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# The Effect of Firm Cash Holdings on Monetary Policy

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### Abstract

Firm cash holdings increased substantially from 1980 to 2013. The overall distribution of firm cash holdings changed in the same period. We study the implications of these changes for monetary policy. We use Compustat data and a model with financial frictions that allows the calculation of the monetary policy effects according to the distribution of cash holdings. We find that the interest rate channel of the transmission of monetary policy has become more powerful, as the impact of monetary policy over real interest rates increased. With the observed changes in firm cash holdings, the real interest rate takes 3.4 months more to return to its initial value after a shock to the nominal interest rate.

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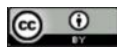
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## 1. Introduction

We obtain predictions for the effect on monetary policy of the changes in the distribution of corporate cash holdings. Bates et al. (2009), Bover and Watson (2005) and other studies have noticed the increase in corporate cash holdings since 1980, both in real terms and as a percentage of total cash. Corporate cash holdings corrected for inflation increased five times from 1980 to 2010. The median cash-sales ratio increased from 3% in 1980 to 12% in 2010. The mean cash-sales ratio increased from 6% to 23% during the same period. Corporate cash holdings, measured as cash and equivalents of the U.S. nonfinancial firms listed in Compustat, amounted to 1.56 trillion dollars in 2010. As M1 amounted to 1.84 trillion, according with the FED of St. Louis data, 1.56 trillion dollars corresponds to 85% of M1. The ratio of corporate cash holdings decreased from 2010 to 2013, as M1 has increased sharply more recently. Even so, corporate cash holdings to M1 was equal to 65% in 2013. Moreover, this ratio was 23% on average from 1980 to 1999 and 75% from 2000 to 2010. The demand for money from corporations is substantial. Changes in corporate cash holdings can affect monetary aggregates and monetary policy significantly.<sup>1</sup>

Our objective is to analyze the implications of the secular increase in corporate cash holdings on the effects of monetary policy. We find that the real interest rate takes 3.4 more months in 2013 than in 1980 to revert to its initial value after a nominal interest rate shock. These findings are obtained in a model that simulates the effects on the real interest rate of a nominal interest rate shock. The main characteristic of the model is that it takes into account the observed distribution of money holdings.

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<sup>1</sup>We restrict our sample to firms with positive cash, positive assets, assets greater than cash, and sales greater than 10 million (CPI adjusted with base 1982-1984). We also truncated the firms at the 1 and 99 percentiles of the cash-sales ratio. With the less stringent constraint of sales greater than zero, the increase in the median cash-sales ratio is from 3.5% to 13.4%, an increase of 3.8 times. There are different measures of cash holdings such as the cash-assets and the cash-net assets ratio. We use the cash-sales ratio because it has a better data counterpart to the variables in the model. We explain this variable in more detail in section 1.

According to the model, the real interest rate takes 1.84 months to revert to its initial value with the distribution of money holdings of 1980, while with the 2013 distribution of money holdings, the real interest rate takes 5.25 months to revert to its initial value. Figure (1) shows how long the real interest rate takes to return to its initial value from 1980 to 2013 according to our simulations.

There is a large literature on the determinants of firm cash holdings. Among the explanations for firm cash holdings, a partial list includes the transactions role of cash (Baumol 1952, Tobin 1956, Miller and Orr 1966, Frenkel and Jovanovic 1980), financial constraints (Almeida et al. 2004, Acharya et al. 2007), tax purposes (Foley et al. 2007), agency problems (Kalcheva and Lins 2007), and corporate governance (Jensen 1986, Blanchard et al. 1994, Dittmar et al. 2003, Pinkowitz et al. 2006, Dittmar and Mahrt-Smith 2007, Harford et al. 2008, Yun 2009). Empirically, the different determinants of firm cash holdings are analyzed by Kim et al. (1998), Opler et al. (1999), and Ozkan and Ozkan (2004).

The increase in cash holdings is surprising, as the evolution of the technology of financial transactions allows firms to sell illiquid assets for cash frequently and to maintain their operations with little cash. It is also surprising to find that firms hold more than half of M1. It would be expected that households would have more difficulty than firms to manage cash, as households face higher transactions costs and have more difficulty in using credit. We do not aim to explain cash holdings or their secular trend (Bates et al. 2009 identify four causes for the increase: an increase in R&D expenditures, a fall in inventories, a fall in capital expenditures, and an increase in cash flow risk). We study the implications of the increase in corporate cash holdings on the effects of monetary policy. As firms hold a large portion of the monetary aggregates, it is important to study the effects of the increase in cash holdings on macroeconomic variables. To the best of our knowledge, we are the first to study the consequences of the changes in the distribution of corporate cash for

monetary policy.<sup>2</sup>

A related paper is Cole and Ohanian (2002), which studies how shifts in the demand for money affect the macroeconomic impact of monetary shocks. We emphasize here the changes in the distribution of cash holdings across firms. Cole and Ohanian (2002) find that changes in the relative demands for money between households and firms have quantitatively important macroeconomic effects in a liquidity model.<sup>3</sup>

We analyze how changes in firm cash holdings affect macroeconomic variables. Fresard (2010) and Palazzo (2012) study the real effects of cash holdings on market share and on equity returns. Here, we study the real effects of the change in the distribution of cash holdings across firms in the aggregate economy. We take the distribution of cash holdings as given and study the effects on the real interest rate.

As we are interested in the effects of the distribution of money holdings, we use a model in which the distribution of money holdings plays an active role. In the first cash-in-advance models such as Lucas and Stokey (1987), Cooley and Hansen (1989), and Hodrick et al. (1991), the distribution of money holdings is degenerate. All participants in the economy behave as a representative agent and they have the same demand for money. We cannot evaluate the impact of the distribution of money with these models because they do not allow any role for the distribution of money.

More recently, the real effects of monetary policy have been studied in new Keynesian models (for example, Clarida et al. 1999, Woodford 2003, and Christiano et al. 2005). These models contain frictions usually in the form of price rigidities. There is a distribution of prices across firms, but the distribution of money is again

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<sup>2</sup>A recent paper that studies the interaction of firm cash holdings and macroeconomic variables is Bacchetta, Benhima, and Poilly (2014).

<sup>3</sup>Cole and Ohanian do not confirm major changes in the effects of money shocks over the postwar period, as indicated with VARs. However, as Chari, Kehoe and McGrattan (2008) point out, the procedure of comparing impulse responses from structural VARs to the theoretical impulse responses from models and rejecting the model if they are different can be misleading. In particular, this procedure cannot be used to reject the liquidity model, as liquidity models do not satisfy the set of identifying assumptions in VARs.

degenerate. A representative agent uses all money carried from the last period to buy products in the current period. As in the cash-in-advance models, the distribution of money holdings in these models does not affect the results of monetary policy. Other kinds of frictions, such as informational frictions (Mankiw and Reis 2002) and menu costs (Golosov and Lucas 2007), have also been introduced to study the real effects of monetary policy. Alternatively, Stein (1998), Kashyap and Stein (2000), and Bolton and Freixas (2006) focused on the role of bank lending.

Here, to take into account the effects of changes in the distribution of cash holdings, we use a market segmentation model. The friction in this kind of model is the separation of markets for liquid and illiquid assets. Liquid assets are used for transactions while illiquid assets receive higher interest yields and are kept mainly as a reserve of value. These markets are separated in the sense that firms cannot exchange illiquid assets for cash with a high frequency.

In the data, there is a nondegenerate cross-sectional distribution of cash. Certainly, the different firm characteristics, such as corporate governance and idiosyncratic risk, are reflected in their behavior toward cash management. We modify the models in Alvarez et al. (2009) and Silva (2012) to match the observed distribution of firm cash holdings in the data. Alvarez et al. (2009) show that the model closely matches the short-run fluctuations in velocity. Here, we use the model to obtain a prediction about the effects of the increase in cash holdings. The prediction is obtained by calculating the response of the real interest rate to a nominal interest rate shock for each year from 1980 to 2013. Our model delivers closed-form solutions for each nominal interest shock. The shocks follow the interest rate dynamics in Christiano et al. (1999) and Uhlig (2005). For each year, we recalibrate the model to fit the distribution of cash holdings. As the distribution of cash holdings changes, the response of the real interest rate changes.

The real effects occur because firms' behavior with respect to the use of cash de-

depends on their cash holdings at the time of the shock. Firms with little cash adapt faster to the shock while firms with large cash holdings take longer to adapt. If the market segmentation friction is removed, the real interest rate does not move after the shock and the real effects vanish. As we want to isolate the effects of the change in cash holdings, we eliminate other mechanisms besides market segmentation that could generate additional real effects. In particular, there are no sticky prices, output is constant, and the only change in the economy during the period is in the distribution of cash holdings. That is, the changes in firm characteristics during the period are reflected in the distribution of cash holdings.

Heterogeneity across cash holdings changes the speed and the size of the adjustment to the shock. If all firms held the same amount of cash, the mean level, for instance, then monetary shocks would have different real effects. This property is not unique to our model, the new Keynesian Phillips curve model shares this property. Carvalho and Nechio (2011) show that heterogeneity in the price setting behavior of firms implies aggregate dynamics substantially different from the case when all firms have the same price setting behavior. Here, after the shock hits the economy, the initial phase of the adjustment process is driven mainly by the set of firms with less cash. The later part of the adjustment process is dominated by the set of firms with larger stocks of cash.

We find that the effects of monetary policy over the real interest rate are now more persistent than in 1980, as there has been a substantial increase in the cash holdings by firms since then. The effects of monetary policy are now stronger, as monetary authorities have more ability to affect real variables. Consistent with this idea, Clarida et al. (2000) state that monetary policy has been more effective after 1980.

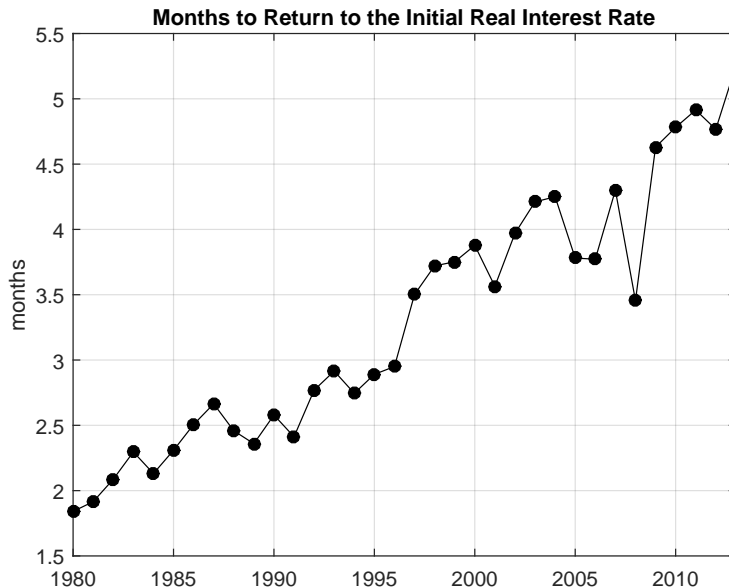


FIG. 1: Simulations with the model of section 3 for a given nominal interest rate shock. The simulations take into account the distribution of the cash-sales ratio for each year.

## 2. The Distribution of Cash Holdings over Time

Figure (2) shows the median and the mean of the cash-sales ratio from 1980 to 2013. Different measures of cash have been used to analyze firm cash holdings such as the cash-net assets ratio (used, for example, by Opler et al. 1999) and the cash-assets ratio (by Bates et al. 2009). The cash-sales ratio has been used, among others, by Mulligan (1997), Harford (1999), Harford et al. (2008), and Bover and Watson (2005). Both, the cash-assets ratio and the cash-sales ratio, have been increasing substantially over time. The cash-assets ratio indicates the way in which a firm allocates cash in its portfolio of assets. The cash-sales ratio indicates how much cash a firm holds with respect to the flow of resources obtained with its operations. It has a more direct interpretation in terms of the use of cash for transactions. The conclusions of this paper are robust to the use of one measure or the other. We use the cash-sales ratio



because its interpretation—cash relative to the flow of resources obtained—allows a better connection between the model parameters and the data.<sup>4</sup>

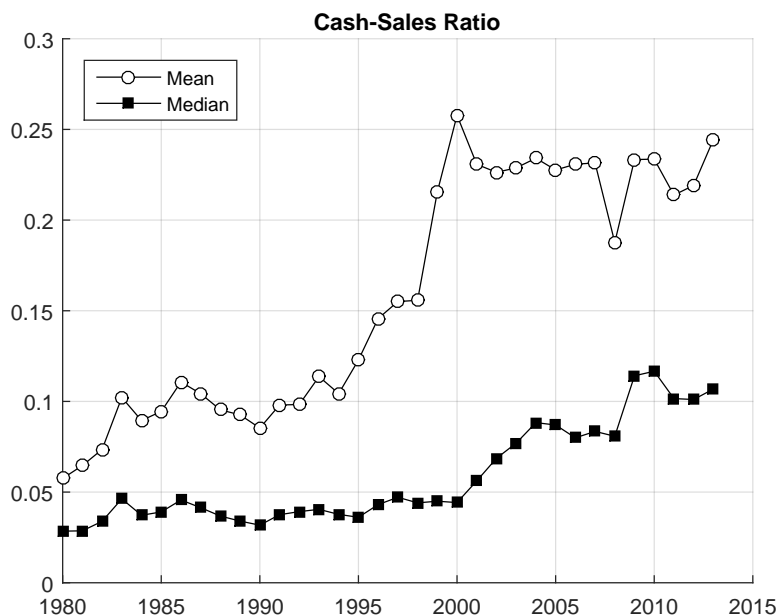


FIG. 2: Mean and median of the cash-sales ratio across firms for each year. The cash-sales ratio state how much firms maintain of their sales in cash. A cash-sales ratio of 0.1, for example, means that firms maintain 10 percent of their yearly sales, or 1.2 months of sales, in cash. Source: Compustat; see note 4 for details.

As cash is measured in dollars and sales is measured in dollars per unit of time, the cash-sales ratio is a variable given in units of time. The median cash-sales ratio of 0.12 year in 2010, for example, means that firms maintained about 1.4 months of their sales in the form of cash. In 1980, this same ratio was only 0.03, or 11 days. The mean cash-sales ratio in the same period increased from 0.06 in 1980 to 0.23 in 2010.

<sup>4</sup>Our measure of cash is cash and equivalents from Compustat, “cash and short-term investments,” CHE, U.S. nonfinancial firms. CHE is not available for utilities, so the dataset removes this sector. To avoid anomalies, we remove observations with cash or assets equal to zero, and observations with cash greater than assets. To avoid extreme cash-sales ratios, we remove observations with sales smaller than 10 million and observations with cash/sales below the 1st and above the 99th percentiles of cash/sales. We later report results without this truncation, which barely changes results. We correct for inflation with the CPI from the FED St. Louis, CPIAUCSL, base 1982-84. For sales, we use SALE in Compustat. Our procedure implies 140,435 firm-years or about 4,130 firms per year.

The distribution of the cash-sales ratio across firms is highly asymmetric as it can be inferred by the difference between its mean and median. The mean was more than two times the median during the whole period and it reached 5.8 times the median in 2000.

If there was no benefit of maintaining cash, firms would choose a cash-sales ratio approximately equal to zero, as holding cash implies an opportunity cost in interest foregone. As the cash-sales ratio is sizeable in economic terms, the data indicate the existence of costs in the management of money. These costs may be in the form of transaction costs or in the form of management costs. A portfolio manager, for example, may schedule sales of long term bonds to coincide with cash needs, but this and other more elaborate schedules of payments are costly. It does not matter the nature of the costs of managing cash holdings for our purposes. What is important is that firm cash holdings are positive and considerable. We take the values of firm cash holdings as given.

Usually, firms maintain cash-sales ratios smaller than one. The 95th percentile of the distribution of the cash-sales ratio reached a maximum of 1.3 in 2000 and it was about 1 during 2002-2007. A cash-sales ratio above one means that a firm keeps more than one year of sales in the form of cash. Firms that maintain high cash-sales ratios tend to be smaller firms in terms of sales; the same is true for the cash-assets ratio. Figure (3) shows the median of the cash-sales ratio over the same period for firms grouped in percentiles of sales. We see that the cash ratio increased for all groups. Moreover, while the cash ratio increased 3 times for all firms as a whole, it increased 5 times for firms in the smaller percentiles. Bates et al. (2009) show a similar evolution for the cash-assets ratio.

In addition to the increase in the cash-sales ratio, firm cash holdings correspond to a large fraction of the monetary aggregates and this fraction has increased substantially. From 1980 to 2010, the ratio between firm cash holdings to M1 increased from 30

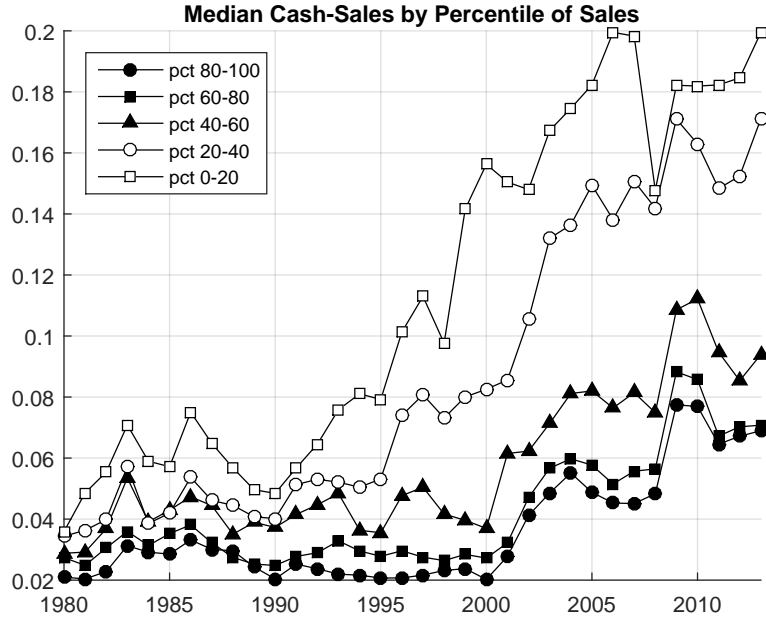


FIG. 3: Median of the cash-sales ratio for different percentiles of sales. Source: Compustat; see note note 4 for details.

percent to 85 percent. This fraction decreased to 65 percent in 2013, still more than two times the ratio in 1980. As we show in this paper, a consequence of the increase in the proportion of firm cash holdings over monetary aggregates is that monetary policy has a much stronger effect than it had in the past.<sup>5</sup>

Figure (4) shows the distribution of the cash-sales ratio for each year. The distributions look symmetric because the figure shows the logs of the cash-sales ratio. The support and the median of the distribution of the cash-sales ratio increased over time. The support of the distribution increased first and the median increased later. In 1980, the maximum cash-sales ratio was equal to 7 months, i.e., below one year.

<sup>5</sup>M1 is defined as currency plus traveler checks plus checkable deposits. In January 2014, currency corresponds to 43.6% of M1 and checkable deposits to 56.3%. The definition of cash and equivalents in Compustat includes the components of M1 and “securities readily transferable to cash,” which includes short term commercial paper, short term government securities, and money market funds. In our sample, the cash portion of cash and equivalents correspond on average to 70% of cash and equivalents.

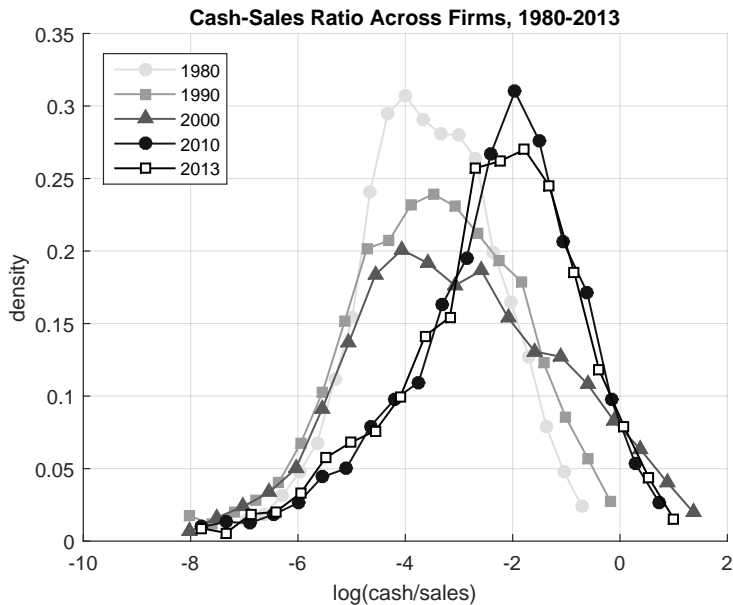


FIG. 4: Distribution of the cash-sale ratio across firms from 1980 to 2013 for selected years. Each curve has the distribution for one year (density histograms with 20 groups). The curves are approximately symmetric because it shows the logs of the cash-sales ratio; the actual distribution is highly asymmetric. Over the years, the support and the median of the cash-sales ratio increased. Source: Compustat; see note 4 for details.

The maximum cash ratio was above 1 after 1983. In 2000, the maximum cash ratio was 5 years (the 95th percentile was 1.3). Figure (2) shows that the increase in the median of the cash ratio accelerated after 2000 and figure (4) shows that the distribution of cash holdings changed substantially after this date. The two figures complement each other as they show that firm cash holdings changed especially after 2000.

As figure (6) shows, the distribution of cash holdings across firms is not uniform; it is far from degenerate; and it has changed over time. Our objective is to calculate the predictions of the effects of monetary policy shocks under different distributions of cash holdings. In order to do so, we need a model that takes into account the different distributions of cash holdings. We introduce this model in the next section.

### 3. The Model

The model combines the cash inventory framework analyzed by Baumol (1952) and Tobin (1956), as agents use cash holdings gradually during holding periods, with the market segmentation framework introduced by Grossman and Weiss (1983), Rotemberg (1984), and later studied, among others, by Grossman (1987), Alvarez et al. (2009), and Silva (2012). The economy is composed by firms with different bond and cash holdings. There is market segmentation between the goods market and the assets market. Firms trade bonds for money in the asset market and goods for money in the goods market. Market segmentation implies that firms sell bonds for money and transfer the proceeds periodically from the asset market to the goods market. As a result, there will be a nondegenerate distribution of cash and bond holdings in the economy.

The model is closely related to Silva (2012). In Silva (2012), there is an explicit cost of transferring money from the assets market to the goods market and the holding period is obtained endogenously. Here, we abstract from this cost and set the holding period exogenously as we focus on the short-run dynamics of interest rates. It is assumed implicitly that the short-run dynamics will not affect the holding periods in an important way.<sup>6</sup> On the other hand, different holding period lengths are allowed for the distinct groups of firms in the economy. In this way, we match the distribution of firms' cash holdings observed in the data.

The groups of firms are indexed by  $i = 1, \dots, I$ . The fraction of firms in each group is given by  $v_i$ , where  $\sum_{i=1}^I v_i = 1$ . Each firm has a bank account and a brokerage account. The bank account is used to hold cash for transactions in the goods market. The brokerage account is used to trade bonds in the asset market. An important

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<sup>6</sup> Alvarez et al. (2009) also keep holding periods fixed in the short run. Adao and Silva (2014) endogenize the decision of the size of the holding period,  $N$ , common to all agents in the model. Our modifications imply closed-form solutions for the effects of shocks.

assumption is that the return of assets in the brokerage account is higher than the return of cash in the bank account. It is assumed to simplify that the return on cash is zero and the return on assets in the brokerage account is positive. Time is continuous,  $t \geq 0$ . Let  $M_{0i}$  denote cash holdings at  $t = 0$  of firms in the group  $i$  and let  $B_{0i}$  denote bond holdings at  $t = 0$  of firms in the group  $i$ . Each firm has an initial portfolio given by  $s_i = (M_{0i}, B_{0i})$ .

The time interval between transfers of money from the asset market to the goods market, the holding period, is the same for all firms that belong to the same group and is denoted by  $N_i$ . Different holding periods express different forms of portfolio management by the firms in the economy, which we do not address in the paper. We do not provide an explanation of why firms have different holding period sizes, instead we take the distribution of cash holding period sizes  $\{N_i\}_{i=1}^I$  that reproduces the distribution of cash holdings over time, which is given by figure (6).

The firms in group  $i$  are distributed uniformly over the interval  $[0, N_i)$ . Alvarez et al. (2009) and Grossman (1987) also dispose agents uniformly over the holding period, but they only consider one holding period size. In contrast, we allow for various holding period with sizes  $\{N_i\}_{i=1}^I$ .

Firms in the group  $i$  produce  $Y_i$  goods at time  $t$  and obtain  $P(t)Y_i$  of sales at time  $t$ , where  $P(t)$  denotes the price level at time  $t$ . The proceeds of sales are deposited directly to the brokerage account and converted into bonds. The price of bonds at time  $t$  is given by  $Q(t)$ , with  $Q(0) = 1$ . The nominal interest rate is  $r(t) \equiv -d \log Q(t) / dt$ . The portfolio choice across different bonds in the brokerage account is not important for our results. For this reason, we simplify the model by having one positive and deterministic rate of return  $r(t)$ . The rate of return on assets in the brokerage account is the opportunity cost of money, and the firms will manage cash over the holding period according with the path of  $r(t)$ .

Let  $T_{ji}(s_i)$ ,  $j = 1, 2, \dots$ , denote the times of the transfers of firm  $s_i$ , and define

$T_{0i}(s_i) \equiv 0$  as that will simplify the notation later. Firms start with initial values of cash and bond holdings given by the pair  $(M_{0i}, B_{0i})$ . At  $T_{ji}(s_i)$ , firm  $s_i$  sells bonds for money and transfers the proceeds to the goods market. The holding period of firm  $s_i$  is  $[T_{j,i}(s_i), T_{j+1,i}(s_i))$ . We have  $T_{j+1,i} - T_{j,i} = N_i$  for  $j = 1, 2, \dots$  for all  $s_i$  firms,  $i = 1, \dots, I$ . Cash holdings are denoted by  $M(t, s_i)$ . Cash just after a transfer is denoted by  $M^+(T_{ji}(s_i), s_i)$  and is equal to  $\lim_{t \rightarrow T_{ji}, t > T_{ji}} M(t, s_i)$ . Analogously, cash just before a transfer is denoted by  $M^-(T_{ji}(s_i), s_i)$  and is equal to  $\lim_{t \rightarrow T_{ji}, t < T_{ji}} M(t, s_i)$ . The transfer amount from the brokerage account to the bank account is given by  $M^+ - M^-$ . Similarly, bonds just before a transfer and just after a transfer are given by  $B^-(T_{ji}(s_i), s_i)$  and  $B^+(T_{ji}(s_i), s_i)$ , respectively. If the amount of cash transferred to the bank account is positive, then  $B^- > B^+$ . Cash holdings in the brokerage account are zero, as cash does not receive interest and it is not possible to purchase goods with cash in the brokerage account. The optimal policy is to keep most of the resources in bonds in the brokerage account and make periodical transfers to the bank account.

The firm manager of firm  $s_i$  starts with assets  $(M_{0i}, B_{0i})$ , receives a flow of funds  $P(t)Y_i$  in the brokerage account, and aims to achieve an optimal amount of transactions  $c_i(t, s_i)$ . The measurement unit for transactions is chosen so that its price is  $P(t)$ . We assume the manager prefers a smooth flow of transactions  $c_i(t, s_i)$  to an uneven one with the same average. Because it simplifies the analysis we take that the objective function the manager seeks to maximize is logarithmic in the amount of transactions. The logarithmic utility allows us to obtain analytical solutions for the dynamics of the real interest rate after shocks.

The problem of the firm manager is to choose transactions  $c_i(t, s_i)$ , cash  $M_i(t, s_i)$ ,

and bonds  $B_i(t, s_i)$  such that

$$\max_{c, B, M} \sum_{j=0}^{\infty} \int_{T_{ji}(s_i)}^{T_{j+1, i}(s_i)} e^{-\rho t} \log(c_i(t, s_i)) dt \quad (1)$$

subject to

$$M_i^+(T_{ji}(s_i)) + B_i^+(T_{ji}(s_i)) = M_i^-(T_{ji}(s_i)) + B_i^-(T_{ji}(s_i)), \quad j = 1, 2, \dots \quad (2)$$

$$\dot{B}_i(t, s_i) = r(t) B_i(t, s_i) + P(t) Y_i, \quad t \geq 0, t \neq T_{1i}(s_i), T_{2i}(s_i), \dots, \quad (3)$$

$$\dot{M}_i(t, s_i) = -P(t) c_i(t, s_i), \quad t \geq 0, t \neq T_{1i}(s_i), T_{2i}(s_i), \dots, \quad (4)$$

$M_i(t, s_i) \geq 0$ ,  $c_i(t, s_i) \geq 0$ , given  $M_{0i}(s_i) \geq 0$ , and where  $\rho > 0$  is the rate of intertemporal discounting. At  $t = T_{1i}(s_i), T_{2i}(s_i), \dots$ , constraint (3) is replaced by  $\dot{B}_i(T_{ji}(s_i), s_i)^+ = r(t) B_i^+(T_{ji}(s_i), s_i) + P(t) Y_i$ , where  $\dot{B}_i(T_{ji}(s_i), s_i)^+$  is the right derivative of  $B_i(t, s_i)$  with respect to time at  $t = T_{ji}(s_i)$ . Similarly, constraint (4) is replaced by  $\dot{M}_i(T_{ji}(s_i), s_i)^+ = -P(t) c_i^+(T_{ji}(s_i), s_i)$ , where  $\dot{M}_i(t, s_i)^+$  is the corresponding right derivative for cash and  $c_i^+(T_{ji}(s_i), s_i)$  is transactions just after the transfer.

With (3), we can write  $B_i^-(T_{ji})$  as a function of the interest payments accrued during  $[T_{j-1}, T_j]$ . Substituting recursively and using the no-Ponzi condition  $\lim_{j \rightarrow +\infty} Q(T_j) \times B_i^+(T_{ji}) = 0$ , we obtain the present value constraint

$$\sum_{j=1}^{\infty} Q(T_{ji}(s_i)) M_i^+(T_{ji}(s_i), s_i) \leq \sum_{j=1}^{\infty} Q(T_{ji}(s_i)) M_i^-(T_j(s_i), s_i) + W_{0i}(s_i), \quad (5)$$

where  $W_{0i}(s_i) \equiv B_{0i}(s_i) + \int_0^{\infty} Q(t) P(t) Y_i dt$ . Constraint (5) states that the present value of cash transfers is equal to the present value of deposits in the brokerage account.

To minimize the cost of holding money, firms adjust cash  $M_i^+(T_j)$  and the use of



cash during holding periods so that  $M_i^-(T_{j+1,i}) = 0$ . That is, cash transfers should be just enough for the transactions during the holding period. Only  $M_i^-(T_{1i})$  might be positive because  $M_{0i}$  is given. Since  $M_i^-(T_{ji}) = 0$ , for  $j \geq 2$ , then from (4), cash at time  $t$  is given by  $M_i(t, s_i) = \int_t^{T_{j+1,i}(s_i)} P(\tau) c_i(\tau, s_i) d\tau$ ,  $T_{ji}(s_i) \leq t < T_{j+1,i}(s_i)$ ,  $j = 1, 2, \dots$ . Cash at the beginning of a holding period is given by

$$M_i^+(T_{ji}(s_i), s_i) = \int_{T_{j,i}(s_i)}^{T_{j+1,i}(s_i)} P(\tau) c_i(\tau, s_i) d\tau, j = 1, 2, \dots \quad (6)$$

Below, instead of solving the problem of maximizing (1) subject to (2)-(4), we consider the simpler problem of maximizing (1) subject to (4) and (5). The transactions,  $c_i(t, s_i)$ , and cash,  $M_i(t, s_i)$ , that solve this simpler problem can be used in (2) and (3) to obtain the bonds,  $B_i(t, s_i)$ .

The government executes monetary policy through open market operations, that is, by exchanging bonds for money with the firm managers in the assets market. The government supplies aggregate cash  $M(t)$ . An increase in the supply of cash generates revenue, or seigniorage, given by  $\dot{M}(t)/P(t)$ . We abstract from government consumption or taxes to concentrate on the effects of monetary policy. As a result, the government budget constraint is given by  $B_0^G = \int_0^\infty Q(t) \dot{M}(t) dt$ , where  $B_0^G$  is the aggregate supply of government bonds.

The market clearing condition for cash is given by  $\sum_i v_i \int M_i(t, s_i) dF(s_i) = M(t)$ , where  $F$  is the distribution of  $s_i$ . As stated above,  $F(s_i)$  is a uniform distribution over  $[0, N_i)$ , for  $i = 1, \dots, I$ . Similarly, the market clearing conditions for bonds and goods are given by  $B_0^G = \sum_i v_i \int B_{0i}(s_i) dF(s_i)$  and  $\sum_i v_i \int c_i(t, s_i) dF(s_i) = Y$ , respectively.

The equilibrium is defined as a vector of prices  $\{P(t), Q(t)\}$ , and allocations  $\{M_i(t, s_i), B_i(t, s_i), c_i(t, s_i)\}_{i=1}^I$  such that  $\{M_i(t, s_i), B_i(t, s_i), c_i(t, s_i)\}_{i=1}^I$  solves the maximization problems (1)-(4) given  $\{P(t), Q(t)\}$  for all  $s_i$  in the support of  $F(s_i)$ ;

the government budget constraint holds; and the market clearing conditions for cash, bonds, and goods hold.

#### 4. The Distribution of Cash Holdings

The opportunity cost of holding cash implies that it is optimal to start a holding period with more cash than in the rest of the holding period and spend this cash gradually until the next transfer, which initiates a new holding period. The firms engage in  $(S, s)$  policies on spending, bonds, and cash. Aggregate variables are obtained by the aggregation of the  $(S, s)$  policies across firms. The aggregate cash holdings at time  $t$ , for example, is obtained by the aggregation of the cash holdings of each firm at  $t$ .

We first characterize an economy in the steady state; an equilibrium in which the inflation rate is constant and the interest rate is constant. We then study shocks to an economy initially in the steady state. With constant inflation and interest rate, the  $(S, s)$  policies of the firms in each group have the same pattern. Transactions, for example, have the same pattern across firms in the same group.

The relevant variable for a firm is its position in the holding period. Let  $n_i \in [0, N_i)$  denote the position of a firm of the group  $i$  in the holding period. Firm  $n_i$  makes transfers from the brokerage account to the bank account at  $T_{1i}(n_i) = n_i$ ,  $T_{2i}(n_i) = n_i + N_i$  and so on.

Consider the pattern of transactions for each firm. The first order condition for  $c_i(t, n_i)$  of the problem of maximizing (1) subject to (4) and (5) implies  $P(t) c_i(t, n_i) = e^{-\rho t} / [\lambda_i(n_i) Q(T_j)]$ , for  $t \in (T_{ji}(n_i), T_{j+1,i}(n_i))$ ,  $j \geq 1$ , where  $\lambda_i(n_i)$  is the Lagrange multiplier of (5). Let  $c_{0i}$  denote transactions at the beginning of a holding period for firms in group  $i$ . Then, transactions during the holding periods of firms in group  $i$  are given by  $c_i(t, n_i) = c_{0i} e^{(r-\pi-\rho)t} e^{-r(t-T_{ji})}$ , for the largest  $j$  such that  $t \in [T_{ji}(n_i), T_{j+1,i}(n_i))$ . Integrating the transactions of all firms in group  $i$ , we ob-

tain the aggregate transactions of group  $i$ ,  $C_i(t) = c_{0i}e^{(r-\pi-\rho)t}(1 - e^{-rN_i})/(rN_i)$ . In the steady state, aggregate transactions for firms in the same group are constant. Therefore, in a steady state equilibrium, the nominal interest rate is equal to the inflation rate plus the real interest rate  $\rho$ , i.e.  $r = \rho + \pi$ .

Given  $r = \rho + \pi$ , then  $c_i(t, n_i) = c_{0i}e^{-r(t-T_{ji})}$ . Transactions during holding periods must be equal to the cash generated by sales during the same holding period. Therefore,  $\int_{T_{ji}}^{T_{j+1,i}} c_i(t, n_i) dt = \int_{T_{ji}}^{T_{j+1,i}} Y_i dt$ , where  $T_{j+1,i} - T_{ji} = N_i$ . Substituting  $c_{0i}e^{-r(t-T_{ji})}$  yields the value of transactions at the beginning of holding periods,  $c_{0i}(1 - e^{-rN_i})/(rN_i) = Y_i$ . As we will parameterize the model using data on the cash-sales ratio of firms, it is useful to characterize the variable transactions-sales ratio. Let  $\hat{c}_i \equiv c_i/Y_i$  denote the transactions-sales ratio of firms in group  $i$ . We have

$$\hat{c}_{0i}(r) \frac{1 - e^{-rN_i}}{rN_i} = 1, \quad (7)$$

which determines  $\hat{c}_{0i}$ , given  $N_i$  and the interest rate. The transactions-sales ratios during  $t \in [T_{ji}(n_i), T_{j+1,i}(n_i))$  for firms in group  $i$  are given by  $\hat{c}_i(t, n_i) = \hat{c}_{0i}e^{-r(t-T_{ji})}$ .

Aggregate cash holdings are equal to  $M(t) = \sum_i v_i \frac{1}{N_i} \int M_i(t, n_i) dn_i$ . Denote the cash-sales ratio by  $m(t) = M(t)/(P(t)Y)$ . In the appendix, we show that the cash-sales ratio in this economy is given by

$$m = \frac{1}{Y} \sum_{i=1}^I v_i \frac{c_{0i}(r)}{\rho} e^{-rN_i} \left[ \frac{e^{rN_i} - 1}{rN_i} - \frac{e^{(r-\rho)N_i} - 1}{(r-\rho)N_i} \right]. \quad (8)$$

The price level at time zero is equal to  $P_0 = M_0/(mY)$ , where  $M_0$  denotes the money supply at time zero and  $Y$  denotes sales. We interpret  $P_0$  as the price before the monetary shock hits the economy.

Cash holdings at time  $t$  for firms of group  $i$  are given by  $M_i(t, n_i) = \int_t^{T_{j+1,i}(n_i)} P(\tau) \times c_i(\tau, s_i) d\tau$ . The cash-sales ratios for firms in group  $i$  are given by  $m_i(n_i) = M_{0i}(n_i) \times$

$(P_0 Y_{0i})^{-1}$ ,  $n_i \in [0, N_i)$ , where  $M_{0i}(n_i)$  is the initial cash holdings. The  $M_{0i}(n_i)$  compatible with an equilibrium where  $r$  and  $\pi$  are constant is obtained by requiring that  $M_{0i}(n_i)$  is just enough to cover transactions from  $t = 0$  until the first transfer of firm  $n_i$ , at  $T_{1i} = n_i$ . The  $M_{0i}(n_i)$  are determined in the appendix. Dividing by  $P_0 Y_i$ , the cash-sales ratio of the firms along  $n_i \in [0, N_i)$  are given by

$$m_i(n_i) = \frac{r N_i e^{-r N_i} n_i e^{r n_i} (1 - e^{-\rho n_i})}{1 - e^{-r N_i} \rho n_i}. \quad (9)$$

According to (9), cash-sales ratios are distributed along  $[0, m_{H_i})$ , where  $m_{H_i} = \lim_{n_i \rightarrow N_i} m(n_i)$ . As firms are distributed uniformly along  $[0, N_i)$ , the density  $f_i(x)$  of firm cash-sale ratios is given by  $\frac{1}{N_i} \frac{\partial m_i^{-1}(x)}{\partial m}$ , where  $m_i^{-1}(x)$  is the value of  $n_i$  such that  $m_i(n_i) = x$ . There exists a unique value of  $m_i^{-1}(x)$ , as  $m_i(n_i)$  is increasing. Therefore,  $f_i(x) = \frac{1}{N_i} [rx + \frac{r N_i}{1 - e^{-r N_i}} e^{-r N_i} e^{(r - \rho) m_i^{-1}(x)}]^{-1}$ ,  $m_i \in [0, m_{H_i})$ . For the aggregate firms in the economy, the density function is  $f(x) = \sum_i v_i f_i(x) dx$ , where  $v_i$  is the fraction of firms distributed along  $[0, N_i)$ , which ensures that  $\int f(x) dx = 1$ . If we observe an economy with constant  $r$  and  $\pi$  at an arbitrary time, the cross section of the cash-sales will be given by  $f(x)$ ,  $x \in [0, \max(m_{H_i})]$ .

In the data, the distribution of real money holdings is concentrated on small quantities of money, but it is close to a uniform. In the parameterization, the values of  $v_i$  and  $m_{H_i}$  are set so that the model distribution of the cash-sales ratio approximates the actual distribution available in the Compustat data. Figure (5) shows an example with  $I = 4$ .

Figure (6) shows the actual distribution and the parameterized distributions for the years 1980 and 2010. The figure shows the typical shapes of the actual and parameterized distributions over the years. The actual distributions in the figure are the same as the ones shown in figure (4) for 1980 and 2010. For each year, the nominal interest rate,  $r$ , is the commercial paper interest rate and the values of  $m_{H_i}$  and  $v_i$  are

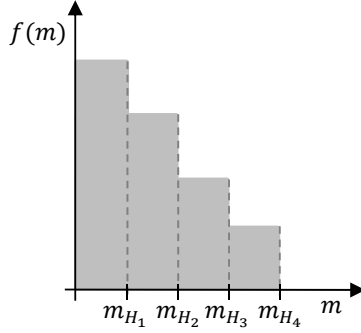


FIG. 5: The parameterization is made by finding the values of  $m_{H_i}$  and  $v_i$ ,  $i = 1, \dots, I$ , so that the model distribution of cash-sales ratios approximates the distribution in the data.  $I$  is set to 50 in the simulations.

found to match the actual distributions of the cash-sales ratios. As explained in the next section, instead of  $v_i$ , we use the fraction of sales of firms in group  $i$  with respect to total sales,  $v_{Y_i}$ , to obtain a counterpart with the data on cash-sales.<sup>7</sup> Figure (6) shows the logs of the cash-sales ratio, as the distribution is highly asymmetric toward small values of the cash-sales ratio.

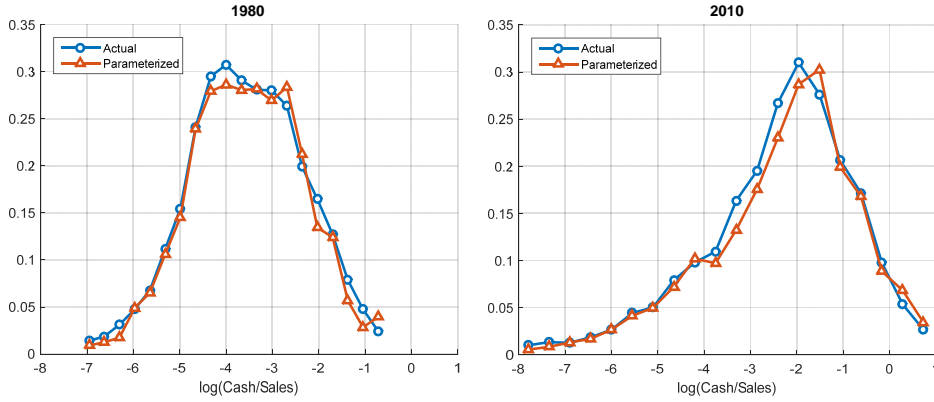


FIG. 6: Actual and parameterized distributions of the cash-sales ratio for 1980 and for 2010.

<sup>7</sup> $v_i = \frac{n_i}{\sum n_i}$  whereas  $v_{Y_i} = \frac{n_i Y_i}{\bar{Y}}$ . Both  $\sum v_i = \sum v_{Y_i} = 1$ . We have that  $v_{Y_i} = \frac{Y_i}{\bar{Y}} v_i$ , where  $\bar{Y} = \frac{\sum Y_i}{\sum n_i}$ . The advantage of using  $v_{Y_i}$  is to characterize the equilibrium variables by using the expressions of the consumption-income ratio.

## 5. Firm Cash Holdings and Monetary Policy Shocks

A monetary policy is summarized by a nominal interest rate path  $r(t)$ ,  $t \geq 0$ . The central bank sets the interest rate path, such as a target for the federal funds rate, and then changes the money supply to obtain the pre-determined target. In the model, a change in  $r(t)$  affects firm  $i$  cash holdings  $M_i(t, n_i)$ . The central bank supplies  $M(t)$  to satisfy the market clearing condition for cash. The interest rate path determines bond prices  $Q(t) = e^{-R(t)}$ , where  $R(t) = \int_0^t r(s) ds$ .

It is equivalent to set  $M(t)$  and obtain the equilibrium  $r(t)$  or to set  $r(t)$  and obtain the equilibrium  $M(t)$ . To obtain equilibrium prices, however, it is simpler to set  $r(t)$  and obtain the other equilibrium variables. The evidence suggests that the practice of central banks is to set monetary policy in terms of interest rates. By focusing on  $r(t)$  as the target for the monetary policy, we follow the literature, for example, Woodford (2003).

When an unexpected increase in the interest rate hits the economy, firms have different cash holdings  $M_{0i}(n)$ . Firms with little cash are about to make a transfer. These firms adapt fast to the shock because they make a transfer soon after the shock. With the bond trade and subsequent transfer, they adjust cash holdings taking into account the new interest rate path. Firms with large cash holdings take longer to make their first transfer after the shock. Until when they make a transfer, they can only adjust transactions.<sup>8</sup>

The different reactions of transactions affect the real interest rate. If the holding periods  $N_i$  are small, the real interest rate changes little. If  $N_i \rightarrow 0$ , we are back to the standard cash-in-advance model with a representative agent and the real interest rate doesn't move. If the values of  $N_i$  are large, there is a large degree of heterogeneity across agents. Their different reactions after the shock make the real interest rate

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<sup>8</sup>Christiano, et al. (1996) present evidence that firms in fact take long on average to adjust their portfolios after an interest rate shock.

change.

The different reactions in transactions make the price level move slowly after an increase in the nominal interest rate. As the real interest rate is equal to the difference between the nominal interest rate and the rate of inflation, the real interest rate increases together with the nominal interest rate just after the shock. Market segmentation, therefore, explains the real effects of monetary policy through the reactions of agents according to their cash holdings at the time of the shock.<sup>9</sup>

Let  $t = 0$  be the time of the interest rate shock. The shock occurs when the economy is initially with constant interest rate  $r$  and constant inflation  $\pi$ . Let  $z(t)$  denote the real interest rate and  $\pi(t)$  denote the rate of inflation,  $\pi(t) \equiv \dot{P}(t)/P(t)$ . The real interest rate at each time is given by  $z(t) = r(t) - \pi(t)$ . To obtain  $z(t)$ , we have to determine the price level at each time  $P(t)$ . Before the shock, the real interest rate is  $\rho$  and  $r = \rho + \pi$ .

Cash and bond holdings of firm  $i$  at the time of the shock,  $M_{0i}(n_i)$  and  $B_{0i}(n_i)$ , are such that the economy would continue in a steady state equilibrium had the shock not happened. These cash holdings represent the choices before the shock. To solve for the transition after the shock, we solve problem (1)-(4), using  $M_{0i}(n_i)$  and  $B_{0i}(n_i)$  as initial conditions. As a result, firms with cash and bond holdings chosen previously are caught by surprise by a different interest rate path  $r(t)$ .

As Grossman (1987), we assume that bonds  $B_{0i}(n_i)$  are contingent to the shock. That is, constraint (2) is extended for two states of nature. In one state of nature, the economy continues in the steady state and in another state of nature the nominal interest rate follows  $r(t)$ . The constraint shares the same Lagrange multiplier across the two states of nature, which can be written as function of the transaction-sales ratio just after transfers  $\hat{c}_{0i}$ . We make the probability of the shock small and use the

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<sup>9</sup>A slow response of prices and an increase in the real interest rate after an increase in the nominal interest rate is found in many empirical studies. Among others, Cochrane (1994), Christiano et al. (1999), Khan et al. (2002), Bernanke et al. (2005), and Uhlig (2005).

values of  $\hat{c}_{0i}$  to solve for the equilibrium after the shock.

The equilibrium price level is obtained through the market clearing condition for goods. After the shock, there are firms in each group  $i$  that have made a transfer taking into account the shock, and other firms that have not made the first transfer yet. Firms that have not made a transfer make transfers out of  $M_{0i}(n_i)$ . Firms that have made the transfer first are firms with smaller values of  $n_i \in [0, N_i)$ , as they make the first transfer at  $T_{1i} = n_i$ . Aggregate transactions for firms in group  $i$  are given by

$$C_i(t) = \frac{1}{N_i} \int_0^t \frac{e^{-\rho t}}{\lambda_i Q(T_1(n_i)) P(t)} dn_i + \frac{1}{N_i} \int_t^{N_i} \frac{e^{-\rho t}}{\mu(n_i) P(t)} dn_i, \quad 0 \leq t < N_i, \quad (10)$$

where  $\lambda_i = 1/(P_0 c_{0i})$  is the Lagrange multiplier associated to (2) and  $\mu(n_i)$  is the Lagrange multiplier associated to the constraint for  $M^-(T_{1i}) \geq 0$ ; the value of  $\mu(n_i)$  depends on  $M_{0i}$  and it is stated in the appendix. The second term in the right of equation (10) determines transactions when  $t$  is close to zero; and the first term dominates transactions when  $t$  is close to  $N_i$ . The interpretation is that most firms cannot react completely to the shock just after the shock arrived but can do it gradually. As a result, prices react slowly to the shock.

The equilibrium price path  $P(t)$  is obtained by equating  $\sum v_i C_i(t)$  to aggregate sales. The logarithm utility allows us to isolate  $P(t)$ . In order to obtain a counterpart with the data on cash-sales, we rewrite the equation in terms of the fraction of sales of firms in group  $i$  with respect to total sales,  $v_{Y_i}$ . Proposition 1 describes the solution for prices obtained with equation (10) and completes the characterization of  $P(t)$  for  $t \geq N_i$ . The version of the market segmentation model that we use gives us great flexibility. Apart from the integral in  $e^{R(n)}$ , which can be solved fast with numerical methods, we have closed-form solutions for the price level for any  $r(t)$ .<sup>10</sup>

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<sup>10</sup>In particular, we don't need to assume an arbitrary initial path for the price level  $P^0(t)$ ,  $t \in [0, +\infty)$ , and iterate  $P^j(t)$  until it converges. That would greatly slow down the solution. The assumption of logarithmic preferences allows us to isolate  $P(t)$ .



**Proposition 1 *Prices after shocks.*** *The equilibrium price level  $P(t)$  after a nominal interest rate shock with path  $r(t)$ ,  $t \geq 0$ , is given by*

$$\begin{aligned}
P(t) &= \sum_{i=1}^I v_{Y_i} e^{-\rho t} P_0 \hat{c}_{0i} \left[ \frac{1}{N_i} \int_0^t e^{R(n_i)} dn_i + \frac{1-e^{-r(N_i-t)}}{rN_i} \right], \text{ for } 0 \leq t < N_1, \\
P(t) &= v_{Y_1} \frac{1}{N_1} P_0 \hat{c}_{01} e^{-\rho t} \int_{t-N_1}^t e^{R(n_1)} dn_1 \\
&\quad + \sum_{i=2}^I v_{Y_i} e^{-\rho t} P_0 \hat{c}_{0i} \left[ \frac{1}{N_i} \int_0^t e^{R(n_i)} dn_i + \frac{1-e^{-r(N_i-t)}}{rN_i} \right], \text{ for } N_1 \leq t < N_2, \\
&\quad \vdots \\
P(t) &= \sum_{i=1}^I v_{Y_i} \frac{1}{N_i} P_0 \hat{c}_{0i} e^{-\rho t} \int_{t-N_i}^t e^{R(n_i)} dn_i, \text{ for } t \geq N_I,
\end{aligned} \tag{11}$$

where  $r$  is the nominal interest rate before the shock and  $R(n_i) \equiv \int_0^{n_i} r(s) ds$ . The real interest rate is given by  $z(t) = r(t) - \pi(t)$ , where  $\pi(t)$  is the inflation rate,  $\pi(t) = \dot{P}(t) / P(t)$ .

As  $M_{0i}(n_i)$  are cash holdings under the initial steady state, they can be too large given the new interest rate path  $r(t)$ . Firms can choose  $M^- > 0$  after the shock. In the proof of proposition 1, we show that  $M^- = 0$  for any  $r(t)$ . When there is a shock, firms adapt to the shock by changing transactions rather than choosing  $M^-(n) > 0$ . An implication of this result is that the effects of a decrease or an increase in the nominal interest rate are symmetric.

Proposition 1 implies that monetary policy affects real interest rates. According to the Fisher relation  $z(t) = r(t) - \pi(t)$ , the real interest rate changes after a nominal interest rate shock only if inflation moves slowly after the shock. In a standard cash-in-advance model,  $\pi(t)$  changes instantaneously after a shock to  $r(t)$  and  $z(t)$  remains constant. Here,  $\pi(t)$  remains constant just after the shock because of the effects of market segmentation. As a result, the real interest rate increases with the nominal interest rate.

We can determine the effects of market segmentation using (11). Suppose, for

example, that the shock is a permanent increase of the nominal interest rate from  $r_1$  to  $r_2$ . Before the shock, inflation is equal to  $r_1 - \rho$  and the real interest rate is equal to  $\rho$ . We have  $e^{R(t)} = e^{r_2 t}$ . Solving for  $\dot{P}(t)/P(t)$ , we obtain that inflation just after the shock is equal to  $r_1 - \rho$ , its value before the shock. The real interest rate increases to  $\rho + r_2 - r_1$ . After  $\bar{t} = \max(N_i)$ , we have  $P(t) = ke^{(r_2 - \rho)t}$ , where  $k$  is a positive constant. Only after  $\bar{t}$  we have that inflation increases to  $r_2 - \rho$ , where  $t \geq \bar{t}$  means that all firms made the first bond trade after the shock, taking into account the new shock. If  $\bar{t}$  is large, it takes longer for inflation to converge to its value at the new steady state. Inflation reacts slowly to the shock. A nominal shock, however, cannot affect real variables indefinitely. In accordance with this idea, the model implies that the real interest decreases gradually to its steady state value,  $\rho$ . The effects on the real interest rate last longer if the values of  $N_i$  are larger.

We emphasize that  $P(t)$  is an equilibrium price. Prices are not assumed to be sticky. We isolate the effects of the monetary policy on the behavior toward the management of cash holdings. We focus on the management of cash holdings because the distribution of cash holdings has greatly changed in the last decades.

Proposition 2 focus the effects of market segmentation. It generalizes the result that the price level reacts slowly to a shock to the nominal interest rate. The proposition shows in particular that the real interest rate increases by the same size of the increase in the nominal interest rate just after the shock. It also shows that the real interest rate does not move if we eliminate market segmentation.

**Proposition 2 *Slow reaction of prices.*** *For any interest rate path  $r(t)$  announced at time  $t = 0$ , the price level and the inflation rate do not move just after the shock,  $P(0) = P_0$ ,  $\pi(0) = r - \rho$ , and the change in the real interest rate at  $t = 0$  is equal to the change in the nominal interest rate,  $v(0) - v = r(0) - r$ . If  $N_i \rightarrow 0$ , the real interest rate is constant and equal to  $\rho$  for any  $r(t)$  and all  $t \geq 0$ .*

Consider now a monetary policy shock as the one estimated by Uhlig (2005). According to figure 2, plot 6, in Uhlig, reproduced in figure (7), a monetary policy shock, described as an increase in the federal funds rate, initially increases the interest rate 0.3 percentage points and gradually decreases the interest rate towards its initial value. On average, the interest rate returns to its initial value in about 2 years and stays below its initial value for some time until it returns to zero. We approximate this shock with the process for the interest rate given by  $r(t) = r_1 + (r_2 - r_1 + Bt) e^{-\eta t}$ , also depicted in figure (7), where  $r_2 - r_1 = 0.3$  percentage points per year. We set  $B$  and  $\eta$  so that  $r(t)$  approximates the average impulse response of the federal funds rate as estimated by Uhlig (2005). We set  $\rho = 3$  percent per year.<sup>11</sup> The estimation in figure (7), as explained by Uhlig, uses a range of OLS estimates of a VAR. We later use different estimates for the monetary policy shock for comparison.<sup>12</sup>

Given the distributions of the cash-sales ratio from 1980 to 2013, we hit the economy with the shock  $r(t)$  and obtain the real interest rate path using the expressions of proposition 1. As explained in section 4, the cash-sales distribution for each year is obtained by determining the values of  $v_{Y_i}$  and  $N_i$  so that the distribution of the cash-sales ratio from the model approximates the actual distribution of the cash-sales ratio given by Compustat data.<sup>13</sup> According to proposition 1, the real interest rate  $z(t)$  implied by the shock to  $r(t)$  depends on the distribution of the cash-sales ratio across firms. The paths for the real interest rates for each year are our predicted effects of shocks to  $r(t)$  given the distributions of the cash-sales ratio.

Figure (8) shows the equilibrium real interest rate obtained from the model for the

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<sup>11</sup>This value for  $\rho$  has been used by Lucas (2000), Silva (2012), among others. It implies that a nominal interest rate of 3 percent per year in the steady state generates zero inflation.

<sup>12</sup>The expression of  $r(t)$  is a result of the differential equation  $m\ddot{r}(t) + \dot{c}r(t) + kr(t) = 0$ ,  $\eta = c/(2m)$ , which describes a dampened shock. We set  $r_1 = 3\%$  p.a. and  $r_2 = 3.3\%$  p.a. Figure (7) expresses the results as the difference from the initial value of the nominal interest rate. In our simulations,  $t$  denotes one day and we divide the year in 360 days.  $B = -0.15\%$  and  $\eta = 0.30$ , for  $r(t)$  given in percentage per year.

<sup>13</sup>Table 1 in the appendix shows the values of  $v_{Y_i}$  and  $N_i$  for 1980 and 2010.

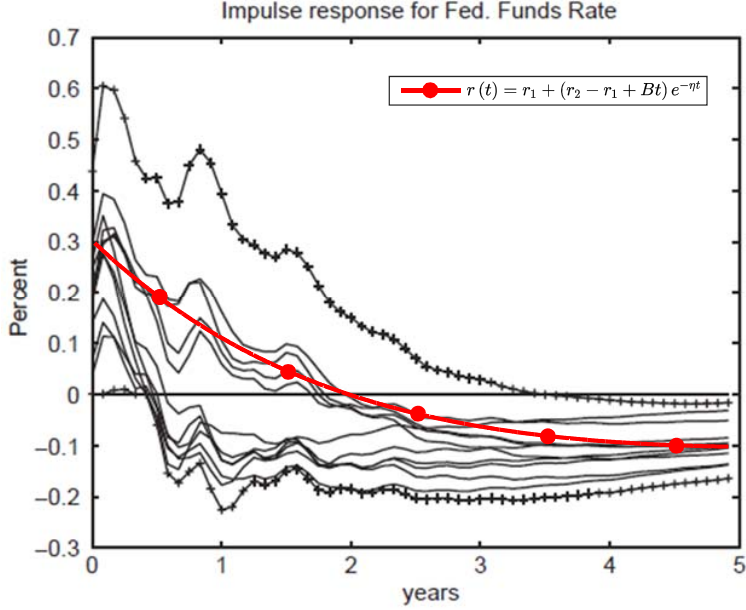


FIG. 7: Process for the nominal interest rate path,  $r(t) = r_1 + (r_2 - r_1 + Bt)e^{-\eta t}$ , with the parameters  $B$  and  $\eta$  used in the simulations,  $B = -0.15\%$  and  $\eta = 0.30$ , for  $r(t)$  given in percentage per year. The parameters  $B$  and  $\eta$  were chosen to approximate the impulse-response function for the monetary policy shock estimated in Uhlig. Figure reproduced from Uhlig (2005), Fig 2, plot 6), with the process  $r(t)$  for the nominal interest rate path added to the figure.

cash-sales distributions of the selected years from 1980 to 2013 as shown in figure (4). We show the difference in percentage points from the initial value of the real interest rate. For a standard cash-in-advance model, we would have a straight line after the shock,  $z(t) = 0$ , as a standard cash-in-advance model implies an instantaneous reaction of prices and no change in real interest rates. Here, with market segmentation, the real interest rate increases after the nominal interest rate shock and returns gradually to its initial value.

We measure the effect of monetary policy by the time that it takes for the real interest rate to reach its initial value. In figure (8), we have, for example, that the real interest rate reaches its initial value in 1.84 month given the cash distribution of 1980. Given the cash distribution of 2013, the real interest rate reaches its initial

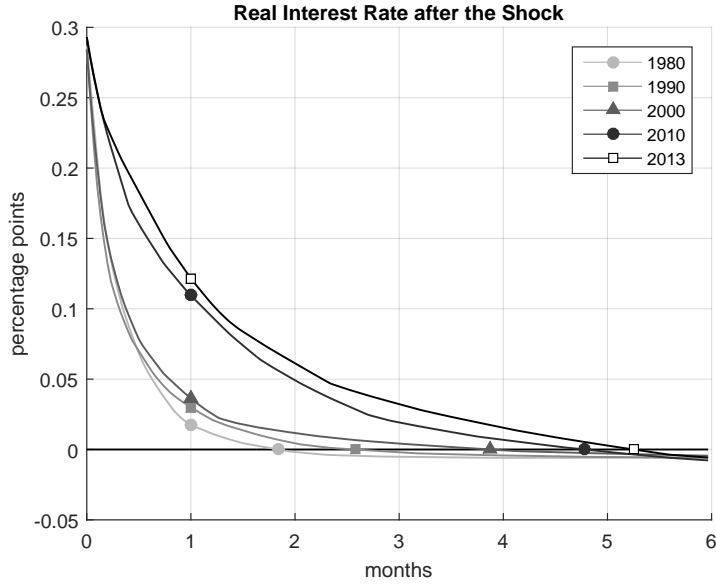


FIG. 8: Response of the real interest rate for selected years given the nominal interest rate shock of figure (7). Results from simulations. The distribution of cash holdings is determined through the data for each year. The markers in the horizontal axis show the time for the real interest rate to return to its initial value. The values are 1.84, 2.58, 3.88, 4.78, and 5.25 months for the selected years. The values for all years are in figure (1).

value in 5.25 months. The values for all years from 1980 to 2013 obtained through the simulations are in figure (1).

The effect on the real interest rate implied by the model changes as the distribution of cash-sales ratio changes from 1980 to 2013. The recent distribution of cash-sales makes the real interest rate take longer to return to its initial value. The monetary authority, therefore, is able to affect the real interest for a longer period.

To check the robustness of our results, we simulate the economy with different paths for the monetary policy shock and with different cash aggregates. We use other identification methods of the shock, recalculate the parameters  $B$  and  $\eta$  of the process for  $r(t)$ , and obtain the effect of the shock for the different estimates.

Besides using different identification methods for the monetary policy shock, we

verify our results with different cash aggregates. Our results in figure (1) use cash and equivalents (CHE) for the distribution of cash across firms, as CHE is the variable usually used for firm cash holdings.<sup>14</sup> It may be argued, however, that CHE contains variables that are not in traditional monetary aggregates such as short-term marketable security, which is part of CHE but not of M1.<sup>15</sup> To check whether we maintain our results with a more restricted variable for firm cash holdings, we repeat the exercise using only the cash component of cash and equivalents (CH instead of CHE).

We use three forms of identification of the monetary policy shock, provided by Uhlig (2005). In the first, used to obtain the results of figure (1), Uhlig generates impulse-response functions, obtained from an OLS estimate of a VAR, that satisfies sign restrictions for the monetary policy shock and the price level for six months after the shock. Figure (7), reproduced from Fig 2 of Uhlig (2005), contains the results of this identification exercise. The figure has ten random draws of the impulse responses that satisfy the sign restrictions and the upper and lower bounds of ten thousand random draws of impulse responses. We added the process  $r(t)$  used in our simulations, with the parameters  $B$  and  $\eta$  chosen to approximate the impulse-response function of the shock.

The second method of identification follows a conventional identification procedure found, for example, in Christiano et al. (1999). This method uses a standard Cholesky decomposition and there is no imposition of sign restrictions. The third method, called pure-sign-restriction approach by Uhlig, also imposes sign restrictions for the identification and uses Bayesian methods. The OLS estimate and the pure-

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<sup>14</sup>Cash and equivalents is used, for example, by Almeida, Campello, and Weisbach (2004), Bover and Watson (2005), Bates, Kahle and Stultz (2009), Bacchetta, Benhima, and Poilly (2014), among others.

<sup>15</sup>A substantial part of CHE, in any case, is comprised by cash, which, by the industrial definition, “represents any immediately negotiable medium of exchange or any instruments normally accepted by banks for deposit and immediate credit to a customer’s account” (Compustat). This definition closely resembles the definition of M1.

sign-restriction approach produce similar results, although the pure-sign-restriction approach satisfies additional technical requirements. The conventional identification implies a larger increase of the interest rate at the time of the shock and a more persistent shock as compared with the OLS estimate. The pure-sign-restriction approach implies a smaller shock at the time of the shock and a somewhat more persistent shock. The shock identified with the OLS estimate is in between the pure-sign-restriction approach and the conventional identification.<sup>16</sup>

With the three different identification methods for the monetary policy shock and the two variables for cash holdings, we have a total of six different simulations. The results of these simulations are in figure (9). For comparison, the results in figure (1) are repeated in the first plot of figure (9) for the case with CHE.

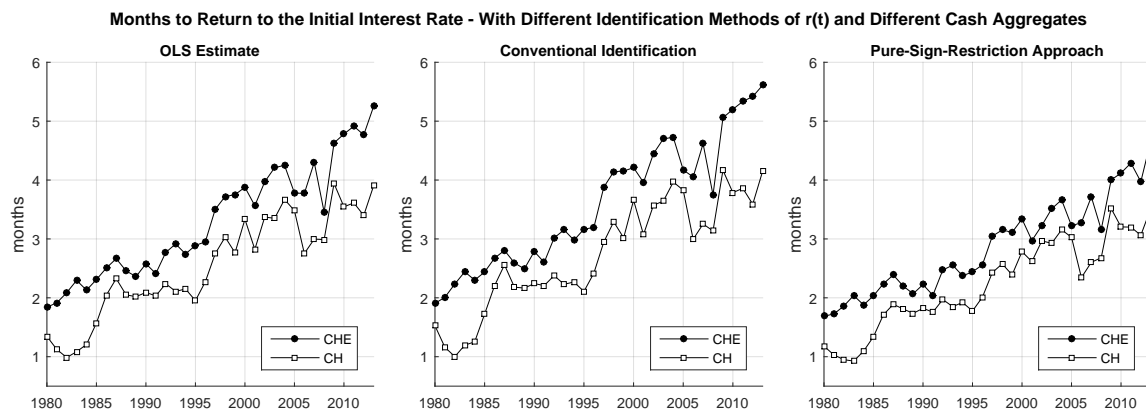


FIG. 9: Results of simulations using different estimates for  $r(t)$  from Uhlig (2005) and different cash aggregates. OLS Estimate: VAR with sign restrictions obtained by OLS and random draws of possible impulse-response functions. Conventional Identification: conventional VAR without sign restrictions. Pure-Sign-Restriction Approach: VAR with sign restrictions obtained with Bayesian methods. CHE: simulations with cash and equivalents. CH: simulations with the cash portion of cash and equivalents. The results in figure (1) are repeated in the first plot with CHE. For all cases, the time to return to the initial value of the real interest rate increases substantially.

<sup>16</sup>For the OLS estimate, conventional identification, and pure-sign-restriction approach:  $B = -0.150$ ,  $\eta = 0.3008$ ;  $B = -0.158$ ,  $\eta = 0.4497$ ; and  $B = -0.167$ ,  $\eta = 0.3852$ .

All simulations imply that the time that it takes for the real interest rate to return to its initial value increases as we change the cross-sectional distribution of cash holdings from 1980 to 2013. The pure-sign-restriction approach implies a smaller monetary policy shock. As a result, the simulations yield smaller effects on the real interest rate. The time during which the real interest rate is above its initial value increases from 1.7 month in 1980 to 4.6 months in 2013, using the CHE aggregate, compared with an increase from 1.8 month to 5.3 months with the OLS estimate. The increase in cash holdings would therefore imply an increase on the effects of the real interest rate of 2.9 months according with the pure-sign-restriction and 3.5 according with the OLS estimate. On the other hand, the conventional identification method for the monetary policy shock implies a larger shock and a more persistent interest rate shock. The simulations then yield longer effects on the real interest rate. They also yield a larger difference between the duration in 1980 and 2013. The real interest rate takes 1.9 month in 1980 and 5.6 months in 2013 to return to its initial value. The effect of the increase in cash holdings is then 3.7 months with the conventional identification.

Working with CH instead of CHE for the cash aggregate also implies the same conclusions on the more prolonged effect of monetary policy using the cash distribution of recent periods. CHE implies larger effects of monetary policy, although the effects in percentage terms are larger with CH. The predictions about the large increase in the time for the real interest rate to return to its initial value are valid for both CH and CHE. The difference of the time for the real interest rate to return to its initial value is less than 1 month on average for the simulations made with CH or with CHE.<sup>17</sup>

Our simulations have the objective of isolating the effect of the changes in the level

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<sup>17</sup>We also used different ways of treating the data, using different constraints on minimum cash holdings, truncation values for the cash-sales ratio, and minimum sales. These modifications do not change conclusions in a significant way.



of cash holdings and in the cross-sectional distribution of cash holdings from 1980 to 2013. Our point is that the increase from 1.9 month in 1980 to 5.6 month in 2013 can be attributed, according to our simulations, to the changes the behavior toward cash holdings over the period. The model used by us, with market segmentation and a non-degenerate distribution of cash holdings, is particularly useful to obtain these predictions.

This exercise with additional shocks and cash aggregates, summarized in figure (9), show that our predictions are not sensitive to changes around the setup of our first results. We still find that the changes in the distribution of cash holdings generates a longer period during which the real interest rate is affected by monetary policy. The conclusion that the current distribution of firm cash holdings implies stronger effects of monetary policy shocks is maintained.

## **6. Conclusions**

We show that the recent increase in cash holdings by firms has strong macroeconomic consequences. We find that it affects the response of the real interest rate to nominal interest rate shocks. The effect of firm cash holdings on monetary policy is substantial. According to our predictions, the changes in the distribution of cash holdings from 1980 to 2013 imply that the real interest rates takes 3.4 months more in 2013 than in 1980 to return to its initial value after a shock.

The current distribution of cash holdings implies that changes in monetary policy have more prolonged effects. There is a current debate about how central banks should increase nominal interest rates back to normal values, when the effects of the financial crisis and of the sovereign debt crisis are mitigated. An implication of our results is that these changes in interest rates should be made gradually in small steps. Given the high current values of the cash-sales distribution as compared to past values, changes in nominal interest rates will imply stronger effects in the economy.

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## Appendix

### Aggregate transactions

*Proof.* Let  $\lambda_i(n_i)$  and  $\mu_i(n_i)$  denote the Lagrange multipliers on (5) and  $M_i^-(T_{1i}(n_i)) \geq 0$ . The first order conditions imply  $P(t)c(t, n_i) = \frac{e^{-\rho t} e^{R(T_{ji})}}{\lambda(n_i)}$  for  $t \in (T_{ji}, T_{j+1i})$ ,  $P(T_{ji})c^+(T_{ji}, n_i) = \frac{e^{-\rho T_{ji}} e^{R(T_{ji})}}{\lambda(n_i)}$  for  $t = T_{ji}$ , and  $P(T_{j+1i})c^-(T_{j+1i}, n_i) = \frac{e^{-\rho T_{j+1i}} e^{R(T_{ji})}}{\lambda(n_i)}$  for  $t \rightarrow T_{j+1i}$ ,  $t < T_{j+1i}$ ,  $j = 1, 2, \dots$ ,  $i = 1, \dots, I$ . Similarly,  $P(t)c(t, n_i) = \frac{e^{-\rho t}}{\mu(n_i)}$  for  $t \in (0, T_{1i})$ ,  $P(0)c^+(0, n_i) = \frac{1}{\mu(n_i)}$ , and  $P(T_{1i})c^-(T_{1i}) = \frac{e^{-\rho T_{1i}}}{\mu(n_i)}$ . The first transfer occurs at  $T_{1i} = n_i$ . For  $M_i^-(n_i)$ ,  $Q(T_{1i})\lambda(n_i) - \mu(n_i) \leq 0$ , with equality if  $M_i^-(n_i) > 0$ . Therefore, the first order conditions for transactions imply that nominal transactions  $P(t)c(t, n_i)$  decrease at the rate  $\rho$  within holding periods. Together with the constraints (2) and (4), the first order conditions imply  $\lambda(n_i) = \frac{1}{W_{0i}(n_i) + Q(T_{1i})M_i^-(n_i)} \frac{e^{-\rho n_i}}{\rho}$  and  $\mu(n_i) = \frac{1}{M_{0i}(n_i) - M_i^-(n_i)} \frac{1 - e^{-\rho n_i}}{\rho}$ .

The values of  $M_{0i}(n_i)$  and  $W_{0i}(n_i)$  such that the economy is in an equilibrium with constant interest rate at  $t = 0$  are such that (1) nominal transactions  $P(t)c(t, n_i)$  evolve at the steady state rate and (2) all firms start a holding period with transactions  $c_{0i}$ , excluding the shorter holding period from  $t = 0$  to  $t = n_i$ . By the first order conditions,  $\pi(t) + \frac{\dot{c}(t, n_i)}{c(t, n_i)} = -\rho$ . So, transactions decrease at the rate  $\rho$  and, in the steady state, transactions decrease at the rate  $\rho + \pi = r$ . For an arbitrary firm  $n_i$ , nominal transactions at  $t = 0$  are  $P_0 c(0, n_i) = P_0 c_{0i} e^{-r(N_i - n_i)}$ , where  $P_0$  is the price level at  $t = 0$  in the steady state before the shock hits the economy. The value  $c_{0i} e^{-r(N_i - n_i)}$  implies that firm  $n_i$  makes transactions  $c_{0i}$  just after the first bond trade. Therefore, from  $\int_0^{n_i} P(t)c(t, n_i) dt + M_i^-(n_i) = M_{0i}(n_i)$ , imposing  $M_i^-(n_i) = 0$ , we obtain  $M_{0i}(n_i) = P_0 c_{0i} e^{-r(N_i - n_i)} \frac{1 - e^{-\rho n_i}}{\rho}$ . Analogously,  $W_{0i}(n_i) = \sum_{j=1}^{\infty} Q(T_{ji}) \int_{T_{ji}}^{T_{j+1i}} P(t)c(t, n_i) dt$ . We have  $T_{ji} = n_i + N_i(j - 1)$ ,  $j = 1, 2, \dots$ . As  $Q(T_{ji}) = e^{-rT_{ji}}$  and transactions decrease at the rate  $\rho$  at the steady state, then  $W_{0i}(n_i) = P_0 c_{0i} \frac{e^{-\rho n_i}}{\rho}$ . Using constraints (2) and (4) with  $K(n_i) = 0$  and the first order conditions, we obtain  $\mu(n_i) = \frac{1 - e^{-\rho n_i}}{\rho M_{0i}(n_i)}$  and  $\lambda(n_i) = \frac{e^{-\rho n_i}}{\rho W_{0i}(n_i)}$ . Substituting  $M_{0i}(n_i)$  and  $W_{0i}(n_i)$  implies  $\mu(n_i) = \frac{e^{r(N_i - n_i)}}{P_0 c_{0i}}$  and  $\lambda_i(n_i) = \frac{1}{P_0 c_{0i}}$ . The condition to verify whether  $M_i^-(n_i) = 0$  is  $\mu(n_i) > Q(T_{1i})\lambda(n_i)$ , which holds as  $e^{rN_i} > 1$ . With  $W_{0i}(n_i)$ , we obtain  $B_{0i}(n_i)$  by  $B_{0i}(n_i) = W_{0i}(n_i) - \int_0^{\infty} Q(t)P(t)Y_i dt$ .

To obtain aggregate transactions, suppose an arbitrary  $t \geq N_i$  (the argument is similar for  $t < N_i$ ). As  $t \geq N_i$ , we know that firm  $n_i$  has already made the first transfer. As nominal transactions decrease at the rate  $r$ , we have  $c(t, n_i) = c_{0i} e^{-r(t - T_{ji}(n_i))}$ , for the highest  $j(n_i)$  such that  $T_{ji}(n_i) \leq t < T_{j+1i}(n_i)$ . Firms with  $n_i \in [0, t - jN_i)$  are in their  $(j + 1)$ th holding period while firms with  $n_i \in [t - jN, N_i)$  are in their

$j$ th holding period. Aggregate transactions are then  $\frac{1}{N_i} \int_0^{t-jN_i} c_{0i} e^{-r(t-T_{j+1i}(n_i))} dn_i + \frac{1}{N_i} \int_{t-jN_i}^{N_i} c_{0i} e^{-r(t-T_{ji}(n_i))} dn_i$ . Changing variables to  $s \equiv T_{j+1i} = n_i + jN_i$  and  $s_i \equiv T_{ji} = n_i + (j-1)N_i$  in the first and second integrals, we obtain  $C_i(t) = \frac{1}{N_i} \int_{t-N_i}^t c_{0i} e^{-r(t-s_i)} ds_i$ . With another change of variables,  $C_i(t) = \frac{1}{N_i} \int_0^{N_i} c_{0i} e^{-rx} dx$ , which implies  $C_i(t) = c_{0i} \frac{1 - e^{-rN_i}}{rN_i}$ . ■

*Cash-sales ratio*

To obtain the cash-sales ratio,  $m = \frac{M(t)}{P(t)Y}$ , first note that aggregate cash holdings grows at the same rate of inflation in the steady state. Therefore, the cash-sales ratio is constant in the steady state. In particular,  $m = \frac{M(0)}{P_0Y}$ . At time zero, aggregate cash holdings are equal to  $M(0) = \frac{1}{N_i} \int_0^{N_i} M_0(n_i) dn_i$ . Substituting the values found for  $M_0(n_i)$  and dividing by  $P_0Y$ , we obtain  $m_i = \frac{e^{-rN_i}}{\rho} \frac{rN_i}{1-e^{-rN_i}} \left[ \frac{e^{rN_i}-1}{rN_i} - \frac{e^{(r-\rho)N_i}-1}{(r-\rho)N_i} \right]$ . Finally, as  $M(0)$  and  $Y$  are normalized to 1, we obtain  $P_0 = 1/m$ . With this final step, we obtain all equilibrium prices and quantities of the steady state. ■

**Proposition 1.** *Proof.* First note that all firms choose  $M_i^-(n_i) = 0$  under the new interest rate path  $r(t)$ , given the initial cash and bond holdings  $M_{0i}(n_i)$  and  $W_{0i}(n_i)$  of the first steady state. As a result, in particular, the Lagrange multipliers  $\lambda(n_i)$  and  $\mu(n_i)$  do not change with the shock. To show this statement, we have to show that the condition for  $M_i^-(n_i) = 0$ , given by  $\mu(n_i) > Q(n_i)\lambda(n_i)$ , holds for every  $n_i$ . We have  $\lambda(n_i) = \frac{e^{-\rho n_i}}{\rho[W_0(n_i)+Q(n_i)M^-(n_i)]}$  and  $\mu(n_i) = \frac{1-e^{-\rho n_i}}{\rho[M_0(n_i)-M^-(n_i)]}$  with the first order conditions and the budget constraints. Substituting the values of  $M_{0i}(n_i)$  and  $W_{0i}(n_i)$  for the initial equilibrium, we have that the condition for  $M_i^-(n_i) = 0$  holds if and only if  $e^{r(N_i-n_i)} > Q(n_i)$ , which is always true as  $Q(n_i) < 1$  (moreover,  $\mu(n_i) = Q(n_i)\lambda(n_i)$  cannot hold for  $K(n_i) > 0$ ).

We obtain the price level at each time with the market clearing condition for transactions. For  $t \geq N_i$ , all firms have already made their first bond trade. Working similarly as above, aggregate transactions are given by  $C(t) = \frac{P_0 c_{0i}}{N_i} \int_0^{t-jN_i} \frac{e^{-\rho t} e^{R(T_{j+1})}}{P(t)} dn_i + \frac{P_0 c_{0i}}{N_i} \int_{t-jN_i}^{N_i} \frac{e^{-\rho t} e^{R(T_j)}}{P(t)} dn_i$ , substituting  $\lambda(n_i) = \frac{1}{P_0 c_{0i}}$ . Firms  $n_i \in [0, t-jN_i)$  are in their  $(j+1)$ th holding period and firms with  $n_i \in [t-jN_i, N_i)$  are in their  $j$ th holding period. We have, therefore,  $C(t) = \frac{P_0 c_{0i}}{N_i} \int_{t-N_i}^t \frac{e^{-\rho t} e^{R(n_i)}}{P(t)} dn_i$ . For  $0 \leq t < N_i$ , firms with  $n_i \in [0, t)$  have already made their first bond trade and firms with  $n_i \in [t, N_i)$  are in the short holding period from zero to  $t = n_i$ . Let real transactions of these two groups be denoted by  $C^1(t) = \frac{P_0 c_{0i}}{N_i} \int_0^t \frac{e^{-\rho t} e^{R(n_i)}}{P(t)} dn_i$  and  $C^0(t) = \frac{1}{N_i} \int_t^{N_i} \frac{e^{-\rho t}}{\mu(n_i)P(t)} dn_i$ . Aggregate real transactions are then  $C(t) = C^1(t) + C^0(t)$ . As  $t \rightarrow N_i$ , the group of firms that has not made a transfer decreases, and so  $C^0(t)$  decreases to zero. Substituting  $\mu(n_i) = \frac{e^{r(N_i-n_i)}}{P_0 c_{0i}}$ , we obtain  $C^0(t) = \frac{P_0 c_{0i} e^{-\rho t} (1-e^{-r(N_i-t)})}{P(t)rN_i}$ , where  $r$  is the nominal in-

terest rate before the shock. With  $C(t) = Y$ , we obtain  $P(t)$  in the statement of the proposition. ■

**Proposition 2.** *Proof.* Obtain  $P(0) = P_0$  by using the formula of  $P(t)$  for  $t = 0$ . Also,  $\lim_{t \rightarrow 0} P(t) = P_0$ , which shows that  $P(t)$  is continuous at  $t = 0$ , and so does not jump at the time of the shock. The derivative of  $P(t)$  with respect to  $t$  is  $\dot{P}(t) = k[-\rho e^{-\rho t} \int_0^t e^{R(n_i)} dn_i + e^{-\rho t} e^{R(t)} - \frac{\rho e^{-\rho t} + (r-\rho)e^{(r-\rho)t} e^{-rN}}{r}]$ , where  $k$  is a constant. So, inflation just after the shock remains equal to inflation before the shock,  $\pi(0) = r - \rho = \pi$  for any  $r(t)$ . As the real interest rate before the shock is  $v = \rho$ , we have  $v(0) - v = r(0) - r$ . We have  $z(t) = r(t) - \pi(t) \Rightarrow z(t) = \rho + r(t) - \frac{e^{R(t)} - e^{R(t-N_i)}}{\int_{t-N_i}^t e^{R(n_i)} dn_i}$ , using the formula of  $P(t)$  for  $t \geq N_i$ . We obtain  $\lim_{N_i \rightarrow 0} z(t) = \rho + r(t) - r(t) = \rho$ , which implies that the real interest rate is constant for any  $r(t)$  if there is no market segmentation and, consequently, no heterogeneity in the distribution of cash holdings. ■

1980											
	$N_i$	$v_{Y_i}$		$N_i$	$v_{Y_i}$		$N_i$	$v_{Y_i}$		$N_i$	$v_{Y_i}$
1	0.8	0.00	11	3.9	0.00	21	7.7	0.35	31	15.4	1.71
2	1.1	0.00	12	4.2	0.00	22	8.2	6.66	32	16.5	7.16
3	1.5	0.00	13	4.5	11.91	23	8.9	0.00	33	17.8	2.80
4	1.9	0.00	14	4.8	0.00	24	9.5	0.00	34	19.1	1.73
5	2.2	0.00	15	5.2	0.00	25	10.2	0.00	35	20.6	1.37
6	2.5	0.00	16	5.5	0.00	26	10.8	7.89	36	22.2	2.17
7	2.8	0.00	17	5.9	6.59	27	11.7	0.08	37	24.0	0.00
8	3.1	0.00	18	6.2	4.24	28	12.6	1.03	38	25.7	8.17
9	3.4	0.00	19	6.7	1.68	29	13.5	3.09	39	27.8	4.60
10	3.6	0.00	20	7.2	0.06	30	14.5	0.00	40	30.1	8.99
41	33.8	0.00	42	37.0	2.27	43	41.0	1.30	44	45.4	3.95
45	52.3	0.00	46	59.2	4.08	47	69.6	3.25	48	85.6	1.31
49	116.9	1.08	50	202.4	0.48						

Average  $N = 23.6$  days.

2010											
	$N_i$	$v_{Y_i}$		$N_i$	$v_{Y_i}$		$N_i$	$v_{Y_i}$		$N_i$	$v_{Y_i}$
1	0.6	0.16	11	11.5	0.00	21	31.4	0.74	31	62.1	2.84
2	1.2	0.02	12	12.7	16.45	22	34.2	0.00	32	66.4	1.32
3	1.9	1.23	13	14.7	0.00	23	37.3	0.00	33	71.0	2.06
4	2.8	0.00	14	16.5	1.66	24	39.6	1.05	34	76.3	0.00
5	3.8	0.00	15	18.4	0.21	25	42.1	3.84	35	81.6	11.34
6	4.6	2.99	16	20.7	0.00	26	45.0	0.00	36	88.0	4.73
7	5.7	1.24	17	22.6	6.30	27	47.7	0.68	37	95.8	0.97
8	6.8	2.20	18	24.8	0.00	28	50.4	7.49	38	104.4	0.31
9	8.5	0.00	19	27.0	0.00	29	54.0	2.13	39	113.2	3.96
10	10.0	0.00	20	29.1	3.52	30	58.2	0.00	40	125.7	0.00
41	138.1	1.83	42	153.0	0.74	43	168.7	3.28	44	188.9	1.31
45	212.3	5.03	46	245.8	2.17	47	292.0	1.31	48	363.4	2.18
49	507.4	2.45	50	974.7	0.26						

Average  $N = 90.4$  days.

Table 1: Values of  $v_{Y_i}$  and  $N_i$  for each percentile  $i = 1, \dots, 50$  of  $m_{H_i}$ .  $v_{Y_i}$  in % and  $N_i$  in days. These values were used to generate the distributions of cash-sales for 1980 and 2010 in figure (6). The average  $N$  is a weighted average of  $N_i$  using  $v_{Y_i}$  as weights.