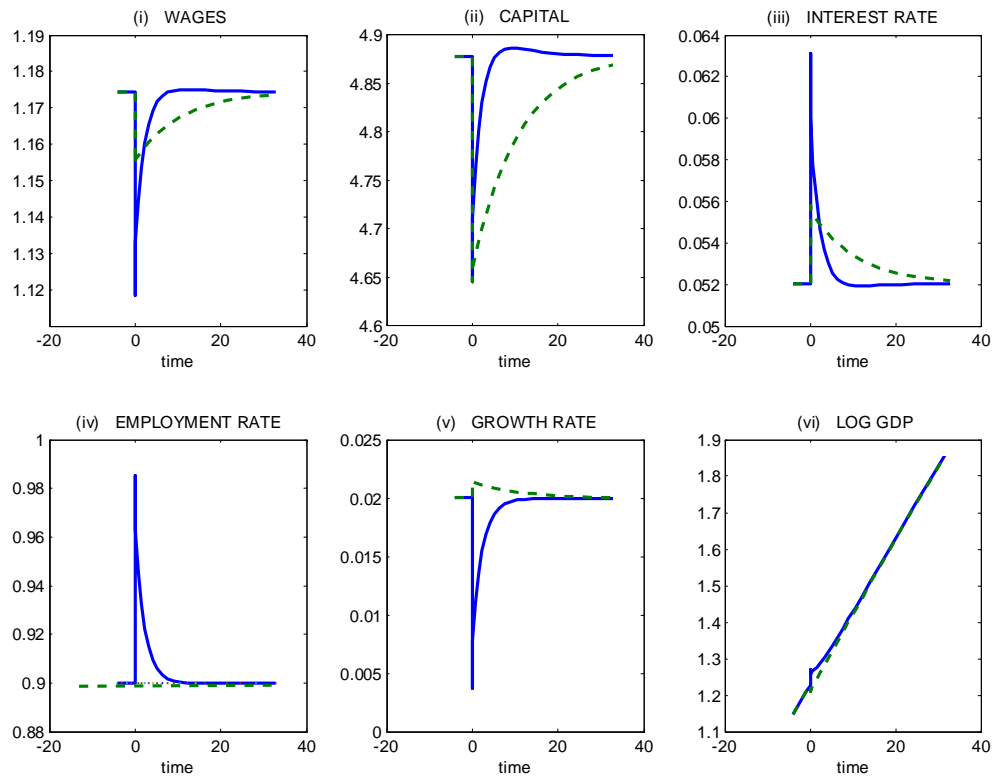


Figure 5. Permanent increase in the long run growth rate

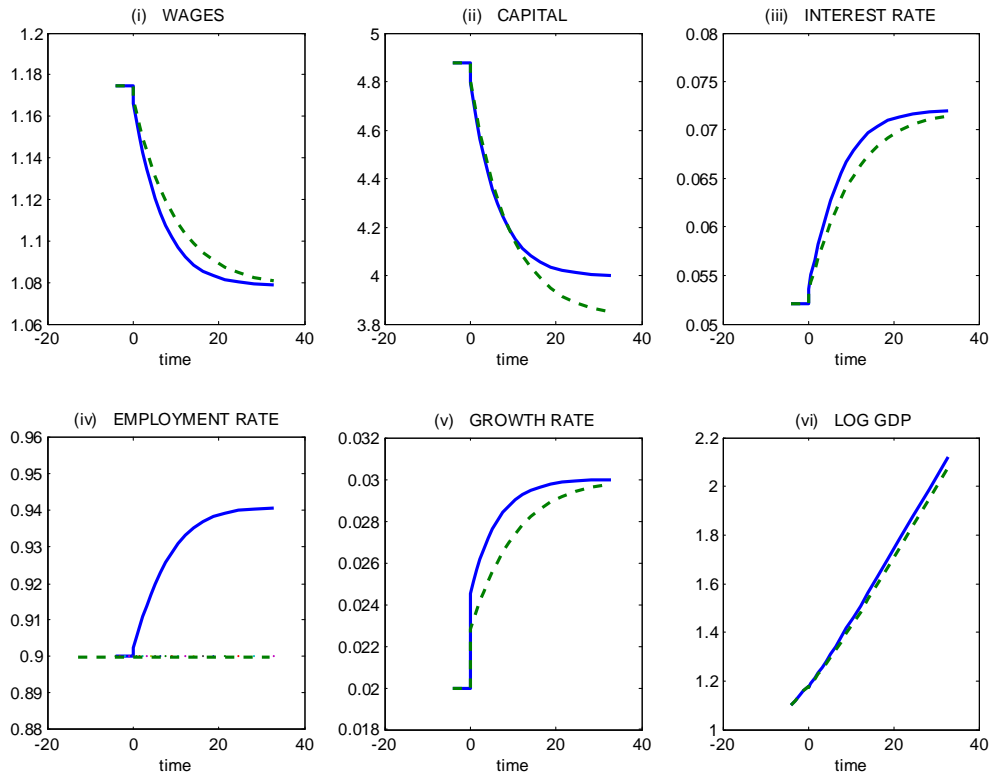


Figure 6. Unemployment benefit

