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Decompositions of Profitability Change Using Cost Functions: A Comment

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Abstract

Recently Diewert (2014) decomposed cost change into the product of four drivers. He then combined three of these drivers with a novel measure of returns to scale to decompose profitability change. We use an implicit Konüs input quantity index to show that his expression for profitability change is the product of a price recovery index and an implicit productivity index, and we extend his analysis by exploiting new relationships between theoretical Konüs and empirical Fisher price indexes to obtain two new decompositions of profitability change. One pairs a Konüs price recovery index with a Fisher implicit productivity index, the other has pure Fisher structure, and we note the advantages of each.

Decompositions of Profitability Change Using Cost Functions: A Comment

1. Introduction

Recently Diewert (2014) decomposed cost change from one period to the next into the product of four drivers. He then combined three of these cost change drivers with a measure of returns to scale, which we criticize, to decompose profitability (or cost recovery) change from one period to the next. We support his focus on profitability, which we believe is an under-utilized financial performance indicator.[1] He expresses profitability change as the product of a price recovery index and an implicit productivity index; his price recovery index pairs a Fisher output price index with a Konüs input price index, and his implicit productivity index pairs a Fisher output quantity index with an implicit Konüs input quantity index. We find the Fisher/Konüs and Fisher/implicit Konüs structures unnatural, and we note that the implicit Konüs input quantity index fails to satisfy the fundamental linear homogeneity property in input quantities.

We extend Diewert's analysis by exploiting new relationships between theoretical Konüs and empirical Fisher price indexes to obtain two new decompositions of profitability change. One has a Konüs price recovery index, an implicit Fisher productivity index, and an output allocative efficiency effect that enforces satisfaction of the product test. The other has a Fisher price recovery index, an implicit Fisher productivity index, and an input allocative efficiency effect that enforces satisfaction of the product test. We point out some virtues of each decomposition.

2. The Cost Change Decomposition

Diewert decomposes cost change as

$$\frac{C^1}{C^0} = \beta_F \times \frac{\alpha_F}{\tau_F \times \varepsilon},$$ (1)

where

$$\beta_F = \left[\frac{c^0(y^0, w^1)}{c^0(y^0, w^0)} \times \frac{c^1(y^1, w^1)}{c^1(y^1, w^0)} \right]^{1/2}$$

is a Konüs input price index $W_K(w^1, w^0, y^1, y^0)$,

$$\alpha_F = \left[\frac{c^0(y^1, w^0)}{c^0(y^0, w^0)} \times \frac{c^1(y^1, w^1)}{c^1(y^0, w^1)}\right]^{1/2}$$

is a Konüs output quantity index $Y_K(y^1, y^0, w^1, w^0)$,

$$\tau_F = \left[\frac{c^0(y^0, w^1)}{c^1(y^0, w^1)} \times \frac{c^0(y^1, w^0)}{c^1(y^1, w^0)}\right]^{1/2}$$

is a Konüs technical change index, and

$$\varepsilon = \frac{c^1(y^1, w^1)/w^{1\mathrm{T}}x^1}{c^0(y^0, w^0)/w^{0\mathrm{T}}x^0}$$

is a measure of cost efficiency change. Cost is increased by increases in input prices and output quantities, and reduced by technical progress and improvements in cost efficiency.

That part of cost change not attributable to the Konüs input price index is an implicit Konüs input quantity index, which we write as

$$XI_K(y^1, y^0, w^1, w^0, x^1, x^0) = \frac{(C^1/C^0)}{\beta_F}$$

$$= \frac{\alpha_F}{\tau_F \times \varepsilon}, \tag{2}$$

and save for future use.

3. The Profitability Change Decomposition

Profitability is the ratio of revenue to cost, and profitability change can be expressed as

$$\frac{\Pi^1}{\Pi^0} = \frac{(R^1/R^0)}{(C^1/C^0)}$$

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$$= \frac{P}{W} \times \frac{Y}{X},$$
(3)

in which P and Y are output price and quantity indexes satisfying the product test with R^1/R^0, W and X are input price and quantity indexes satisfying the product test with C^1/C^0, and profitability change becomes the product of price recovery change P/W and productivity change Y/X. In his expression (34) Diewert decomposes profitability change into "five separate explanatory factors that help to explain profitability growth" as

$$\frac{\Pi^1}{\Pi^0} = \frac{P_F}{\beta_F} \times [\varepsilon \times \rho_F \times \tau_F],$$
(4)

in which $P_F(p^1, p^0, y^1, y^0)$ is a Fisher output price index, β_F, ε and τ_F are defined beneath (1), and

$$\rho_F = \left[\frac{p^{0T} y^1}{p^{0T} y^0} \times \frac{p^{1T} y^1}{p^{1T} y^0} \right]^{1/2} \Big/ \left[\frac{c^0(y^1, w^0)}{c^0(y^0, w^0)} \times \frac{c^1(y^1, w^1)}{c^1(y^0, w^1)} \right]^{1/2}$$
(5)

is a measure of returns to scale to which we return below.

Decomposition (4) has some interesting features. First, since $\beta_F = W_K(w^1, w^0, y^1, y^0)$, the price recovery index in (3) is $P/W = P_F(p^1, p^0, y^1, y^0)/W_K(w^1, w^0, y^1, y^0)$. Second, if the product test is to be satisfied, the productivity index in (3) must be the implicit productivity index $Y/X = Y_F(y^1, y^0, p^1, p^0)/XI_K(y^1, y^0, w^1, w^0, x^1, x^0)$, with $XI_K(y^1, y^0, w^1, w^0, x^1, x^0)$ defined in (2). Third, the implicit productivity index decomposes as

$$\frac{Y_F(y^1, y^0, p^1, p^0)}{XI_K(y^1, y^0, w^1, w^0, x^1, x^0)} = \varepsilon \times \rho_F \times \tau_F$$

$$= \frac{\rho_F \times \alpha_F}{\alpha_F/(\tau_F \times \varepsilon)}$$

$$= \frac{Y_F(y^1, y^0, p^1, p^0)}{\alpha_F/(\tau_F \times \varepsilon)}.$$
(6)

The first equality states that productivity change is driven by cost efficiency change, returns to scale as defined by Diewert, and technical change. The second equality multiplies and divides the first by α_F. The third equality exploits (2), $XI_K(y^1, y^0, w^1, w^0, x^1, x^0) = \alpha_F/(\tau_F \times \varepsilon)$, which means that the numerator $\rho_F \times \alpha_F = Y_F(y^1, y^0, p^1, p^0)$. The drivers of implicit productivity change in (6) are the same as the non-price drivers of cost change in (1): size change (α_F), which replaces Diewert's returns to scale term, technical change (τ_F) and cost efficiency change (ε).

We reconsider Diewert's returns to scale measure ρ_F in (5). From (6), $Y_F(y^1, y^0, p^1, p^0) = \rho_F \times \alpha_F$. From the definition beneath (1), $\alpha_F = Y_K(y^1, y^0, w^1, w^0)$. Consequently

$$\rho_F = \frac{Y_F(y^1, y^0, p^1, p^0)}{Y_K(y^1, y^0, w^1, w^0)}, \tag{7}$$

which is also clear from (5). Diewert thus defines returns to scale as the ratio of our best empirical output quantity index to our best theoretical output quantity index, a definition that is as intriguing as it is unconventional.

We use (6) to rewrite Diewert's profitability change decomposition as

$$\frac{\Pi^1}{\Pi^0} = \frac{P_F(p^1, p^0, y^1, y^0)}{W_K(w^1, w^0, y^1, y^0)} \times \frac{Y_F(y^1, y^0, p^1, p^0)}{XI_K(y^1, y^0, w^1, w^0, x^1, x^0)}, \tag{8}$$

and we observe that the implicit productivity index is the ratio of a Fisher output quantity index to an implicit Konüs input quantity index. Expression (8) is not entirely satisfactory, since both the price recovery index and the implicit productivity index have mixed Fisher/Konüs structure rather than pure Fisher or pure Konüs structure, and since $XI_K(y^1, y^0, w^1, w^0, x^1, x^0)$ does not satisfy the fundamental property of linear homogeneity in input quantities.[2]

4. Alternative Decompositions

We suggest two alternative restructurings of (8); both exploit analysis in Grifell-Tatjé and Lovell (2015;144-150).

In the first restructuring we convert the price recovery index in (8) to purely Konüs structure by relating the Fisher output price index to a Konüs output price index means of

$$P_F(p^1, p^0, y^1, y^0) = \left[\frac{r^0(x^0, p^1)}{r^0(x^0, p^0)} \times \frac{r^1(x^1, p^1)}{r^1(x^1, p^0)}\right]^{\frac{1}{2}}$$

$$\times \left[\frac{p^{1T}(y^0/D_O^0(x^0, y^0))/r^0(x^0, p^1)}{p^{0T}(y^0/D_O^0(x^0, y^0))/r^0(x^0, p^0)} \times \frac{p^{1T}(y^1/D_O^1(x^1, y^1))/r^1(x^1, p^1)}{p^{0T}(y^1/D_O^1(x^1, y^1))/r^1(x^1, p^0)}\right]^{1/2}$$

$$= P_K(p^1, p^0, y^1, y^0) \times \gamma_p, \tag{9}$$

in which $P_K(p^1, p^0, y^1, y^0)$ is a Konüs output price index and

$$\gamma_p = \left[\frac{AE_O^0(x^0, p^1, y^0)}{AE_O^0(x^0, p^0, y^0)} \times \frac{AE_O^1(x^1, p^1, y^1)}{AE_O^1(x^1, p^0, y^1)}\right]^{1/2} \tag{10}$$

provides a clear economic interpretation of the relationship between the empirical Fisher output price index and the theoretical Konüs output price index. γ_p is the geometric mean of two output-oriented allocative efficiency ratios. If y^0 is more allocatively efficient relative to p^0 than to p^1 on base period technology, the first ratio is bounded above by unity. If y^1 is more allocatively efficient relative to p^1 than to p^0 on comparison period technology, the second ratio is bounded below by unity. Accordingly we expect their geometric mean to approximate unity, and $P_F(p^1, p^0, y^1, y^0)$ and $P_K(p^1, p^0, y^1, y^0)$ to be approximately equal.

Substituting (9) into (8) yields

$$\frac{\Pi^1}{\Pi^0} = \frac{P_K(p^1, p^0, y^1, y^0)}{W_K(w^1, w^0, y^1, y^0)} \times \frac{Y_F(y^1, y^0, p^1, p^0)}{XI_K(y^1, y^0, w^1, w^0, x^1, x^0)} \times \gamma_p, \tag{11}$$

and substituting (2) into (11) generates our first decomposition of profitability change

$$\frac{\Pi^1}{\Pi^0} = \frac{P_K(p^1, p^0, y^1, y^0)}{W_K(w^1, w^0, y^1, y^0)} \times \frac{Y_F(y^1, y^0, p^1, p^0)}{\alpha_F/(\tau_F \times \varepsilon)} \times \gamma_p, \tag{12}$$

which should be contrasted with Diewert's decomposition (34). In (11) the price recovery index has Konüs structure and the implicit productivity index has mixed Fisher/Konüs structure. In (12) we replace the implicit Konüs input quantity index with

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the three non-price drivers of cost change, and hence productivity change, identified by Diewert. It is apparent from (9) that (12) satisfies the product test with R^1/R^0.

In the second restructuring we convert the price recovery index in (8) to purely Fisher structure by relating the Konüs input price index to a Fisher input price index by means of

$$W_K(y^1, y^0, w^1, w^0) = \left[\frac{x^{0T}w^1}{x^{0T}w^0} \times \frac{x^{1T}w^1}{x^{1T}w^0} \right]^{\frac{1}{2}}$$

$$\times \left[\frac{w^{0T}(x^0/D_i^0(y^0,x^0))/c^0(y^0,w^0)}{w^{1T}(x^0/D_i^0(y^0,x^0))/c^0(y^0,w^1)} \times \frac{w^{0T}(x^1/D_i^1(y^1,x^1))/c^1(y^1,w^0)}{w^{1T}(x^1/D_i^1(y^1,x^1))/c^1(y^1,w^1)} \right]^{1/2}$$

$$= W_F(w^1, w^0, y^1, y^0) \times \gamma_W, \qquad (13)$$

in which $W_F(w^1, w^0, y^1, y^0)$ is a Fisher input price index and

$$\gamma_W = \left[\frac{AE_i^0(y^0, w^0, x^0)}{AE_i^0(y^0, w^1, x^0)} \times \frac{AE_i^1(y^1, w^0, x^1)}{AE_i^1(y^1, w^1, x^1)} \right]^{1/2} \qquad (14)$$

provides an economic interpretation of the relationship between the theoretical Konüs input price index and the empirical Fisher input price index. γ_w is the geometric mean of two input-oriented allocative efficiency ratios. If x^0 is more allocatively efficient relative to w^0 than to w^1 on base period technology, the first ratio is bounded below by unity. If x^1 is more allocatively efficient relative to w^1 than to w^0 on comparison period technology, the second ratio is bounded above by unity. We therefore expect their geometric mean to approximate unity, and $W_K(y^1, y^0, w^1, w^0)$ and $W_F(w^1, w^0, y^1, y^0)$ to be approximately equal.[3]

Substituting (13) into (8) yields

$$\frac{\Pi^1}{\Pi^0} = \frac{P_F(p^1, p^0, y^1, y^0)}{W_F(w^1, w^0, y^1, y^0)} \times \frac{Y_F(y^1, y^0, p^1, p^0)}{(\gamma_W \times XI_K(y^1, y^0, w^1, w^0, x^1, x^0))}, \qquad (15)$$

which pairs a Fisher price recovery index with a Fisher productivity index, since the product test requires $\gamma_W \times XI_K(y^1, y^0, w^1, w^0, x^1, x^0) = X_F(x^1, x^0, w^1, w^0)$. More

importantly, the Fisher input quantity index can be decomposed. Substituting (2) into the implicit Konüs input quantity index component of the Fisher input quantity index yields

$$X_F(x^1, x^0, w^1, w^0) = \frac{\gamma_W \times \alpha_F}{\tau_F \times \varepsilon}, \tag{16}$$

and

$$\frac{\Pi^1}{\Pi^0} = \frac{P_F(p^1, p^0, y^1, y^0)}{W_F(w^1, w^0, y^1, y^0)} \times \frac{Y_F(y^1, y^0, p^1, p^0)}{(\gamma_W \times \alpha_F)/(\tau_F \times \varepsilon)}, \tag{17}$$

which also should be contrasted with Diewert's decomposition (34). In (17) both the price recovery index and the productivity index have Fisher structure, and the productivity index decomposes into the cost-oriented drivers of productivity change.

5. Conclusions

We have derived a pair of cost-based decompositions of profitability change to complement a decomposition proposed by Diewert. Both are based on economically meaningful analytical expressions for the relationships between theoretical Konüs and empirical Fisher output and input price indexes. We believe our two decompositions have some advantages over that proposed by Diewert.

Both decompositions (12) and (17) have "pure" structures, with either Konüs or Fisher price recovery index and Fisher output quantity index. Both contain the three non-price drivers of cost change, and hence productivity change, and neither contains Diewert's returns to scale measure. Both contain an allocative efficiency effect, either γ_p or γ_W, that provides an easily interpreted link between the respective Konüs and Fisher price indexes. The two effects have the added advantage of enabling one to quantify the magnitude of allocative inefficiency and to generate a statistical test of the null hypothesis of allocative efficiency.

An advantage of (12) is that, in principle, a Konüs price recovery index can be decomposed into the product of theoretical drivers of price recovery. To the best of our knowledge this exercise has not been undertaken, but it is well worth exploring.

An advantage of (17) is that, in practice, a Fisher price recovery index can be decomposed into the product of M+N individual price drivers of price recovery. An added advantage of (17) is that its allocative efficiency effect is cost-oriented, which is consistent with the cost-oriented decomposition of productivity change.

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Endnotes

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[1] Georgescu-Roegen (1951;103) introduced profitability (which he called return to the dollar) as a financial performance indicator into the economics literature, and noted its independence of the scale of production, a virtue not shared by cost, revenue or profit.

[2] Diewert (1981;174).

[3] The reasoning behind (14), and (10) above, does not require within-period allocative efficiency; contrast Balk (1998;36). In addition, (14) becomes an equality if either w is a scalar or $w^1 = \lambda w^0$, $\lambda > 0$, and (10) becomes an equality if either p is a scalar or $p^1 = \lambda p^0$, $\lambda > 0$. There is a structural resemblance, but not equality, between our γ_p and γ_w and the allocative efficiency terms of Ray and Mukherjee (1996; (19b)) and Kuosmanen and Sipiläinen (2009; (25)).