Bank Opacity and Financial Crises

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Abstract

This paper studies a model of endogenous bank opacity. In the model, bank opacity is costly for society because it reduces market discipline and encourages banks to take on too much risk. This is true even in the absence of agency problems between banks and the ultimate bearers of the risk. Banks choose to be inefficiently opaque if the composition of a bank’s balance sheet is proprietary information. Strategic behavior reduces transparency and increases the risk of a banking crisis. The model can explain why empirically a higher degree of bank competition leads to increased transparency. Optimal public disclosure requirements may make banks more vulnerable to a run for a given investment policy, but they reduce the risk of a run through an improvement in market discipline. The option of public stress tests is beneficial if the policy maker has access to public information only. This option can be harmful if the policy maker has access to banks’ private information.

Keywords: bank opacity, bank runs, market discipline, bank competition, stress tests.

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1. Introduction

The risk exposure of banks is notoriously hard to judge for the public. Bank supervisors try to address this problem through public disclosure requirements which regulate how much information banks need to reveal about their investment behavior. Transparent bank balance sheets are supposed to allow financial markets to discipline bank risk taking.¹ During the recent financial crisis, public information about the risk exposure of individual banks appears to have been particularly scarce. Bank regulators in the U.S. and in Europe responded with the publication of bank stress test results. This information seems to have been valuable to the public.² Former Fed Chairman Ben Bernanke even called the 2009 U.S. Stress Test one of the critical turning points in the financial crisis (Bernanke, 2013).

Motivated by these observations, this paper seeks to identify the market failure which justifies the regulation of bank transparency through public disclosure requirements or the publication of bank stress test results. Should we force banks to be more transparent than they choose to be? The policy debate on bank transparency often sketches a supposed trade-off between market discipline and proprietary information.³ This trade-off has not yet been formally studied in the literature.

In the model, bank opacity is costly for society because it reduces market discipline and encourages banks to take on too much risk. A bank has an incentive to subject itself to market discipline through public disclosure of its asset holdings. But the bank faces a trade-off here if it uses private information to select the composition of its balance sheet. A bank chooses to be inefficiently opaque if disclosure of its private information benefits its competitors at its own expense. The associated reduction in market discipline results in an inefficiently high level of bank risk taking. This suggests that proprietary information can simultaneously explain bank opacity and justify its reduction through policy.

In the model described below, strategic banks try to avoid information leakage to their competitors. A higher number of banks investing in a given market segment reduces the importance of strategic considerations. This mechanism can rationalize the positive empirical relationship between bank competition and transparency found by Jiang, Levine, and Lin (2014). They document that the removal of regulatory impediments to bank

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³Proprietary information is private information by which a firm can obtain an economic advantage over its competitors. For the trade-off between market discipline and proprietary information, see Basel Committee on Banking Supervision (2006, Part 4.I.G, page 228), French et al. (2010, page 33), or Bartlett (2012). The idea of such a trade-off is criticized by Chamley, Kotlikoff, and Polemarchakis (2012).
competition by individual states in the U.S. has improved the informational content of banks’ financial statements.

If a bank’s private information is useful for its rivals, the composition of its balance sheet is proprietary information. The model predicts that the supply of public information about banks’ risk exposure is inefficiently low in this case. Increasing transparency through public disclosure requirements (“transparency ex-ante”) affects financial stability in two distinct ways. (1.) Opacity renders solvent and insolvent banks observationally equivalent. If not all banks are run at the same time, none of them will. In contrast, transparency allows to identify insolvent banks. This may increase the incidence of bank runs for a given level of risk taking by banks.\(^4\) (2.) Transparency reduces risk taking by banks through an improvement of market discipline. This lowers the risk of a bank run.

The publication of bank stress test results in the U.S. and in Europe at the peak of the crisis arguably did not aim at improving market discipline. It was too late for that. For this reason, I consider the policy maker’s option to increase transparency after a bank’s portfolio is chosen (“transparency ex-post”). In the model, this option reduces the incidence of bank runs if the policy maker’s decision to disclose a bank’s risk exposure is based on public information only. If it is based on banks’ private information, a second equilibrium appears in which “suspicious” bank creditors force the regulator to reveal more information than she would like to. This reduces risk sharing opportunities among banks and triggers additional bank runs.\(^5\)

The paper makes an additional contribution by extending the concept of market discipline. Commonly, public information about banks’ balance sheets is deemed useful because of a misalignment of incentives between the bank and the ultimate bearers of the risk (depositors, creditors, shareholders, or the deposit insurance system).\(^6\) I show that even in the absence of agency problems of any kind, bank transparency is generally necessary to achieve efficiency. The role of public information in this model is not to keep bank managers from acting on their own behalf, but rather to allow them to use their portfolio choice to influence public expectations. In the absence of transparency, banks have an incentive to deviate from the announced portfolio policy and take on too much risk. They face a credibility problem similar to the problem of time-inconsistency in Kydland and Prescott (1977). Bank opacity creates a credibility problem which is absent under transparency.

This paper also differs from the existing theoretical literature because it studies trans-

\(^4\)Hirshleifer (1971) provides an early example of how public information reduces risk sharing opportunities among agents.

\(^5\)In practice, stress test exercises seldom trigger runs because under-capitalized banks are often required to raise new equity. To the extent that this additional capital has an opportunity cost, these bank re-capitalizations are socially costly.

\(^6\)Incentive problems in banking are emphasized by Calomiris and Kahn (1991), and Diamond and Rajan (2001). In these models, market discipline serves as a rationalization of demandable debt. In my model, the financial structure of the bank is exogenous. Market discipline can be improved through an increase in transparency. Holmström (1979) describes the central role of information in limiting inefficiencies which arise from agency problems. For the relationship between transparency and risk taking when incentives are misaligned, see Cordella and Yeyati (1998), Matutes and Vives (2000), and Blum (2002).
parency as public information about a bank’s risk exposure, that is, its portfolio choice. This is the kind of information which matters for market discipline and which is regulated by Pillar 3 of Basel II. In contrast, existing models of endogenous bank transparency study public information about a bank’s realized losses (see the Related Literature section below). In these models, bank losses are exogenous and banks do not choose the risk of their portfolio. Obviously, these models cannot address the issue of market discipline. In Section 2.5, I briefly show that transparency of a bank’s risk exposure may generate very different results from transparency of bank losses. One reason for the absence of models of endogenous transparency which include banks’ portfolio choice might be that this analysis can be technically quite challenging. Positive bank run risk implies non-linear payoffs. In a standard portfolio problem with risk aversion, this non-linearity complicates the analysis of the optimal portfolio choice. In a modified portfolio problem under risk neutrality, I am able to derive an interior solution in closed-form. This makes the analysis of transparency as public information about a bank’s risk exposure highly tractable.

Model Assumptions

The intermediary in the model is called a bank because of a maturity mismatch between its assets and liabilities. In particular, some part of its funding is short-term debt. This assumption is meant to capture banks’ exposure to the risk of a sudden dry-up of funds. In case a bank is unable to roll-over, it needs to liquidate some part of its assets prematurely which reduces their value.

The structure of the model is similar to Allen and Gale (1998). Banks use investors’ funds to select a portfolio of riskless and risky projects. This portfolio choice introduces a relationship between risk taking and transparency, which is essential for the model to address the concept of market discipline. The fundamental uncertainty about risky projects is resolved in an interim period before projects have actually paid off. The analysis also shares with Allen and Gale (1998) a focus on fundamental-based bank runs as opposed to the panic-based runs of Diamond and Dybvig (1983).

A bank can acquire private information about a risky project in its neighborhood. This local intelligence is proprietary information. Distance need not be an exclusively geographical factor though. Banks might differentiate themselves from competitors by specializing in separate types of loans. Projects are assumed to be bank-specific. This assumption allows me to abstract from competition between banks for assets in order

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7For U.S. banks, Shibut (2002) calculates that uninsured deposits account for 15 percent of overall liabilities. This ratio is very stable across size groups. She also documents the increasing importance of non-depository sources of credit. Beatty and Liao (2014) report that ‘noncore funding’ (which largely consists of short-term uninsured liabilities) accounts for roughly 20 percent of U.S. bank funding. For an empirical account of short-term funding of U.S. bank holding companies (including broker-dealers), see Hanson, Kashyap, and Stein (2011). An international view on bank liabilities is provided by the IMF (2013).

8Empirical results by Agarwal and Hauswald (2010) suggest that banks’ ability to collect private information about loan applicants declines with physical distance. For instance, a bank commits more errors in granting credit to small businesses which are farther away.
to focus on competition between banks for liabilities (i.e. household savings). I study competition between a finite number of banks. Market power of banks seems to be empirically relevant.

A bank in the model decides on its level of transparency by selecting ex-ante the probability that its portfolio choice becomes public information. In practice, a bank can set the level of transparency by choosing the frequency and the level of detail of publicly disclosed information about asset holdings. This disclosure becomes credible through external auditing. Bank regulators lend some credibility as well, since they know more about individual banks than the public. This prevents banks from reporting materially false information.

Banks choose their desired level of transparency ex-ante when they are still perfectly identical. If a bank could decide about disclosure after its portfolio is chosen and news about risky projects have arrived, solvent banks could avoid bank runs by disclosing their asset holdings. I assume that the transparency choice is made ex-ante to account for the fact that in a crisis situation an opaque bank arguably cannot decide to become transparent instantaneously. When bad news raise doubts about the solvency of financial institutions, each bank tries to convince the public that its own exposure to the bad shock is small. But it takes time to communicate this information in a credible way. Disclosed information needs to be verified by external auditors or bank supervisors. The “quickest” of the recent stress test exercises was the 2009 Supervisory Capital Assessment Program in the U.S., which took three months from the official announcement until the release of the results.

**Bank Transparency in Practice**

The empirically observed level of public disclosure by banks appears to be largely determined by regulatory minimum requirements. National regulations (such as the mandatory quarterly call reports in the U.S.) are harmonized and supplemented by the international Basel Accords. Pillar 3 of Basel II foresees public disclosure of bank-specific credit risk broken down by geographical area, risk class, residual maturity, business sector, and counterparty type. Pillar 3 disclosures are highly aggregated and published on a semi-annual or quarterly basis.

Banks are required to report loan loss provisions as part of their income statements. Loan loss provisions are a source of public information about risk exposure, since they are estimates of probable loan losses. The empirical accounting literature finds that banks exercise a substantial amount of discretion in managing loan loss provisions.

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9The median distance between a lending bank and its small-firm customer is 4 miles in the U.S. (Petersen and Rajan, 2002) and 1.4 miles in Belgium (Degryse and Ongena, 2005). Competition between banks for assets is modeled by Hauswald and Marquez (2003, 2006).

10Scherer (2015) reports that the average market share of the biggest three retail banks is 61.2 percent for U.S. metropolitan areas, and 85.5 percent for rural counties.

11See Table 1 in Candelon and Sy (2015).


13See Beatty and Liao (2014).
The publication of bank stress test results is an additional channel through which the public learns about individual banks’ risk exposure. The impact of selected macroeconomic scenarios on a bank’s equity contains information about its asset portfolio. The level of detail of published stress test results differs across stress test exercises.\textsuperscript{14}

**Related Literature**

A formal analysis of efficiency in the supply of public information about banks’ risk exposure is practically absent. This is surprising because a sound and consistent economic argument is needed to justify the observed regulatory interventions.

Cordella and Yeyati (1998), Matutes and Vives (2000), and Blum (2002) show that banks take on more risk if their portfolio choice is not publicly observable. But the level of transparency is taken as exogenous in these studies and they do not address the question whether policy intervention is warranted. Similarly, Sato (2014) studies the behavior of an investment fund whose portfolio is unobservable for outsiders. Again, opacity is not a choice in this model.\textsuperscript{15}

A number of contributions addresses public disclosure of bank losses. In these models, bank performance is largely exogenous and there is no or no interesting role for the bank’s portfolio or risk choice. Obviously, these papers cannot directly address market discipline or the relationship between bank opacity and risk taking.

In Chen and Hasan (2006), banks decide to delay the disclosure of losses in order to avoid efficient bank runs. Mandatory disclosure may be beneficial because it increases the probability of a bank run. This result is in contrast to the conventional wisdom that transparency should serve to reduce the likelihood of a banking crisis. Spargoli (2012) offers a complementary analysis by studying costly forbearance. He shows that the incentives of banks to hide bad loans from the public are stronger in crisis times. Alvarez and Barlevy (2014) examine banks’ transparency choice in a network of interbank claims. They find that mandatory disclosure of bank losses may improve upon the equilibrium outcome because of contagion effects. Moreno and Takalo (2014) study the precision of private signals to asymmetrically informed creditors. This notion of bank transparency differs from the focus on public information shared by the rest of the literature.

A number of contributions stresses the social benefits of limited disclosure. Opacity allows to pool investment risks. Liquidity insurance may be reduced if losses of investment projects become public. Examples of this mechanism are Kaplan (2006), or Dang,\textsuperscript{16}

\textsuperscript{14}In the U.S., the 2009 Supervisory Capital Assessment Program was the first of a series of bank stress tests. The Dodd-Frank Act of 2010 requires the Federal Reserve to publish a summary of the results of its yearly stress tests. The first European Union-wide stress test was conducted by the Committee of European Banking Supervisors in 2009. Since its establishment in 2011, these tasks have been inherited by the European Banking Authority. Bank-specific results were not published after all exercises. See Schuermann (2014) and Candelon and Sy (2015) for a description of bank stress tests in the U.S. and in Europe.

\textsuperscript{15}Sato (2014) shows that in his model fund managers would prefer to make their portfolio choice observable if they could. In Section 5 of his paper, he conjectures that fund managers might prefer to hide their portfolio choice if it was proprietary information.
Gorton, Holmström, and Ordoñez (2014). These results rely on the Hirshleifer effect.\footnote{Hirshleifer (1971) provides an early example of how public information reduces risk sharing opportunities among agents. See also Andolfatto and Martin (2013), Monnet and Quintin (2013), Andolfatto, Berentsen, and Waller (2014), or Gorton and Ordoñez (2014). The benefits of symmetric ignorance relative to the case of asymmetric information are described by Gorton and Pennacchi (1990), Jacklin (1993), Pagano and Volpin (2012), and Dang, Gorton, and Holmström (2013).} Also in Bhave (2014), investment in opaque assets insures against idiosyncratic shocks. But it results in a bank run whenever the aggregate state is bad. Banks’ choice of transparency is generally inefficient due to fire sales in crisis times.

Without exception, the models cited above study a bank’s incentive to hide losses from the public. The result that a bank would like to prevent bad news about risky projects from hitting the market is also present in the model which I describe below. Costly bank runs imply that the cost of bad news in the interim period is higher than the benefit of good news. The social planner would pay a price to prevent the arrival of news in the interim period in order to exclude the possibility of a bank run.

I contribute to the literature by studying a case in which the arrival of news about risky projects in the interim period is unavoidable and not a choice of the bank. Bad things happen. Asset prices drop. Economic indicators reveal bad news about certain business sectors or regions. Whenever this happens, investors would like to know if a bank is exposed to this shock. I show that ex-ante the market may reward banks which choose not to provide this information even if this increases the probability of a banking crisis. In a simple example, I also discuss why the disclosure of a bank’s risk exposure is quite different from the disclosure of losses (Section 2.5).\footnote{In Dang, Gorton, Holmström, and Ordoñez (2014), the bank tries to hide bad news about investment projects. At the same time, the authors show in Section 3.2 of their paper that the bank has no incentive at all to hide its portfolio choice between risky and riskless assets.} This different kind of market failure generally requires a different kind of policy intervention.

My paper also relates to two contributions which study how an intermediary can protect its informational advantage about investment projects from free-riding by competitors. In Anand and Galetovic (2000), a dynamic game between oligopolistic banks can sustain an equilibrium without free-riding on the screening of rivals. Breton (2011) shows that intermediaries which fund more projects than they have actually screened can appropriate more of the value created by their screening effort. In these two contributions, investment projects last for more than one period. Funding a project reveals information about its quality to rivals which intensifies competition for projects at an interim stage. In contrast, in my model projects require only initial funding. It is the size of the investment in one project which reveals information about another project. This intensifies competition among banks for funding at the initial stage and can be counteracted by hiding the portfolio choice.

Another strand of literature studies models of stress tests and disclosure by bank regulators. These papers do not address the question whether banks themselves might be able to supply the efficient amount of transparency to the public. Recent examples of this literature are Bouvard, Chaigneau, and de Motta (2014), or Goldstein and Leitner (2015).
In summary, the problem of a bank which deliberately hides its portfolio choice from the public has not yet been formally studied. In contrast, informed trading on asset markets has been extensively analyzed building on the seminal contributions by Grossman and Stiglitz (1980) and Kyle (1985). The key difference between that literature and my model is that a bank’s informed portfolio choice is not reflected by publicly observable asset prices.

2. Market Discipline

This section describes the role of market discipline in a three-period model: \( t = 0, 1, 2 \). There are many households and two banks: \( A \) and \( B \). Transparency is measured as the probability that a bank’s portfolio choice becomes public information. I show that transparency affects risk taking even if the preferences of bankers and households are perfectly aligned. In contrast to the rest of the paper, banks cannot acquire private information about investment projects in this section.

**Households.** There is a representative household with endowment \( w \). Households are risk neutral and have a discount factor of one. They can invest their endowment in bank equity and short-term debt.

**Bank Liabilities.** There are three periods: \( t = 0, 1, 2 \). At the end of period 0, bank \( j \) has collected some quantity \( k_j \) of resources from households. Some part of the bank’s liabilities is in the form of equity and some part is in the form of short-term debt. The quantity of equity capital \( q \) is exogenously given and identical for both banks. The remainder of a bank’s balance sheet \( k_j - q \) is financed by short-term debt. Short-term debt needs to be rolled over in the interim period at \( t = 1 \). If short-term creditors refuse to roll-over, their claims are served sequentially. Claims of debt holders are senior to the claims of shareholders. The face value of short-term debt with maturity at \( t = 1 \) is denoted by \( D_j \).

**Projects.** Each bank has access to two projects: a safe and a risky one. Both projects are started at the end of period 0. The safe project pays off a return \( S \geq 1 \) at \( t = 2 \) with certainty. Also the risky project pays off in \( t = 2 \). This project has a higher marginal return \( R > S \). The risky project is bank-specific. It is risky because the maximum project size \( \theta_j \) is uncertain. If the amount of resources invested in the risky project \( i_j \) by bank \( j \) is higher than \( \theta_j \), the surplus amount \( i_j - \theta_j \) is pure waste. Accordingly, the gross value of bank \( j \)’s portfolio at \( t = 2 \) after the two projects have been completed is given by:

\[
V_j \equiv S (k_j - i_j) + R \min\{ i_j, \theta_j \}.
\]

This setup is designed to generate a non-trivial portfolio problem under risk neutrality. The uncertain project size of the two bank-specific risky projects has one common and one idiosyncratic component:

\[
\theta_j = \Theta + \varepsilon_j, \quad j = A, B,
\]
where $\varepsilon_j$ is a uniform random variable with:

$$\varepsilon_j \sim U(-a,a), \quad j = A, B.$$ 

The common component $\Theta$, which affects both $\theta_A$ and $\theta_B$, is uniform as well:

$$\Theta = \mu + \eta, \quad \text{with: } \eta \sim U(-b,b).$$

A high value of $b$ relative to $a$ implies that $\theta_A$ and $\theta_B$ are strongly correlated. I consider the case that the first best portfolio choice is always interior for all realizations of $\theta_j$: $\mu - a - b > 0$ and $\mu + a + b < k_j$ for $j = A, B$.

**Bank Runs.** If projects are stopped and liquidated in the interim period 1, there is a cost. In this model, premature liquidation occurs in equilibrium only in case of a bank run. For this case I assume a fixed cost $\Phi$. The net value of the bank’s portfolio in $t = 2$ is equal to $V_j - \mathbb{1}_{\text{run}_j} \Phi$, where $\mathbb{1}_{\text{run}_j}$ is an indicator function with value one in case of a run on bank $j$.

**Multiple Equilibria.** As in Allen and Gale (1998), I assume that if there are multiple equilibria at the roll-over stage in $t = 1$, the bank is allowed to select the equilibrium that is preferred by creditors. This means that a bank run occurs only if it is the unique equilibrium. This assumption is made purely for analytical convenience. Alternatively, equilibrium selection by sunspots could easily be accommodated and would not change the key results of the paper.

**Bankers.** The two bankers $A$ and $B$ compete for household funds in period 0. Banker $j = A, B$ has the following preferences:

$$u(c_j, i_j) = c_j - \mathbb{1}_{i_j > 0} \xi,$$

where $c_j$ denotes consumption by banker $j$ and $\xi$ is a small positive number. Bankers are risk neutral and incur a fixed cost of exerting the effort of selecting a portfolio different from the default choice of $i_j = 0$. Since I assume the cost $\xi$ to be small, compensating banker $j$ with a positive share $\tau_j \in [0, 1]$ of the net value of the bank’s portfolio at $t = 2$ is sufficient to perfectly align the banker’s and households’ preferences for the portfolio choice. Claims of bankers are senior to the claims of households (i.e. short-term debt and outside equity).

**Information.** The realization of $\theta_A$ and $\theta_B$ becomes public knowledge at $t = 1$ after the two banks have chosen their portfolio but before projects have actually paid off. Bank $j$’s portfolio choice $i_j$ becomes public information at the end of period 0 with probability $\pi_j$. This probability is endogenous. Bank $j$ publicly chooses its level of transparency $\pi_j$ at the beginning of period 0. If $i_j$ becomes public information at the end of period 0, households know the exact value of $V_j$ already in the interim period.

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18 As Diamond and Dybvig (1983, p. 405) put it: “One interpretation of the technology is that long-term capital investments are somewhat irreversible, which appears to be a reasonable characterization.”

19 As in Diamond and Dybvig (1983), a lender of last resort could rule out the Pareto-inferior equilibrium at zero cost. For models of sunspot-driven bank runs, see Cooper and Ross (1998), or Peck and Shell (2003).
If \( i_j \) does not become public information, households need to form a belief about \( V_j \) based on their date 1 information set \( Q_1 \).

**Timing.** The timing of the setup is summarized below:

\[ t=0 \] Bankers \( A \) and \( B \) publicly choose transparency \( \pi_A \) and \( \pi_B \) and offer prices for their services \( \tau_A \) and \( \tau_B \). They collect funds from households.

The two banks select a portfolio: \( i_A \) and \( i_B \). The portfolio choices \( i_A \) and \( i_B \) become public information instantaneously with probability \( \pi_A \) and \( \pi_B \), respectively.

\[ t=1 \] All agents observe \( \theta_A \) and \( \theta_B \). Creditors decide whether to roll-over the short-term debt of the respective bank.

\[ t=2 \] The payoff of projects net of liquidation costs is distributed among households and bankers. All agents consume.

### 2.1. Bank Runs

Without loss of generality, I focus attention on banker \( A \). The problem of banker \( B \) is entirely symmetric. I solve for the equilibrium allocation by backward induction. In period 2 all agents consume their entire wealth. The interim period at \( t = 1 \) is more interesting. At this point, creditors choose whether to roll-over banks’ short-term debt. They already know the realization of \( \theta_A \) and \( \theta_B \), but projects have not paid off yet. Creditors also know \( i_A \) and \( i_B \) if they are publicly observable.

If a creditor considers bank \( A \)’s short-term debt as riskless, she will roll-over at the same conditions as before: a payment of \( D_A \) due at \( t = 2 \). If after the observation of \( \theta_A \) (and possibly \( i_A \)) a creditor assumes that bank \( A \) will not be able to fully serve \( D_A \) in period 2, she may demand immediate repayment in \( t = 1 \). If creditors do not roll-over, the bank needs to prematurely liquidate some part of its projects in order to pay out \( D_A \). Because of the costs of early liquidation \( \Phi \), roll-over is efficient.

If banker \( A \)’s portfolio choice \( i_A \) is publicly observed, creditors know the exact value of \( V_A \) at \( t = 1 \) already. If \( i_A \) is not observed, creditors have to form a belief about \( i_A \) and about \( V_A \) conditional on their period 1 information set \( Q_1 \). The bank’s portfolio choice problem studied below has a deterministic solution. Agents have rational expectations. It follows that even if \( i_A \) is not publicly observed, creditors’ belief about \( i_A \) will be a degenerate probability distribution with mass one on a single value \( \hat{i}_A \). The same is true for creditors’ belief about \( V_A \) which puts probability mass one on the value \( \mathbb{E}[V_A | Q_1] \).

If short-term creditors believe in period 1 that the gross value of bank \( A \)’s portfolio \( V_A \) is not sufficient to cover the claims both of the banker and of creditors, a bank run occurs as shown in the following Lemma.

**Lemma 2.1.** A bank run is the only Nash equilibrium if and only if:

\[
\mathbb{E}[V_A | Q_1] < \tilde{D}_A,
\]

where \( \tilde{D}_A = \frac{D_A}{1-\tau_A} \).

Proofs which are omitted from the body of the text can be found in the appendix. Lemma 2.1 states a familiar result. Even though collectively short-term creditors have
no interest in the early liquidation of projects, each creditor individually may find it optimal not to roll-over her credit claim. In particular, this is the case if a creditor expects that the bank will not be able to fully serve all debt claims in period 2.

Transparency

With probability $\pi_A$, bank A’s portfolio choice is public information. In this case, both $i_A$ and $V_A$ are directly observed at $t = 1$. Intuitively, a bank with little exposure to the risky project (i.e. a low value of $i_A$) should face a small risk of a bank run. This is the case if the following condition holds:

(A0) $Sk_A \geq \tilde{D}_A$.

If condition (A0) is satisfied, an observable portfolio choice $i_A = 0$ implies that bank A can never face a bank run. A run can only occur if the bank has overinvested: $i_A > \theta_A$. This yields the following Corollary.

Corollary 2.2. If $i_A$ is public information and condition (A0) holds, a bank run occurs if and only if:

$$\theta_A < \frac{1}{R} [Si_A - (Sk_A - \tilde{D}_A)].$$

A high value of $i_A$ increases the range of realizations of $\theta_A$ which trigger a bank run. This makes a bank run more likely.

Opacity

If bank A’s portfolio choice is not public information, $i_A$ and $V_A$ are not directly observable at $t = 1$. In this case, short-term creditors have to rely on their beliefs $\hat{i}_A$ and $E[V_A|Q_1]$.

Corollary 2.3. If $i_A$ is not public information and condition (A0) holds, a bank run occurs if and only if:

$$\theta_A < \frac{1}{R} [S\hat{i}_A - (Sk_A - \tilde{D}_A)].$$

This expression makes use of the fact that creditors’ belief about $i_A$ puts a probability mass of one on the value $\hat{i}_A$. In contrast to the case of an observable portfolio choice, it is now the public’s belief about bank A’s risk exposure $\hat{i}_A$ which matters for the risk of a bank run. The actual value of $i_A$ is not important.

2.2. Portfolio Choice

We have seen that the bank’s actual or perceived portfolio choice can affect the risk of a bank run. Continuing to proceed by backward induction, I study the bank’s portfolio choice problem at the end of period 0. Bank A has collected some quantity $k_A$ from households in order to invest it in a portfolio of a risky and a riskless project. Banker
A knows that with probability $\pi_A$ her portfolio choice $i_A$ becomes public information instantaneously. With probability $\pi_B$, banker $A$ observes banker $B$’s portfolio choice $i_B$ at the time when she chooses $i_A$. Since in this section I assume that both bankers have the same information about the probability distribution of $\theta_A$ and $\theta_B$, a possible observation of $i_B$ does not contain additional information for banker $A$. Therefore, it does not affect banker $A$’s portfolio choice.

Banker $A$ is compensated by a positive share of the net value of bank $A$’s portfolio. She chooses $i_A$ by solving:

$$
\max_{i_A} \quad \tau_A \mathbb{E} [V_A - 1_{\text{run}_A} \Phi],
$$

subject to:

$$
V_A = S (k_A - i_A) + R \min \{ i_A, \theta_A \},
$$

$$
1_{\text{run}_A} = \begin{cases} 
1 & \text{if } \theta_A < \frac{1}{R} [Sx - (Sk_A - \tilde{D}_A)], \\
0 & \text{otherwise},
\end{cases}
$$

$$
x = \begin{cases} 
i_A & \text{with probability } \pi_A, \\
\hat{i}_A & \text{with probability } 1 - \pi_A.
\end{cases}
$$

Obviously, banker $A$’s choice maximizes the expected net value of bank $A$’s portfolio. For any $\tau_A \in (0, 1)$, the preferences of households and the banker about the optimal portfolio choice are perfectly aligned. It is as if banker $A$ would choose the portfolio in order to maximize household utility. The optimal portfolio choice is described by the following Lemma.

**Lemma 2.4.** The optimal portfolio choice is:

$$
i_A^* = \mu - a \left( \frac{2S}{R} - 1 \right) - \pi_A \Phi \frac{S}{R^2},
$$

if the following conditions are jointly satisfied:

(A0) $Sk_A \geq \tilde{D}_A$,

(A1) $a > b$,

(A2) $\mu - a + b < \frac{1}{R} \left[ S \left( \mu - a \left( \frac{2S}{R} - 1 \right) - \Phi \frac{S}{R^2} \right) - (Sk_A - \tilde{D}_A) \right]$, and

(A3) $\mu - a \left( \frac{2S}{R} - 1 \right) < \mu + a - b$.

Conditions (A1), (A2), and (A3) are new. The maximum project size $\theta_A$ is the sum of the two independent uniform variables $\Theta$ and $\varepsilon_A$. Condition (A1) holds if the uncertainty about $\theta_A$ is primarily driven by the idiosyncratic component $\varepsilon_A$ instead of the common component $\Theta$. Conditions (A2) and (A3) imply that the probability density of $\theta_A$ is flat both at the threshold value of $\theta_A$ which triggers a bank run and at the point $\theta_A = i_A^*$. 

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If these conditions do not hold, the solution has a different expression which is more cumbersome to derive. The results of the paper do not depend on the particular form of this expression. The one important assumption used in the analysis below is that banker $A$’s portfolio choice is sufficiently aggressive to allow for a bank run if $i_A$ is observed and $\theta_A$ turns out to be too low.\footnote{As shown in the proof of Lemma 2.4 in the appendix, the probability density of $\theta_A$ is linearly increasing for low values of $\theta_A$, flat in the middle, and then linearly decreasing for high values. The flat part in the middle may cover almost the entire range of $\theta_A$ if $b$ is sufficiently small relative to $a$. Therefore, condition (A2) is compatible with a low risk of a bank run.}

The optimal portfolio choice $i^*_A$ has an interior solution even though all agents are risk neutral. As long as $\theta_A$ is higher than $i_A$, the marginal return of investment in the risky project $R$ is higher than the safe return $S$. But if $\theta_A$ turns out to be smaller than $i_A$, the marginal return of the risky project is zero. A higher choice of $i_A$ makes it more and more likely that the maximum project size $\theta_A$ is smaller than $i_A$. Accordingly, the expected marginal return of investment in the risky project is falling in $i_A$.

The optimal portfolio choice $i^*_A$ is increasing in the expected maximum project size $\mu$. An increase of uncertainty $a$ decreases $i^*_A$ if and only if:

$$\frac{2S}{R} \geq 1 \iff S \geq R - S. \quad (1)$$

If $S = R - S$, investing too much in the risky project and earning zero at the margin instead of the safe return $S$ is just as costly as investing too little and missing out on $R - S$. If $S > R - S$, overinvestment is more costly than underinvestment. In this case, an increase in uncertainty $a$ lowers $i^*_A$.

Importantly, the optimal portfolio choice $i^*_A$ is falling in the level of transparency $\pi_A$. Why is this the case? Consider a bank which knows that $i_A$ will be public information with certainty: $\pi_A = 1$. By Corollary 2.2, a higher value of $i_A$ increases the likelihood of a bank run. This reduces the expected marginal benefit of $i_A$.

Consider now a bank which knows that $i_A$ will remain hidden: $\pi_A = 0$. The bank is of course free to choose the value of $i_A$ which is optimal under full transparency. If the bank did so and creditors expected that, there would be no difference between the allocation under full transparency ($\pi_A = 1$) and the one under complete opacity ($\pi_A = 0$). Bank runs would happen whenever $\theta_A$ turned out to be too low, but the bank would choose the risk of a crisis optimally by trading off the benefits of a high risky return against the potential costs of early liquidation.

However, the bank has no incentive to select the same portfolio under opacity as under transparency. If the bank’s portfolio choice is not observable, the risk of a bank run does not depend directly on $i_A$ anymore, but on creditors’ expectations $\hat{i}_A$. An opaque bank does not change these expectations through its portfolio choice because $i_A$ is not observed by creditors. Choosing a higher value of $i_A$ does not increase the likelihood of a bank run, since creditors’ expectations $\hat{i}_A$ must be taken as given by an opaque bank. This is why the potential costs of early liquidation do not affect the bank’s optimal portfolio choice if $\pi_A = 0$. Only a transparent bank has an incentive to take the risk of a bank run.
run into account.

2.3. Bank Opacity

We have seen that transparency matters for the bank’s portfolio choice. An opaque bank chooses a riskier portfolio. The bank’s portfolio choice is deterministic and households know a bank’s level of transparency. If households have rational expectations, they know the bank’s portfolio choice even if it is not publicly observable: \( \hat{i}_A = i_A \). It follows that the threshold realization of \( \theta_A \) which triggers a bank run increases as a bank becomes more opaque and chooses a higher value of \( \hat{i}_A \). This implies a higher risk of a bank run.

Proposition 2.5 summarizes the effect on the expected net value of bank A’s portfolio.

Proposition 2.5. Assume that conditions (A0)-(A3) hold. A transparent bank is worth more than an opaque bank:

\[
\frac{\partial E [V_A - \mathbb{1}_{\text{run}_A} \Phi]}{\partial \pi_A} = (1 - \pi_A) \Phi^2 \frac{S^2}{2aR^3}.
\]

There are diminishing returns to transparency:

\[
\frac{\partial^2 E [V_A - \mathbb{1}_{\text{run}_A} \Phi]}{\partial \pi_A^2} = -\Phi^2 \frac{S^2}{2aR^3}.
\]

The intuition is the following. Because of rational expectations, a bank is subject to the constraint \( i_A = \hat{i}_A \) in equilibrium regardless if the portfolio choice is observable or not. A transparent bank internalizes this constraint and takes the effect of its observable portfolio choice on creditors’ expectations into account. The transparent bank acts as a Stackelberg leader who knows that its action \( i_A \) influences the expectations of its followers \( \hat{i}_A \). A fully opaque bank (\( \pi_A = 0 \)) does not internalize the constraint \( i_A = \hat{i}_A \) because it knows that its portfolio choice is unobservable. The opaque bank and creditors move simultaneously. The opaque bank takes \( \hat{i}_A \) as given and chooses \( i_A \) as a best response. Since the solution of this problem is different from the one of the Stackelberg problem, the opaque bank faces a credibility problem. This results in an inefficiently high probability of a bank run and a low expected net value of its portfolio.\(^{21}\)

Transparency has value in this model. Note however that the role of market discipline is not to mitigate an agency problem. Banker A’s preferences about the portfolio choice are perfectly aligned with households’ preferences.\(^{22}\) This is different from the models of Calomiris and Kahn (1991) or Diamond and Rajan (2001). In these models, there is an agency problem between bank managers and outside investors. Short-term funding can

\(^{21}\)The bank’s credibility problem is similar to the problem of time-inconsistency in Kydland and Prescott (1977). There is an important difference however: the bank’s portfolio choice does not depend on its timing, but on its observability.

\(^{22}\)In this sense, the bank forms a team with each one of its individual investors. In contrast to the team problems analyzed by Marschack (1955) and Radner (1962), communication (i.e. transparency) between team members (i.e. between the bank and its creditors) does not simply serve as to coordinate behavior.
be used to discipline the bank manager’s behavior. In my model, there is no benefit of short-term funding. The bank’s financial structure is exogenous. Proposition 2.5 states that for a bank which is susceptible to runs, transparency is necessary for a portfolio choice which takes the risk of a bank run into account. This is true even in the absence of agency problems.

2.4. Bank Competition

So far, I have taken banker $A$’s portfolio size $k_A$, the price of her banking services $\tau_A$, and the level of transparency $\pi_A$ as given. These values are determined by supply and demand in the market for banking services. In the beginning of period 0, banker $A$ and banker $B$ simultaneously decide on the price charged for their banking services as well as on the transparency of their balance sheet. Households decide how to allocate their wealth across the two banks. Both bankers are identical at the competition stage in the beginning of period 0.

2.4.1. Demand for Banking Services

The representative household owns an endowment of quantity $w$. Some part $k_j$ he hands over to banker $j$ who promises an expected return $r_j$ ($j = A, B$).

\[
\max_{k_A, k_B \geq 0} \quad k_A r_A + k_B r_B \quad (1a)
\]

subject to:

\[
k_A + k_B \leq w. \quad (1b)
\]

Whoever of the two bankers can credibly promise the higher expected return, captures the entire market. If $r_A = r_B$, households are indifferent with respect to any feasible choice of $k_A$ and $k_B$. The household only cares about the expected return of a security. To him it does not matter if this security is bank equity or a short-term debt claim.

2.4.2. Supply of Banking Services

We continue to focus on banker $A$. She is choosing $\tau_A$ and $\pi_A$ in order to maximize her objective function:

\[
\max_{\tau_A, \pi_A \in [0,1]} \quad \tau_A \mathbb{E} [V_A - \mathbb{1}_{\text{run}_A} \Phi] = \tau_A \mathbb{E} \left[ S(k_A - i^*_A) + R \min\{i^*_A, \theta_A\} - \mathbb{1}_{\text{run}_A} \Phi \right]. \quad (2a)
\]

She needs to take households’ demand for her services into account:

\[
k_A = \begin{cases} 
  w & \text{if } r_A > r_B, \\
  [0, w] & \text{if } r_A = r_B, \\
  0 & \text{if } r_A < r_B,
\end{cases} \quad (2b)
\]
where the expected return $r_A$ which banker $A$ can credibly offer to households is given as:

$$r_A = \frac{(1 - \tau_A) E[V_A - 1_{\text{run}_A} \Phi]}{k_A}.$$  \hfill (2c)

Through market discipline, banker $A$’s choice of transparency $\pi_A$ affects $i^*_A$ and the likelihood of a bank run. The corresponding expressions hold symmetrically for banker $B$.

2.4.3. Equilibrium

At the competition stage in the beginning of period 0, banker $A$ and banker $B$ play a perfect-information simultaneous-move price-setting game. We are interested in the following standard Nash allocation.

**Definition** A Nash equilibrium consists of a combination of values $\pi^*_A$, $\pi^*_B$, $\tau^*_A$, and $\tau^*_B$, such that banker $A$ solves (2), while banker $B$ simultaneously solves her corresponding constrained maximization problem.

Transparency matters for banks. Market discipline raises the value of a bank because it solves the bank’s credibility problem and induces a prudent portfolio choice. This is reflected by the equilibrium outcome in this economy.

**Proposition 2.6.** Full transparency is the unique equilibrium: $\pi^*_A = \pi^*_B = 1$. The two banks charge: $\tau^*_A = \tau^*_B = \frac{\mu - S}{\rho - \frac{S}{2}}$ with:

$$\rho = \frac{E[V_A - 1_{\text{run}_A} \Phi]}{k_A} = \frac{E[V_B - 1_{\text{run}_B} \Phi]}{k_B}, \text{ and: } k_A = k_B = \frac{w}{2}.$$

The two bankers compete for household funds in order to invest them. I consider the case that $\mu + a + b < k_A$. In equilibrium, the marginal unit of resources collected by banker $A$ is invested in the safe project yielding a return $S$. Market power allows banker $A$ to pay households an equilibrium return $r_A = (1 - \tau_A^*)\rho$ smaller than the marginal return $S$. Each banker has monopoly access to one risky project of limited size, while the safe project is perfectly scalable. The difference between the expected social return on her portfolio $\rho$ and the safe return $S$ measures the social value-added of the risky project which the banker has exclusive access to.

A banker agrees with households that the expected net value of her portfolio should be maximized. Since market discipline has value in this economy, full transparency is the unique equilibrium.

2.5. Disclosure of Losses

As mentioned above, most contributions to the literature have studied the disclosure of bank losses. Results which apply to that problem do not extend to the disclosure of a bank’s portfolio choice. To make this point, I briefly show that it is possible that public
information about the risky project is harmful while at the same time disclosure of the bank’s portfolio choice is beneficial.

So far I have assumed that at \( t = 1 \) the realization of \( \theta_A \) is public information. Now I assume that the realization of \( \theta_A \) becomes public information with some exogenous probability \( \psi \in [0, 1] \). Proposition 2.7 follows.

**Proposition 2.7.** Assume that \( S > R - S \). The expected net value of bank A’s portfolio is falling in \( \psi \) and increasing in \( \pi \):

\[
\frac{\partial \mathbb{E}[V_A - 1_{\text{run}_A} \Phi]}{\partial \psi} < 0, \quad \text{and} \quad \frac{\partial \mathbb{E}[V_A - 1_{\text{run}_A} \Phi]}{\partial \pi_A} = (1 - \pi_A) \psi^2 \Phi^2 \frac{S^2}{2aR^3} > 0.
\]

The intuition is simple. The assumption \( S > R - S \) is sufficient to guarantee that \( i^*_A < \mu \). This means that public news about above-average realizations \( \theta_A \geq \mu \) do not trigger a bank run. If \( \theta_A \) is not revealed in the interim period, creditors expect \( \mathbb{E}[\theta_A|Q_{t=1}] = \mu \) for lack of additional information. Again, nobody has a reason to run and the bank is stable. Only public news about a low realization of \( \theta_A \) can trigger a run.

There is nothing to gain from positive news about \( \theta_A \) while there are potential costs from negative news about \( \theta_A \). This is why public information about \( \theta_A \) in the interim period is harmful.

High risk exposure increases the range of realizations \( \theta_A \) which trigger a run given that \( \theta_A \) is observable in the interim period. To the extent that this may happen (\( \psi > 0 \)), market discipline continues to matter as it induces a prudent portfolio choice. For this reason, public information about \( i_A \) is beneficial.

### 3. Private Information

The previous section has described the role of market discipline. Banks have a strong incentive to be transparent in order to reap the benefits of market discipline and financial stability. This section departs from the previous one by introducing private information. Private information by banks means that creditors do not know an opaque bank’s portfolio choice even under rational expectations. As we will see below, this may give rise to a benefit of opacity because it allows to pool weak banks with strong banks when public news about risky projects are bad. Information spillovers introduce a second motive for opacity. The equilibrium choice of transparency may have an interior solution now. I modify the setup by the following assumption.

**Screening.** Bankers can screen their risky project before they choose their portfolio. Screening is costless. With probability \( p \), banker \( j \) learns the true value of \( \theta_j \) already at the end of period 0. With probability \( 1 - p \) screening fails and the banker learns nothing. Success in screening is not observable by outsiders and statistically independent across the two bankers. The information derived from screening is private. As before, \( \theta_A \) and \( \theta_B \) become public knowledge at \( t = 1 \).

**Timing.** The timing of the new setup is summarized below:
t=0 Bankers $A$ and $B$ publicly choose transparency $\pi_A$ and $\pi_B$ and offer prices for their services $\tau_A$ and $\tau_B$. They collect funds from households.

Both bankers screen their respective risky project.

The two banks select a portfolio: $i_A$ and $i_B$. The portfolio choices $i_A$ and $i_B$ become public information instantaneously with probability $\pi_A$ and $\pi_B$, respectively. This allows banks to react to the portfolio choice of their rival.

t=1 All agents observe $\theta_A$ and $\theta_B$. Banker $A$ offers a new face value of debt $D_A + d_A$ due in period 2. Banker $B$ offers $D_B + d_B$. Creditors decide whether to roll-over the short-term debt of the respective bank.

t=2 The payoff of projects net of liquidation costs is distributed among households and bankers. All agents consume.

3.1. Bank Runs

As before, I focus attention on banker $A$ and proceed by backward induction. In the interim period $t = 1$, banker $A$ holds a portfolio of size $k_A$. Creditors’ date 1 information set $Q_1$ includes $\theta_A$ and $\theta_B$. If $i_A$ or $i_B$ are publicly observable, they are also included in $Q_1$. If bank $A$’s portfolio choice $i_A$ is publicly observed, creditors know the exact value of $V_A$ at $t = 1$ already. If $i_A$ is not observed, creditors have to form a belief about $i_A$ and about $V_A$. Creditors know that with probability $p$ banker $A$ has successfully screened the risky project in period 0. If $i_A$ is not observed, creditors assign probability $p$ to the event $i_A = \theta_A$ and $V_A(\theta_A) = Sk_A + (R - S)\theta_A$. With probability $1 - p$, banker $A$ has failed at screening and chosen $i_A \neq \theta_A$. This implies a lower portfolio value for any value of $\theta_A$: $V^u_A(Q_1) < V^i_A(\theta_A)$.

If a creditor considers bank $A$’s short-term debt as riskless, she will roll-over at the same conditions as before: a payment of $D_A$ due at $t = 2$. If after the observation of $\theta_A$ (and possibly $i_A$ and $i_B$) a creditor assumes that bank $A$ may not be able to fully serve $D_A$ in period 2, she may only roll-over if banker $A$ offers a higher face value $D_A + d_A$ as payout in $t = 2$. The term $d_A$ is a risk premium. If the amount which creditors believe banker $A$ will actually be able to pay in $t = 2$ is too low, no risk premium is high enough and creditors will prefer immediate repayment of $D_A$ in $t = 1$.

In Section 2, there was no uncertainty about $V_A$ in $t = 1$ even if $i_A$ was not public information. Short-term debt was either riskless or default in period 2 was certain. This is different now and introduces a role for risk premia. Just as in Section 2, a bank run occurs whenever creditors believe that the gross value of bank $A$’s portfolio $V_A$ is not sufficient to cover the claims both of the banker and of creditors.

Lemma 3.1. A bank run is the only Nash equilibrium if and only if: $\mathbb{E}[V_A|Q_1] < \hat{D}_A$, where $\hat{D}_A = \frac{D_A}{1 - \tau_A}$.
Transparency

With probability \( \pi_A \) banker \( A \)'s portfolio choice \( i_A \) is public information. Both \( i_A \) and \( V_A \) are directly observed at \( t = 1 \) in this case. The incidence of a bank run follows from the analysis in Section 2.1:

\[
Pr_o(i_A) \equiv \Pr \left[ \theta_A < \frac{1}{R} [Si_A - (Sk_A - \tilde{D}_A)] \right].
\]

Opacity

If bank \( A \)'s portfolio choice is not public information, \( i_A \) and \( V_A \) are not directly observable at \( t = 1 \). In this case, creditors take into account the possibility that with probability \( p \) screening was successful and banker \( A \) has chosen \( i_A = \theta_A \) at the end of period 0. Creditors expect banker \( A \)'s unobservable portfolio to have a value of:

\[
E[V_A|Q_1] = p \left( Sk_A + (R - S)\theta_A \right) + (1 - p) \left( S(\hat{i}_A - \hat{i}_A) + R \min \{\hat{i}_A, \theta_A\} \right),
\]

where \( \hat{i}_A \) is the portfolio choice which creditors at date 1 believe banker \( A \) has selected if screening was unsuccessful. If banker \( A \)'s portfolio choice is not public information, the probability of a bank run is equal to:

\[
Pr_u(\hat{i}_A) \equiv \Pr \left\{ p \left( Sk_A + (R - S)\theta_A \right) + (1 - p) \left[ S(k_A - \hat{i}_A) + R \min \{\hat{i}_A, \theta_A\} \right] < \tilde{D}_A \right\}.
\]

This probability does not directly depend on banker \( A \)'s portfolio choice \( i_A \), but rather on creditors’ date 1 belief about banker \( A \)'s portfolio choice. Screening efficiency \( p \) is important as well. A high value of \( p \) lowers the risk of a bank run.

3.2. Portfolio Choice

I continue to proceed by backward induction. Anticipating the contingency of a bank run, banker \( A \) divides her funds between the risky and the riskless project. Since she is compensated by a positive share of the net value of the bank’s portfolio, her preferences and the preferences of households about the optimal portfolio choice are perfectly aligned.

If screening was successful, this problem has a simple solution: \( i_A = \theta_A \). If screening has failed, there are two additional cases to consider: (1.) Banker \( B \)'s portfolio choice might not be public information. This happens with probability \( 1 - \pi_B \). This case is identical to the bank’s problem studied in Section 2.2. Lemma 2.4 gives the solution to banker \( A \)'s portfolio choice. (2.) Banker \( B \)'s portfolio choice might be public information. Now there are two possibilities:
(2.a) Banker B’s portfolio choice might be the solution to banker B’s portfolio choice problem subject to public information only (that is, knowledge of i_A if it is public information as well). This reveals that banker B has failed at screening. Her portfolio choice i_B does not contain additional information for banker A. Again, Lemma 2.4 gives the solution to banker A’s portfolio choice.

(2.b) Banker B’s portfolio choice might be different from the solution to banker B’s portfolio choice problem subject to public information only. This reveals that banker B has screened her risky project successfully. Optimal behavior by banker B dictates that 

\[ i_B = \theta_B \]  

Banker A does not know \( \theta_A \) but she learns the realization of \( \theta_B \) from \( i_B \). She can adjust her portfolio choice to the information contained in \( \theta_B \) about \( \theta_A \).

Lemma 3.2 describes banker A’s optimal portfolio choice in this latter case. She does not know \( \theta_A \) but she has learned \( \theta_B \) from \( i_B \). This happens with probability \((1 - p)p\pi_B\).

**Lemma 3.2.** Define: \( \hat{\mu} \equiv E[\theta_A|\theta_B] \). If \( \theta_B \) is known, the optimal portfolio choice is:

\[ i^{**}_A = \hat{\mu} - a \left( \frac{2S}{R} - 1 \right) - \pi_A \Phi \frac{S}{R^2}, \]

if conditions \((A0)-(A3)\) are jointly satisfied.

Now the optimal portfolio choice differs from the solution in Section 2.2. To the extent that \( \theta_A \) and \( \theta_B \) are correlated, banker A’s posterior mean of \( \theta_A \) reacts to the information contained in \( \theta_B \). Her portfolio choice shifts up or down together with the updated expected maximum project size \( \mu \). Her uncertainty about the common component \( \Theta \) may decrease after the observation of \( \theta_B \), but under conditions \((A1)-(A3)\) this does not affect the optimal portfolio choice. The uncertainty which matters for banker A at the margin is the one stemming from the idiosyncratic component \( \varepsilon_A \) measured by parameter \( a \). Importantly, \( i^{**}_A \) is falling in the level of transparency \( \pi_A \) just as \( i^*_A \).

### 3.3. Bank Opacity

In order to keep the analytical expressions as simple as possible, I substitute assumption \((A2)\) with a more restrictive condition, which requires the screening efficiency \( p \) to be sufficiently low. This makes sure that, even if \( i_A \) is unobserved, the unconditional probability density of \( \theta_A \) is still flat at the threshold value which triggers a run.

\[ \mu - a + b < \frac{1}{R-pS} \left[ (1-p)S \left( \mu - a \left( \frac{2S}{R} - 1 \right) - \Phi \frac{S}{R^2} \right) - (S k_A - \tilde{D}) \right]. \]

**Lemma 3.3.** Assume that conditions \((A0), (A1), (A2'), and (A3)\) hold. Transparency affects the expected net value of banker A’s portfolio according to:

\[ \frac{\partial E[V_A - 1_{runA} \Phi]}{\partial \pi_A} = (1-\pi_A) \frac{1-p}{R-pS} \Phi^2 \frac{S^2}{2aR^2} - \Phi \left[ (1-p)Pr_o(i_A^*) - Pr_o(i_A^*) \right] \equiv \Delta(\pi_A). \]
There are diminishing returns to transparency:

\[
\frac{\partial^2 \mathbb{E}[V_A - 1_{\text{run}_A}\Phi]}{\partial \pi_A^2} = -\frac{1 - p}{R - pS} \Phi^2 \frac{S^2}{2aR^2} \frac{R + pS}{R}.
\]

The first term of the marginal benefit of transparency is similar to the one derived in Section 2.3. It captures the benefit of market discipline: a portfolio choice which takes the risk of a bank run into account. Note that this term is equal to the marginal benefit in Proposition 2.5 multiplied by the factor \( \frac{R(1-p)}{R-pS} \). This factor is smaller than 1 and decreasing in \( p \). If \( p = 0 \), there is no difference between this term and the term in Proposition 2.5. Market discipline is useful only if there is a risk of a bank run. The probability \( p \) that screening is successful reduces the risk of a bank run and therefore also the benefit of market discipline.

The second term of the marginal benefit of transparency is new and measures the difference between the expected bank run costs if \( i_A \) is observed and if \( i_A \) is unobserved. Note that \( i_A^{**} \) does not show up in this expression. This is because of the following Corollary.

**Corollary 3.4.** The probability of a bank run is independent of the observability of \( \theta_B \):

\[
\Pr_o(i_A^*) = \Pr_o(i_A^{**}) \quad \text{and} \quad \Pr_u(i_A^*) = \Pr_u(i_A^{**}).
\]

Corollary 3.4 follows from the derivation of Lemma 3.3 in the appendix. What is this second role of transparency besides market discipline? The risk of a bank run depends on the properties of creditors’ date 1 expectation of the gross value of banker A’s portfolio. Consider now this expectation conditional on some realization \( \theta_A \):

\[
\mathbb{E}[V_A|\theta_A] = p \mathbb{E}[V_A|\theta_A, i_A = \theta_A] + (1 - p) \mathbb{E}[V_A|\theta_A, i_A = i_A^*].
\]

For simplicity, I assume here that \( i_B \) is unobservable. The left hand side of the equation gives creditors’ expectation of \( V_A \) if \( i_A \) remains hidden. The right hand side gives creditors’ average expectation of \( V_A \) if \( i_A \) is observable. It is averaged across the two possible values of \( i_A \) which creditors may observe. Actually, creditors’ expectation of \( V_A \) is equal to its true value if \( i_A \) is observed since in this case all uncertainty is revealed at \( t = 1 \). The equality follows from the assumption that creditors have rational expectations. Whether creditors observe \( i_A \) at date 1 does not affect their average expectation of \( V_A \). Note however that \( \mathbb{E}[V_A|\theta_A, i_A = \theta_A] \neq \mathbb{E}[V_A|\theta_A, i_A = i_A^*] \). Whether creditors’ observe \( i_A \) at date 1 does affect the variability of their expectation of \( V_A \). Variability of expectations may be beneficial or not depending on the realization of \( \theta_A \).

1. \( \mathbb{E}[V_A|\theta_A, i_A = i_A^*] < \mathbb{E}[V_A|\theta_A] < \bar{D}_A < \mathbb{E}[V_A|\theta_A, i_A = \theta_A] \): Here \( \theta_A \) is very low. A bank run is triggered even if \( i_A \) remains hidden. The probability of being in this region is \( \Pr_u(i_A^*) \). Transparency is beneficial because it allows to avoid a bank run with probability \( p \). For very low values of \( \theta_A \), banker A has nothing to lose from observability but everything to gain. Adding variability to creditors’ expectation is beneficial in this case.
2. $\mathbb{E}[V_A|\theta_A, i_A = i_A^*] < \tilde{D}_A < \mathbb{E}[V_A|\theta_A] < \mathbb{E}[V_A|\theta_A, i_A = \theta_A]$: Here $\theta_A$ is high enough to avoid a run if $i_A$ remains hidden but still too low to avoid a run if $i_A$ is observed and equal to $i_A^*$. The probability of this case is $\Pr_o(i_A^*) - \Pr_u(i_A^*)$. Opacity is beneficial because it rules out bank runs completely. Adding variability to creditors’ expectation is harmful as it introduces the possibility of a bank run with probability $1 - p$.

3. $\tilde{D}_A < \mathbb{E}[V_A|\theta_A, i_A = i_A^*] < \mathbb{E}[V_A|\theta_A] < \mathbb{E}[V_A|\theta_A, i_A = \theta_A]$: If $\theta_A$ is sufficiently high, the observability of $i_A$ does not matter for the risk of a bank run since it is always zero.

This explains the second effect of transparency on the expected net value of banker $A$’s portfolio. For a given portfolio choice policy, an increase in $\pi_A$ raises the probability of a bank run by:

$$(1 - p)[\Pr_o(i_A^*) - \Pr_u(i_A^*)] - p \Pr_u(i_A^*) = (1 - p)\Pr_o(i_A^*) - \Pr_u(i_A^*)$$

$$= \frac{p}{2a} \left[ \mu - a - \frac{1 - p}{R - pS} S^2 \right] + \frac{R + (1 - p)S}{R(R - pS)} (Sk_A - \tilde{D}_A).$$

If $p = 0$, the value of this expression is zero and we are back in Section 2. For positive values of $p$ close to zero, the term inside of the square brackets may be negative. In particular, this is the case if $\frac{R}{S}$ and $Sk_A - \tilde{D}$ are not too high. For this range of parameter values, a bank gains more by demonstrating its unlikely success in screening than by insuring itself against the likely screening failure through opacity. An increase in transparency not only improves banker $A$’s portfolio choice but it also reduces the risk of a bank run for a given portfolio choice policy. As $p$ is increased, the term inside of the square brackets eventually turns positive. Efficient screening allows opacity to reduce the incidence of bank runs by pooling the large probability mass of informed banks with the small mass of uninformed ones. This introduces a cost of transparency.

**Lemma 3.5.** There is some value $\bar{p} \in (0, 1)$ such that the following is true: If $p \in [0, \bar{p}]$, then $\Delta(\pi_A) > 0$ for some range of positive values $\pi_A \in [0, \bar{\pi}(p))$ with $\bar{\pi}(0) = 1$.

This Lemma simply states that as long as screening is not too efficient, the expected net value of banker $A$’s portfolio is maximized by choosing a strictly positive level of transparency. In contrast to the result of Section 2.3, this level may now be different from one. Market discipline is still important but there is an additional role of transparency. The assumption of screening introduces a new source of risk. Opacity allows to insure banks against the risk of screening failure. This insurance has value if screening efficiency $p$ is sufficiently high. Transparency precludes this risk sharing opportunity. The Hirshleifer effect of public information creates a social role for opacity which must be weighed against the associated reduction of market discipline.

As stated in Lemma 3.3, the returns to transparency are decreasing. An increase in $\pi_A$ moves $i_A$ closer towards the level which maximizes the expected net value of banker $A$’s portfolio. As $\pi_A$ tends towards one, the associated marginal gain becomes infinitely
small. Also the Hirshleifer term is concave in $\pi_A$. It measures the difference between the expected cost of a bank run if $i_A$ is observed and if $i_A$ is unobserved. This difference is increasing in $\pi_A$. Insurance through opacity becomes more efficient as transparency is increased.

Lemma 3.3 describes how the introduction of private information changes the effect of a bank’s level of transparency on its own portfolio value. Lemma 3.6 shows that with private information transparency also matters for the competitor’s portfolio value.

**Lemma 3.6.** Assume that conditions (A0), (A1), (A2), and (A3) hold. The value of a bank is increasing in its competitor’s level of transparency:

$$\frac{\partial \mathbb{E} [V_A - 1_{runA}] }{\partial \pi_B} = (1-p) p \left( \mathbb{E} [V_A | i_A = i^*_A] - \mathbb{E} [V_A | i_A = i^*_{\pi_A}] \right)$$

$$= (1-p) p \frac{R}{4a} \mathbb{E} \left( \left[ \mathbb{E} [\Theta | \emptyset] - \Theta \right]^2 - \left[ \mathbb{E} [\Theta | \theta_B] - \Theta \right]^2 \right)$$

$$= (1-p) p \frac{Rb^3}{24a^2} .$$

The intuition for this result is straightforward. Whenever banker A has failed in screening project A but banker B has successfully screened project B, there is a benefit for banker A in observing $i_B$. This happens with probability $(1-p)p$. Banker B’s informed portfolio choice reveals the value of $\theta_B$. The benefit for banker A of observing $\theta_B$ is proportional to the reduction in the error variance of her forecast of $\Theta$. If $b$ is high and $a$ is low, the variation in $\theta_B$ is driven to a large extent by variation in $\Theta$. In this case, $\theta_B$ is very informative with respect to $\Theta$ and banker A benefits a lot from observing $\theta_B$.23

Banker A’s returns to $\pi_A$ are decreasing while the returns to $\pi_B$ are constant. This is because a marginal increase in $\pi_A$ changes $i^*_A$ and $i^{**}_A$ only by a marginal amount. In contrast, the observation of $\theta_B$ causes a discrete change from $i^*_A$ to $i^{**}_A$. The expected welfare gain of this discrete change is not decreasing in the level of $\pi_B$. The key difference for banker A between $\pi_A$ and $\pi_B$ is of course that she can choose $\pi_A$ but not $\pi_B$. These values are determined by strategic competition as described below.

### 3.4. Bank Competition

As in Section 2, banker A and banker B compete for households’ funds at the beginning of period 0. Each of them sets a price and a level of transparency in order to maximize her expected payout subject to the constraint set by households’ demand for intermediation services.

---

23A slightly modified version of the bank’s portfolio choice problem with discrete support for $\theta_A$ and $\theta_B$ can be written down. Theorem 4 of Blackwell (1951) applies to this modified version and demonstrates that the expected net value of banker A’s portfolio must be increasing in the precision of $\theta_B$ as a signal of $\theta_A$. 

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3.4.1. Supply of Banking Services

We continue to focus on banker A. Just as in Section 2, she is choosing \( \tau_A \) and \( \pi_A \) in order to maximize her objective function:

\[
\tau_A \mathbb{E}[V_A - \mathbb{1}_{\text{run}_A}\Phi] = \tau_A \mathbb{E}
\left[
  k_A - i_A + R \min\{i_A, \theta_A\} - \Pr_{\text{run}_A}\Phi
\right].
\]

With respect to Section 2, there are two changes. First of all, banker A’s portfolio choice \( i_A \) depends on her own uncertain screening success, as well as on the uncertain success in screening of banker B and whether \( i_B \) is observable or not:

\[
i_A = \begin{cases}
  \theta_A & \text{with probability } p, \\
  i^{**}_A & \text{with probability } (1 - p)p\pi_B, \\
  i^*_A & \text{with probability } (1 - p)(1 - p\pi_B).
\end{cases}
\]

The probability of a run by banker A’s short-term creditors in the interim period depends on banker A’s screening success and the observability of \( i_A \):

\[
\Pr_{\text{run}_A} = \begin{cases}
  0 & \text{with probability } \pi_A p, \\
  \Pr_o(i^*_A) & \text{with probability } \pi_A(1 - p), \\
  \Pr_u(i^*_A) & \text{with probability } 1 - \pi_A.
\end{cases}
\]

The corresponding expressions hold symmetrically for banker B. Banker A’s choice of transparency \( \pi_A \) affects \( i_A \) through market discipline. It also affects \( \Pr_{\text{run}_A} \) both through market discipline and through the Hirshleifer effect. In addition to these two effects, there is an information externality on banker B which affects \( i_B \). Banker A understands that her actions affect both \( r_A \) and \( r_B \).

3.4.2. Equilibrium

Private information has introduced two additional roles of transparency. Market discipline raises the value of a bank because it induces a prudent portfolio choice. But transparency also punishes unlucky banks who are surprised by a low value of \( \theta_A \) or \( \theta_B \) in the interim period. And, finally, transparency gives rise to informational spillovers to a bank’s competitor. These three roles of transparency taken together determine the equilibrium levels of \( \pi_A \) and \( \pi_B \).

**Proposition 3.7.** The equilibrium is unique and symmetric. An interior solution \( \pi^*_A = \pi^*_B \in (0, 1) \) is characterized by:

\[
\frac{\partial \mathbb{E}[V_A - \mathbb{1}_{\text{run}_A}\Phi]}{\partial \pi^*_A} - (1 - \tau^*_B) \frac{\partial \mathbb{E}[V_B - \mathbb{1}_{\text{run}_B}\Phi]}{\partial \pi^*_A} = 0,
\]

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where:

\[
\frac{\partial E [V_A - 1_{\text{run}_A} \Phi]}{\partial \pi^*_A} = (1 - \pi^*_A) \left( 1 - p \right) \frac{\Phi^2 S^2}{R - pS} \cdot \frac{R(1-p)}{2aR^2} \left[ \mu - a - \frac{(1-p)S^2}{R(R - pS)} \right] + \frac{R + (1-p)S}{R(R - pS)} (S k_A - \tilde{D}_A) \],
\]

and \[
\frac{\partial E [V_B - 1_{\text{run}_B} \Phi]}{\partial \pi^*_A} = (1 - p) p Rb^{\frac{3}{2}}. \]

The two banks charge: \( \tau^*_A = \tau^*_B = \frac{\rho - S}{\rho - \frac{S}{R}} \) with:

\[
\rho = \frac{E [V_A - 1_{\text{run}_A} \Phi]}{k_A} = \frac{E [V_B - 1_{\text{run}_B} \Phi]}{k_B}, \quad \text{and:} \quad k_A = k_B = \frac{w}{2}.
\]

Each of the two bankers wants to increase households’ demand for her own services as this allows her to charge a higher price. Demand can be increased by offering a higher expected return than the rival bank. This can be done in two ways: (1.) A banker can increase the expected net value of her own portfolio, or (2.) she can reduce the expected net value of her rival’s portfolio. Proposition 3.7 describes the optimal weight which a banker assigns to each of these two options. For a given return \( r_A \) offered to households, an increase in the expected net value of her own portfolio is fully captured by banker \( A \) through a corresponding increase in the price \( \tau^*_A \). At the same time, a reduction in the expected net value of her rival’s portfolio benefits banker \( A \) only to the extent that it reduces \( r_B \). This is why in the first order condition of \( \pi^*_A \) in Proposition 3.7 the marginal reduction of the expected net value of banker \( B \)’s portfolio is weighted by the fraction \( (1 - \pi^*_B) \) which is passed on to households by banker \( B \) and which enters \( r_B \).

In contrast to Section 2, the bank’s equilibrium choice of transparency may have an interior solution now. The expected net value of banker \( A \)’s portfolio is concave in her own level of transparency, while the expected net value of banker \( B \)’s portfolio is linear in \( \pi_A \).

The introduction of private information has profoundly changed the way transparency is chosen in this model. First of all, opacity insures unlucky banks against the risk of unfavorable news about the risky project (the Hirshleifer effect). The second change is even more fundamental. A banker does not choose transparency anymore to maximize the expected net value of her portfolio. Because of an information externality, a banker reduces this value if this hurts the rival more than it hurts herself. This is the case if the composition of her balance sheet is proprietary information, that is, if \( \theta_A \) and \( \theta_B \) are correlated \((b > 0)\). The two bankers interact as in a prisoner’s dilemma and both end up with an expected net portfolio value which is lower than it could be.

Comparative statics are ambiguous as parameters have or may have opposing effects on market discipline, the Hirshleifer effect, and information spillovers. Through the social return \( \rho \), changes in parameter values also affect the equilibrium prices \( \tau^*_A, \tau^*_B, \)
well as the values of $\tilde{D}_A$ and $\tilde{D}_B$.\footnote{For a fixed amount of bank equity $q$, the equilibrium amount of short-term debt financing of bank $A$ is given as $\frac{w^2}{2} - q$. The equilibrium return on short-term debt and bank equity must both be $r_A$. This gives as the equilibrium face value of short-term debt: $D_A = (1 - \tau^*_A)\rho(\frac{w^2}{2} - q)$. Parameter changes affecting the social return $\rho$ also affect $\tilde{D}_A = \frac{D_A}{1 - \tau^*_A} = \rho(\frac{w^2}{2} - q)$, even though the amount of short-term debt financing remains fixed.} If $R$ is sufficiently close to $S$, $\rho$ is close to $S$ and $\tau^*_A$ is close to zero for all parameter values. This allows us to draw conclusions with respect to some (but not all) model parameters.

**Corollary 3.8.** If $R$ is sufficiently close to $S$, the equilibrium level of transparency $\pi^*_A$ is falling in:

- the average maximum size of the risky project $\mu$,
- the common component of project risk $b$,
- the amount of bank equity $q$.

The model predicts that, ceteris paribus, a bank with profitable investment opportunities (high $\mu$) which are strongly correlated to the projects funded by its competitor (high $b$) will choose to be particularly opaque in equilibrium. Similarly, better capitalized banks (high $q$) are expected to be more opaque. High values of $\mu$ and $q$ render a bank less vulnerable to a run. This makes insurance through opacity more efficient and strengthens the Hirshleifer effect. A high value of $b$ increases the correlation between $\theta_A$ and $\theta_B$. This intensifies the proprietary nature of private information. The comparative statics of the remaining parameters are less conclusive. For instance, an increase in the cost of early liquidation $\Phi$ raises the importance of market discipline, but at the same time it also makes insurance through the Hirshleifer effect more valuable.

Private information implies that full transparency ($\pi_A = 1$) does not necessarily minimize the risk of a bank run anymore as it did in Section 2. Nevertheless, in equilibrium the marginal effect of transparency may still be stabilizing as stated by the following Corollary.

**Corollary 3.9.** If $\pi^*_A$ has a positive solution, a marginal increase in $\pi_A$ from its equilibrium level lowers the probability of a bank run.

This Corollary follows from the observation that an increase in $\pi_A$ always decreases the expected gross value of banker $A$’s portfolio while it increases the expected net value of banker $B$’s portfolio. Banker $A$ has no incentive to set a positive level of transparency $\pi_A > 0$ unless this lowers the risk of a run on her own bank.

### 3.5. More Than Two Banks

To study the role of bank competition, I extend the analysis from above and assume that there are $N = 3, 4, 5, \ldots$ banks competing for households’ funds. Apart from this, the environment remains unchanged. Each banker has access to a riskless project and a
bank-specific risky project. The maximum project size of each risky project is the sum of the component \( \Theta \), which is common to all \( N \) bankers, and the idiosyncratic part \( \varepsilon_j \), which is specific to the risky project of banker \( j \) only. A banker \( j \)'s information set \( Q^j_0 \) at the end of period 0 may consist of up to \( N \) different realizations \( \theta_l = \Theta + \varepsilon_l \). As before, I consider the case that in equilibrium the first best portfolio choice is always interior for all realizations of \( \theta_j : \mu + a + b < k_j = \frac{w}{N} \).

**Proposition 3.10.** For any number \( N = 3, 4, 5, \ldots \) of banks, the equilibrium is unique and symmetric. An interior solution \( \pi^*_A \in (0, 1) \) is characterized by:

\[
\frac{\partial \mathbb{E} [V_A - 1_{\text{run}_A} \Phi]}{\partial \pi^*_A} - (1 - \pi^*_B) \frac{\partial \mathbb{E} [V_B - 1_{\text{run}_B} \Phi]}{\partial \pi^*_A} = 0,
\]

where:

\[
\frac{\partial \mathbb{E} [V_A - 1_{\text{run}_A} \Phi]}{\partial \pi^*_A} = (1 - \pi^*_A) \frac{1 - p}{R - pS} \frac{\Phi^2 S^2}{2a R^2} - \frac{p \Phi}{2a} \left[ \mu - a - \frac{(1 - p) S^2}{R(R - pS)} S^* + \frac{R + (1 - p) S}{R(R - pS)} (S k_A - \hat{D}_A) \right],
\]

and

\[
\frac{\partial \mathbb{E} [V_B - 1_{\text{run}_B} \Phi]}{\partial \pi^*_A} = (1 - p) p \frac{R}{4a} \sum_{j=0}^{N-2} \binom{N-2}{j} (1 - \pi^*_N p)^{N-2-j} (\pi^*_N p)^j \left[ \mathbb{E} [e(j)^2] - \mathbb{E} [e(j + 1)^2] \right].
\]

The variable \( e(j) = \mathbb{E} [\Theta | Q^B_j] - \Theta \) denotes banker \( B \)'s forecast error with respect to \( \Theta \) after the observation of \( j \) signals \( \theta_j : |Q^B_0| = j \).

Banker \( A \) charges: \( \tau^*_A = \frac{\rho - S}{\rho - \frac{S}{N}} \) with: \( \rho = \frac{\mathbb{E} [V_A - 1_{\text{run}_A} \Phi]}{k_A} \), and: \( k_A = \frac{w}{N} \).

The impact of transparency on the own bank does not depend on the number of competitors. Neither the role of market discipline nor the Hirshleifer effect are affected by \( N \). This is different for information spillovers. Knowing the realization of \( \theta_A \) is valuable for all bankers. Without loss of generality, as in the case of two bankers I focus on banker \( A \) and banker \( B \). It must be true that banker \( A \) has screened successfully (probability \( p \)) and that banker \( B \) had no success in screening herself (probability \( 1 - p \)) in order for an increase in transparency by banker \( A \) to have an effect on the expected return offered by banker \( B \) (and all other bankers in the economy). But the expected benefit which banker \( B \) derives in this case from the observation of \( \theta_A \) depends on the number of other signals which banker \( B \) observes besides of \( \theta_A \).

**Lemma 3.11.** The expected benefit of observing one additional signal is falling in the total number of banks \( N \):

\[
\left. \frac{\partial \mathbb{E} [V_B - 1_{\text{run}_B} \Phi]}{\partial \pi^*_A} \right|_{N=j} > \left. \frac{\partial \mathbb{E} [V_B - 1_{\text{run}_B} \Phi]}{\partial \pi^*_A} \right|_{N=j+1}.
\]
For a given equilibrium level of transparency $\pi^*_A$, the expected number of signals observed by banker $B$ is increasing in $N$. The higher is the total number of signals observed, the more precise banker $B$’s posterior of $\Theta$ becomes and the smaller becomes the expected benefit of observing the additional signal $\theta_A$. The error variance of her forecast error is falling in the number of signals, but it is falling at a declining rate. This property of the model is key for the following result about the role of competition for bank transparency.

**Proposition 3.12.** If $R$ is sufficiently close to $S$, the equilibrium level of transparency is monotonically increasing in the number of banks while the probability of a bank run for any given bank is falling.

We know from Lemma 3.11 that information spillovers are decreasing in the number of banks. This reduces the private cost of transparency. However, the trade-off given in Proposition 3.10 also includes the equilibrium price of banking services. Only the fraction $1 - \tau^*_B$ of banker $B$’s portfolio value is passed on to households and enters the return $r_B$. If banker $B$ charges a low price $\tau_B$, the return $r_B$ offered to households is particularly sensitive to changes in the expected net value of banker $B$’s portfolio. This makes it easier for banker $A$ to reduce $r_B$ through hiding $i_A$. The equilibrium price $\tau^*_B$ is falling in $N$. Only if this effect does not outweigh the reduction of information spillovers described in Lemma 3.11, transparency is increasing with bank competition. If $R$ is close to $S$, the equilibrium price of banking services is close to zero for all values of $N$. This is a sufficient condition for a positive effect of bank competition on transparency.

This positive effect of bank competition on transparency is remarkable for two reasons. First, this prediction is in line with the empirical evidence. Jiang, Levine and Lin (2014) estimate that the removal of regulatory impediments to bank competition by individual states in the U.S. has improved the informational content of banks’ financial statements. Proposition 3.12 suggests that this might have been driven by a reduction of strategic concerns as regional markets became more competitive. Secondly, as strategic concerns are reduced and transparency increases, the risk of a bank run is falling for any given bank. A higher number of banks increases financial stability in the model through an improvement in market discipline. This is in contrast to the widely held view of a trade-off between financial stability and bank competition.\(^{25}\)

### 4. Efficiency and Policy

Until now, the analysis of the model has been purely positive. We have learned that a bank’s equilibrium choice of transparency may have an interior solution if its portfolio choice is based on private information. Transparency is monotonically increasing in the number of competitors while the risk of a bank run is falling (as long as the rents of bankers are not too high). We also have learned that the equilibrium level of transparency does not maximize the expected net value of a bank’s portfolio because

\(^{25}\)This view is well exemplified by Keeley (1990).
of an information externality. The fact that the market equilibrium described above is reminiscent of a prisoner’s dilemma suggests that there is room for policy.

4.1. Transparency ex-ante

There are various sources of inefficiency in the model: banks have market power, their portfolio choice is subject to a credibility problem, and costly bank runs occur in equilibrium. In order to study the potential benefit of policy interventions, I use a notion of constrained efficiency. For the case of two banks, the social planner solves:

$$\max_{\pi_A, \pi_B} \mathbb{E}\left\{ V_A - 1_{\text{run}_A} \Phi + V_B - 1_{\text{run}_B} \Phi \right\}. $$

The planner maximizes the expected value of aggregate consumption. She cannot directly change banks’ portfolio choices or creditors’ decision to roll-over in the interim period. She can only control the two banks’ levels of transparency $\pi_A$ and $\pi_B$ set in the beginning of period 0. Changing bank A’s level of transparency $\pi_A$ affects $i_A$ through market discipline. It also affects the probability of a bank run on bank A through market discipline and the Hirshleifer effect. It potentially affects bank B’s portfolio choice through the information externality.

The first order condition for a constrained efficient choice of transparency $\pi_A$ reads as:

$$\frac{\partial \mathbb{E}[V_A - 1_{\text{run}_A} \Phi]}{\partial \pi^{SB}_A} + \frac{\partial \mathbb{E}[V_B - 1_{\text{run}_B} \Phi]}{\partial \pi^{SB}_A} = 0. $$

The contrast to the equilibrium choice of $\pi_A$ described in Proposition 3.7 is evident. The planner internalizes the information externality from an increase in $\pi_A$ on the expected net value of banker B’s portfolio. This information externality increases the marginal social benefit of transparency and contributes to a high value of $\pi^{SB}_A$. For bank A’s equilibrium choice the opposite is true: the information externality on banker B’s portfolio value reduces the equilibrium value of transparency $\pi^*_A$.

Proposition 4.1. The gap between the second best level of transparency $\pi^{SB}_A$ and the market outcome $\pi^*_A$ is positive if and only if $b > 0$ and $p \in (0, 1)$. If $R$ is sufficiently close to $S$, this gap is:

- increasing in the common component of project risk $b$,
- falling in the idiosyncratic component of project risk $a$,
- increasing in the return of the risky project $R$.

The market failure in the supply of public information about banks’ risk exposure is severe if information spillovers are large. This is the case if the observation of $\theta_B$ helps to predict $\theta_A$ (high $b$ and low $a$), and if this prediction is valuable (high $R$).

A high value of $b$ characterizes environments in which one banker’s private information about a borrower is a close substitute to another banker’s private information about her
own borrower. This might be the case because both bankers are lending funds to firms or homeowners in the same region. The random variable \( \Theta \) captures a factor which all firms or homeowners in the region are exposed to and which the information acquired by a local banker helps to predict. A high value of \( b \) could also apply to the situation of two banks which lend to different firms in the same industry. By screening one firm, a banker might learn industry-specific information which is valuable to a second banker who has to decide on a loan to a different firm of the same sector.

Both if \( p = 0 \) or \( p = 1 \), the inefficiency in \( \pi_A \) disappears. These are the two extreme cases of a total absence of private information (\( p = 0 \)) and full information (\( p = 1 \)). In both intermediate values \( p \in (0,1) \), there is a private and social benefit from observing the portfolio choice of other banks. These information spillovers result in a gap between the second-best level of transparency and banks’ equilibrium choice. This mechanism rationalizes the puzzling observation that real-world investors choose to lend money to opaque banks while many commentators argue that bank opacity is higher than it should be. In the model, information spillovers create an environment in which the market rewards banks which set an excessive level of opacity.

If a policy maker has the option to increase \( \pi_A \) in this environment, for instance through minimum public disclosure requirements or through periodic and standardized public stress tests, this is beneficial as it improves the portfolio choice of all banks. It may also reduce the risk of a bank run as stated by the following Proposition.

**Proposition 4.2.** If \( R \) is sufficiently close to \( S \) and \( \pi_A^* > 0 \), the risk of a bank run is higher for the equilibrium level of transparency \( \pi_A^* \) than for the the second-best level \( \pi_A^{SB} \).

This result is remarkable for the following reason. As described above, several models of optimal bank opacity stress the role of the Hirshleifer effect in creating social costs of transparency. These contributions study environments where there is no need for market discipline. The analysis above demonstrates that even if the Hirshleifer effect is operating and public disclosure requirements increase the risk of a bank run for a given investment policy, they may actually reduce the risk of a bank run through the associated improvement in market discipline.

### 4.2. Transparency ex-post

The model assumes that a bank cannot decide to increase its level of transparency in the interim period after public news about \( \theta_A \) and \( \theta_B \) has arrived. The reason for this assumption is that it takes time to credibly communicate information to outsiders. Disclosed information needs to be verified by external auditors or bank supervisors. The “quickest” of the recent stress test exercises was the 2009 Supervisory Capital Assessment Program in the U.S., which took three months from the official announcement until the release of the results.\(^{26}\)

\(^{26}\)See Table 1 in Candelon and Sy (2015).
The stress tests recently performed in the U.S. and in Europe at the peak of the Financial Crisis may be viewed as the supply of public information about banks’ risk exposure after unfavorable news about $\theta_A$ and $\theta_B$ has arrived. Through liquidity assistance and public lending, policy makers were able to buy the time necessary to perform these stress tests. The primary purpose of these stress test exercises was not to improve market discipline in order to avoid future crises, but rather to deal with the ongoing financial crisis. In order to study how the implementation of transparency “ex post” (i.e. emergency stress tests) differs from transparency “ex ante” (i.e. public disclosure requirements or periodic and standardized stress tests), I now assume that the policy maker can increase transparency at the beginning of the interim period. In order to capture the novel character of the stress test exercises performed during the crisis, I assume that the policy maker’s option to disclose in the interim period is not anticipated by agents.\footnote{27}{The problem of anticipated disclosure ex-post can be studied as well. In this case, “transparency ex-post” affects banks’ portfolio choice and the equilibrium choice of “transparency ex-ante”.}

As before, the policy maker maximizes the expected value of aggregate consumption. I consider the case of bank $A$. The problem of bank $B$ is symmetric. Assume that $i_A$ has not been revealed at the end of period 0. The policy maker and bank creditors play a game. The policy maker decides whether to disclose $i_A$. Creditors choose whether to roll-over the bank’s short-term debt. The timing of the new sub-game starting from $t = 1$ is:

$t=1$ All agents observe $\theta_A$. The policy maker decides whether to reveal $i_A$.

Banker $A$ offers a new face value of debt $D_A + d_A$ due in period 2. Creditors decide whether to roll-over the short-term debt of bank $A$.

t=2 The payoff of projects net of liquidation costs is distributed among households and bankers. All agents consume.

There is an upper range of realizations of $\theta_A$ for which even public information about screening failure by bank $A$ does not trigger a run. In this upper range of $\theta_A$, the policy maker’s problem is trivial. Transparency “ex post” neither benefits nor hurts anyone. The problem is more interesting within a lower range of $\theta_A$:

$$\mu - a - b \leq \theta_A < \bar{\theta} \equiv \frac{1}{R} \left[ S \hat{i}_A - (S k_A - \tilde{D}_A) \right],$$

where $\hat{i}_A$ is the solution to bank $A$’s portfolio choice problem based on public information only, that is, based on $i_B$ if it happened to be observable at the end of period 0. This is the optimal portfolio choice if bank $A$ has failed to screen successfully. If $\theta_A < \bar{\theta}$, disclosure of $i_A = \hat{i}_A$ triggers a bank run whereas no-disclosure might avoid it. Disclosure of $i_A = \theta_A$ always avoids a run. Note that $\bar{\theta}$ depends on the value of $i_B$ if it happened to be observable at the end of period 0. Assume that $i_B$ was public information. If $i_B$ happened to be different from $i_B^*$, it has publicly revealed the value of $\theta_B$ at the end of period 0 already. In case bank $A$ has failed at screening, it optimally has chosen $\hat{i}_A = i_A^{**}$.
A high value of $\theta_B$ implies a high value of $i_A^*$. This in turn implies a wider range of realizations of $\theta_A$ over which transparency “ex post” may make the difference between a run and roll-over.

There is a second important threshold value $\theta$. This is the lower limit of $\theta_A$ for which a bank run is avoided if $i_A$ remains hidden. It is implicitly defined by:

$$p[Sk_A + (R - S)\theta] + (1 - p)[S(k_A - \hat{i}_A) + R \min\{\hat{i}_A, \theta\}] = \tilde{D}_A.$$  

This threshold value $\theta$ is strictly smaller than $\bar{\theta}$ if $p > 0$. Just as $\bar{\theta}$, it depends on a potential observation of $i_B$ through its impact on $\hat{i}_A$.

### 4.2.1. Uninformed Policy Maker

Here I consider a policy maker who has access to public information only. She does not know $i_A$. The true value of $\theta_A$ is revealed at $t = 1$ and happens to lie within the critical range: $\mu - a - b < \theta_A < \bar{\theta}$. Since the policy maker does not know $i_A$, her action depends on public information only. This game has a simple equilibrium as described by the following Proposition.

**Proposition 4.3.** An uninformed policy maker’s option to disclose $i_A$ at $t = 1$ results in a perfect-information game with a unique equilibrium in pure strategies: Partial Revelation. If $\theta \leq \theta_A < \bar{\theta}$, $i_A$ remains hidden. If $\mu - a - b \leq \theta_A < \theta$, $i_A$ is revealed. The option of transparency “ex post” reduces the probability of a run on an opaque bank from $Pr_u(\hat{i}_A)$ to $(1 - p)Pr_u(\hat{i}_A)$. This increases expected aggregate consumption by the amount $pPr_u(\hat{i}_A) \Phi$.

The uninformed policy maker performs a public stress test exercise if news about $\theta_A$ is sufficiently bad. This allows to prevent a run on all solvent banks. A number of insolvent banks remains opaque and benefits from the Hirshleifer effect. Note that the optimal disclosure policy is bank-specific. In case both $i_A$ and $i_B$ were not revealed at the end of period 0, the event $\theta_A < \theta < \theta_B < \bar{\theta}$ implies that a public stress test is carried out for bank $A$ but not for bank $B$. This allows to save bank $A$ in case $i_A = \theta_A$, while bank $B$ is safe irrespective of $i_B$. Opaque banks which do not face a run optimally remain opaque.

### 4.2.2. Informed Policy Maker

In practice, bank regulators know more about individual banks than the public. To capture this information asymmetry, I assume now that the policy maker knows $i_A$ even if the public does not observe it. The game played between the policy maker and bank creditors turns Bayesian. The policy maker’s decision whether to disclose $i_A$ now may depend on the value of $i_A$ which is observed by the policy maker but not by the public. In particular, it may depend on bank $A$’s success in screening. Creditors base their roll-over decision not only on $\theta_A$ but also on the policy maker’s decision whether to disclose.
Proposition 4.4. An informed policy maker’s option to disclose \( i_A \) at 1 results in a Bayesian game with two Perfect Bayesian equilibria in pure strategies: (1.) Full Revelation, or (2.) Partial Revelation (as in the case of an uninformed policy maker). In the Full Revelation equilibrium, \( i_A \) is disclosed for all values of \( \theta_A \in [\mu - a - b, \bar{\theta}] \). In this case, the option of transparency “ex post” changes the probability of a run on an opaque bank from \( \Pr_u(i_A) \) to \( (1 - p)\Pr_o(i_A) \). This reduces expected aggregate consumption by the amount \( ((1 - p)\Pr_o(i_A) - \Pr_u(i_A)) \Phi \).

In both equilibria, the informed policy maker performs a public stress test exercise if news about \( \theta_A \) is sufficiently bad. But the threshold value triggering a stress test is higher in the Full Revelation Equilibrium. Disclosure occurs more often. The difference between the two equilibria is how bank creditors interpret the policy maker’s decision not to disclose \( i_A \). In the Full Revelation Equilibrium, creditors interpret the absence of disclosure as disclosure of insolvency. If \( i_A \) is not disclosed, creditors run. The policy maker’s best response to these beliefs is to disclose \( i_A \) whenever screening was successful. In this equilibrium, creditors are able to discriminate perfectly between solvent and insolvent banks. In the Partial Revelation Equilibrium, creditors do not interpret the absence of disclosure as a sure sign of insolvency. This allows the policy maker to avoid banks runs on insolvent banks if \( \theta_A \) is not too low \( (\theta \leq \theta_A < \bar{\theta}) \). It is straightforward to verify that both Perfect Bayesian equilibria survive the Cho-Kreps Intuitive Criterion.

As discussed in Section 3.3, the term \( (1 - p)\Pr_o(i_A) - \Pr_u(i_A) \) is positive if \( p \) is sufficiently high. The Hirshleifer effect implies a social cost of transparency in this case. If the Full Revelation Equilibrium prevails, it would be better not to have the option of transparency “ex post”. “Suspicious” creditors force the policy maker to reveal too much information in the interim period, which increases the probability of a bank run. Adding the option of a stress test exercise to the policy maker’s choice set may make her worse off. While this is impossible for a single player’s decision problem, this possibility arises in this setup because of the strategic interaction between the informed policy maker with uninformed short-term creditors.

The Full Revelation Equilibrium may be empirically relevant as real-world stress tests have not been bank-specific. Regulators have performed stress test exercises for a large set of banks regardless of their individual standing on credit markets. This feature of real-world stress tests is in line with Full Revelation as opposed to Partial Revelation. The success of the 2009 U.S. Supervisory Capital Assessment Program is not necessarily inconsistent with this interpretation. It came along with a detailed recapitalization plan for under-capitalized banks. Such a credible mechanism of equity injections was absent for the early EU-wide stress test exercises of 2010 and 2011 which triggered less positive or even negative market reactions overall. The stabilizing role of the 2009 U.S. stress test might therefore be attributed to its credible recapitalization plan rather than to disclosure per se.

28 There is full “unraveling” as in Grossman (1981) and Milgrom (1981).
5. Conclusion

This paper studies a model of endogenous bank transparency. In contrast to the existing literature, I study transparency as public information about a bank’s risk exposure. This is the kind of information which matters for market discipline and which is regulated by Pillar 3 of Basel II. In the model, market discipline has value even in the absence of agency problems. The reason for this result is that an opaque bank faces a credibility problem which does not exist under transparency.

Given the importance of market discipline in preventing excessive risk taking, this paper asks the positive question why banks choose not to be as transparent as possible. A simple model highlights two incentives for bank opacity. (1.) Opacity allows for risk sharing among strong and weak banks. This may reduce the incidence of bank runs for a given degree of risk taking. (2.) Opacity allows banks to prevent information leakage to competitors.

Motivated by real-world regulators’ usage of public disclosure requirements (e.g. Pillar 3 of Basel II), the paper also asks the normative question if we should force banks to be more transparent than they choose to be. The analysis suggests that this is desirable to the extent that a bank’s asset composition is proprietary information. Optimal public disclosure requirements lower the risk of a bank run through an improvement in market discipline. Full transparency is generally undesirable as it destroys valuable risk sharing opportunities among strong and weak banks. As an alternative to public disclosure requirements, an increase in bank competition induces an endogenous rise of transparency in the model.

The model also allows to study emergency stress test exercises such as the ones implemented at the peak of the Financial Crisis in the U.S. and in Europe. The analysis suggests that the option to increase transparency in a crisis situation may not be a reliable substitute for market discipline. This is especially true if “suspicious” financial markets force the policy maker to reveal more information about bank solvency than she would like to. In this case, the mere existence of the stress test option may actually make matters worse and contribute to financial instability.
A. Proofs and Derivations

Proof of Lemma 2.1

Consider an individual atomistic creditor. Debt claims are served sequentially. If all other creditors choose not to roll-over and projects need to be liquidated prematurely, roll-over by one individual agent means that she is repaid only if bank $A$ can fully serve all other creditors at $t = 1$. If bank $A$ cannot fully serve all other creditors at $t = 1$ ($D_A > (1 - \tau_A)(V_A - \Phi)$), roll-over by one individual agent yields zero. This is the case if $\tilde{D}_A > V_A - \Phi$, with $\tilde{D}_A \equiv \frac{D_A}{1 - \tau_A}$. If creditors do not roll-over but $\tilde{D}_A \leq V_A - \Phi$, roll-over by one individual agent yields $D_A$ (the bank can always serve an individual atomistic debt claim as long as $\tilde{D}_A \leq V_A - \Phi$). Running like everyone else yields $D_A$ if $\tilde{D}_A \leq V_A - \Phi$. If bank $A$ is unable to fully serve all other creditors at $t = 1$ (if $\tilde{D}_A > V_A - \Phi$), the payout to an individual creditor who runs is random. Given that debt claims are served sequentially, the payout of an individual creditor who runs depends on her position in line. This position is random. Therefore, running like everyone else yields an expected amount of $(1 - \tau_A)(V_A - \Phi)$ if all other creditors choose to roll-over, projects need not be liquidated prematurely. Roll-over yields a payoff in period 2 of $\min\{D_A, (1 - \tau_A)V_A\}$. Running yields $D_A$. We can distinguish three cases:

1. $E[V_A|Q_1] \geq \tilde{D}_A + \Phi$: If all other creditors run, both roll-over and running yield $D_A$. If all creditors roll-over, both running and roll-over yield $D_A$. By offering a debt claim which pays out a tiny bit more than $D_A$ at $t = 2$, the bank can achieve roll-over as the only equilibrium.

2. $\tilde{D}_A \leq E[V_A|Q_1] < \tilde{D}_A + \Phi$: If all other creditors run, roll-over yields zero while running yields an expected payoff of $(1 - \tau_A)E[V_A - \Phi|Q_1]$. If all creditors roll-over, both running and roll-over yield $D_A$. There are two pure strategy Nash equilibria: one with and one without a bank run.

3. $E[V_A|Q_1] < \tilde{D}_A$: If all other creditors run, roll-over yields zero while running yields $(1 - \tau_A)E[V_A - \Phi|Q_1]$. If all creditors roll-over, roll-over yields $(1 - \tau_A)E[V_A|Q_1]$ while running yields $D_A$. A bank run is the only equilibrium.

Proof of Lemma 2.4

The maximum project size is the sum of two independent uniform variables: $\theta_A = \Theta + \varepsilon_A$. The common component of $\theta_A$ and $\theta_B$ has a deterministic and a random part: $\Theta = \mu + \eta$. It is convenient to separate the two and consider $z_A = \theta_A - \mu = \eta + \varepsilon_A$. The probability density of $z_A$ is determined by the convolution of the densities of $\eta$ and $\varepsilon_A$:

$$\varphi(z_A) = \int_{-\infty}^{\infty} \nu(z_A - \varepsilon_A) \omega(\varepsilon_A) \, d\varepsilon_A,$$

30 This result implicitly uses the assumption that in the interim period banker $A$ cannot credibly promise to forgo some part of her claim $\tau_A[V_A - \Phi]$. There are realizations of $\theta_A$ which would induce banker $A$ to do so if she could in order to avoid a bank run.
where:

\[
v(x) = \begin{cases} 
\frac{b}{2a}, & \text{if } -b \leq x \leq b, \\
0, & \text{otherwise}, 
\end{cases}
\quad \text{and} \quad \omega(y) = \begin{cases} 
\frac{1}{2a}, & \text{if } -a \leq y \leq a, \\
0, & \text{otherwise}. 
\end{cases}
\]

I consider the case of a relatively weak common component:

(A1) \( a > b \).

This gives us:

\[
\varphi(z_A) = \begin{cases} 
\frac{a+b+z_A}{4ab}, & \text{if } -a - b \leq z_A \leq b - a, \\
\frac{1}{2a}, & \text{if } b - a < z_A \leq a - b, \\
\frac{a+b-z_A}{4ab}, & \text{if } a - b < z_A \leq a + b, \\
0, & \text{otherwise}. 
\end{cases}
\]

If (A2) (i.e. the portfolio choice is not too low for any level of \( \pi_A \)) and (A3) (i.e. the portfolio choice is not too high for any level of \( \pi_A \)) both hold, banker A's objective can be written as:

\[
\mathbb{E}[V_A - 1_{\text{run}_A} \Phi] = S(k_A - i_A) + R \left[ \int_{-a-b}^{b-a} (\mu + z_A) \frac{a+b+z_A}{4ab} \, dz_A \right. \\
+ \left. \int_{b-a}^{i_A-\mu} (\mu + z_A) \frac{1}{2a} \, dz_A + \int_{i_A-\mu}^{a-b} i_A \frac{1}{2a} \, dz_A + \int_{a-b}^{a+b} i_A \frac{a+b-z_A}{4ab} \, dz_A \right] \\
- \pi_A \mathbb{E} \left[ \int_{-a-b}^{b-a} a+b+z_A \, dz_A + \int_{b-a}^{i_A-\mu} i_A \frac{1}{2a} \, dz_A \right] \\
- (1 - \pi_A) \mathbb{E} \left[ \theta_A < \frac{1}{R} [S\hat{i}_A - (Sk_A - \hat{D}_A)] \right] \\
= S(k_A - i_A) + R \left[ \frac{b}{2a} \left( \mu + \frac{b}{3} - a \right) \\
+ \frac{\mu(i_A+a-b) - \mu^2}{2a} + \frac{(i_A-\mu)^2 -(a-b)^2}{4a} + \frac{i_A(\mu + a - b) - i_A^2}{2a} + \frac{i_A b}{2a} \right] \\
- \pi_A \mathbb{E} \left[ \frac{b}{2a} \frac{Si_A - (Sk_A - \hat{D})}{2aR} - \frac{\mu - a + b}{2a} \right] \\
- (1 - \pi_A) \mathbb{E} \left[ \theta_A < \frac{1}{R} [S\hat{i}_A - (Sk_A - \hat{D}_A)] \right].
\]

The derivative with respect to \( i_A \) reads as:

\[
\frac{\partial \mathbb{E}[V_A - 1_{\text{run}_A} \Phi]}{\partial i_A} = -S + R \frac{\mu + a - i_A}{2a} - \pi_A \Phi \frac{S}{2aR}.
\]
Clearly, banker A’s objective is strictly concave in $i_A$. The first order condition gives us a closed-form solution for the optimal choice of $i_A$:

$$i_A^* = \mu - a \left( \frac{2S}{R} - 1 \right) - \pi_A \Phi \frac{S}{R^2}.$$ 

**Proof of Proposition 2.5**

The unconditional expected net value of banker A’s portfolio is equal to:

$$E[V_A - 1_{\text{run}_A} \Phi] = E[V_A | i_A = i_A^*] - \pi_A \Phi \Pr \left[ \theta_A < \frac{1}{R} \left[ Si_A^* - (Sk_A - \tilde{D}_A) \right] \right] - (1 - \pi_A) \Phi \Pr \left[ \theta_A < \frac{1}{R} \left[ Si_A - (Sk_A - \tilde{D}_A) \right] \right] = E[V_A | i_A = i_A^*] - \Phi \left[ \frac{b}{2a} + \frac{S_i^* A - (Sk_A - \tilde{D})}{2aR} - \frac{\mu - a + b}{2a} \right],$$

where the last equality is a direct implication of rational expectations: $\hat{i}_A = i_A^*$.

1. First I evaluate the effect of $\pi_A$ on the gross value of banker A’s portfolio.

$$\frac{\partial E_{\theta_A} [V_A | i_A = i_A^*]}{\partial \pi_A} = \frac{\partial E_{\theta_A} [V_A | i_A = i_A^*]}{\partial i_A} \frac{\partial i_A}{\partial \pi_A} = \left[ -S + R \frac{\mu + a - i_A^*}{2a} \right] \left( -\Phi \frac{S}{R^2} \right).$$

The last equality follows from conditions (A1), (A2), and (A3).

$$\frac{\partial E [V_A | i_A = i_A^*]}{\partial \pi_A} = -\pi_A \Phi^2 \frac{S^2}{2aR^3}.$$

The gross value of banker A’s portfolio is reduced to the extent that banker A takes into account the possibility of a bank run.

2. The probability of a bank run depends on transparency through her portfolio choice $i_A^*$:

$$\frac{\partial Pr[1_{\text{run}_A} = 1]}{\partial \pi_A} = -\Phi \frac{S^2}{2aR^3}.$$ 

It follows for the expected net value of bank A’s portfolio:

$$\frac{\partial E [V_A - 1_{\text{run}_A} \Phi]}{\partial \pi_A} = -\pi_A \Phi^2 \frac{S^2}{2aR^3} - \Phi \left( -\Phi \frac{S^2}{2aR^3} \right) = (1 - \pi_A) \Phi^2 \frac{S^2}{2aR^3}.$$ 

**Proof of Proposition 2.6**

The proof proceeds in several steps.
1. $r_A = r_B$: Assume otherwise. For instance: $r_A > r_B$. In this case, banker $A$ can benefit from raising $\tau_A$ by a little. It follows that $r_A \neq r_B$ cannot hold in equilibrium.

2. From $r_A = r_B$, it follows:

$$
\tau_A \mathbb{E} [V_A - 1_{run_A} \Phi] = \mathbb{E} [V_A - 1_{run_A} \Phi] - \frac{k_A}{w - k_A} (1 - \tau_B) \mathbb{E} [V_B - 1_{run_B} \Phi].
$$

The first derivative with respect to $\tau_A$ reads as:

$$
\frac{\partial \tau_A \mathbb{E} [V_A - 1_{run_A} \Phi]}{\partial \tau_A} = \frac{dk_A}{d\tau_A} \left[ \frac{\partial X_A}{\partial k_A} (1 - \tau_B) \left( \frac{k_A}{w - k_A} \frac{\partial X_B}{\partial k_A} + X_B \frac{w}{(w - k_A)^2} \right) \right],
$$

where:

$$
X_A \equiv \mathbb{E} [V_A - 1_{run_A} \Phi], \quad \text{and:} \quad X_B \equiv \mathbb{E} [V_B - 1_{run_B} \Phi].
$$

Furthermore, since $\mu + a + b < k_A$, a marginal unit of funds is always optimally invested in the riskless project: $\frac{\partial X_A}{\partial k_A} = S$. Since $\mu + a + b < k_B$, we also have: $\frac{\partial X_B}{\partial k_A} = -S$. From $r_A = r_B$, we can derive:

$$
\frac{dk_A}{d\tau_A} = \left( 1 - \tau_A \right) S - \left( 1 - \tau_B \right) X_A \frac{w}{(w - k_A)^2} \left( w - k_A \right) \neq 0.
$$

It follows that a first order condition for an interior choice of $\tau_A$ is:

$$
S - (1 - \tau_B) \left( \frac{w X_B}{(w - k_A)^2} - S \frac{k_A}{w - k_A} \right) = 0.
$$

Evaluated at a value $\tau_A^*$ which satisfies this first order condition, the second derivative of banker $A$’s objective with respect to $\tau_A$ is:

$$
\frac{\partial^2 \tau_A X_A}{\partial \tau_A^2} = -2(1 - \tau_B) \left( \frac{dk_A}{d\tau_A} \right)^2 \frac{w}{(w - k_A)^2} \left[ X_B - S(w - k_A) \right] < 0.
$$

It follows that banker $A$’s objective is strictly concave in $\tau_A$. There is a unique solution $\tau_A^*$.

3. The first derivative of banker $A$’s objective with respect to $\pi_A$ is given as:

$$
\frac{\partial \tau_A X_A}{\partial \pi_A} = \frac{\partial X_A}{\partial \pi_A} + \frac{dk_A}{d\pi_A} \frac{d\tau_A X_A}{d\pi_A}.
$$

If $\tau_A$ is chosen optimally, this becomes:

$$
\frac{\partial \tau_A X_A}{\partial \pi_A} = \frac{\partial X_A}{\partial \pi_A}.
$$
By Proposition 2.5, this expression is strictly positive for all values $\pi_A < 1$. The unique solution is $\pi_A^* = 1$.

4. $k_A = k_B$: Assume that banker $B$ sets some values $\hat{\tau}_B$ and $\hat{\pi}_B$. Now consider the values $\tau_A^*$ and $\pi_A^*$ which maximize banker $A$’s objective given $\hat{\tau}_B$ and $\hat{\pi}_B$. If $\hat{\tau}_B = \tau_A^*$ and $\hat{\pi}_B = \pi_A^*$, then both banker $A$ and banker $B$ maximize their respective objective. This follows from the symmetry of the two bankers’ maximization problems. Concavity implies that there is a unique equilibrium. It follows that the symmetric equilibrium is the unique one. It follows: $k_A = k_B = \frac{w}{2}$.

5. Combining the symmetry of the equilibrium with banker $A$’s first order condition for $\tau_A$ yields:

$$\tau_A^* = \tau_B^* = \frac{X_A}{\frac{2}{3} - \frac{s}{R}} - \frac{X_B}{\frac{2}{3} - \frac{s}{R}}.$$}

**Proof of Proposition 2.7**

The assumption $S > R - S$ implies: $i_A^* < \mu$. In this case, a bank run can only occur if at date 1 $\theta_A$ is revealed to be lower than $\mu$. Conditions (A0)-(A3) continue to hold.

The unconditional expectation at date 0 of the bank’s net portfolio value is given by:

$$E[V_A - \mathbb{1}_{\text{run}_A} \Phi] = S(k_A - i_A) + R \left[ \frac{b}{2a} \left( \mu + \frac{b}{3} - a \right) + \frac{\mu(i_A + a - b) - \mu^2}{2a} + \frac{(i_A - \mu)^2 - (a - b)^2}{4a} + \frac{i_A(\mu + a - b) - i_A^2}{2a} + i_A \frac{b}{2a} \right] - \pi_A \psi \Phi \left[ \frac{b}{2a} + S_i_A - \left( \frac{S k_A - \tilde{D}}{2aR} \right) - \frac{\mu - a + b}{2a} \right] - (1 - \pi_A) \psi \Phi \Pr \left[ \theta_A < \frac{1}{R} \left[ S_i_A - \left( S k_A - \tilde{D}_A \right) \right] \right].$$

The derivative with respect to $i_A$ reads as:

$$\frac{\partial}{\partial i_A} E[V_A - \mathbb{1}_{\text{run}_A} \Phi] = -S + R \frac{\mu + a - i_A}{2a} - \pi_A \psi \Phi \frac{S}{2aR}.$$ 

This gives us a closed-form solution for the optimal choice of $i_A$:

$$i_A^* = \mu - a \left( \frac{2S}{R} - 1 \right) - \pi_A \psi \Phi \frac{S}{R^2}.$$ 

The bank’s choice of $i$ is decreasing both in $\psi$ and in $\pi_A$. The expected net value of bank $A$’s portfolio is a function of $\psi$ and $\pi_A$:

$$\frac{\partial}{\partial \pi_A} E[V_A - \mathbb{1}_{\text{run}_A} \Phi] = (1 - \pi_A) \psi^2 \Phi \frac{S^2}{2aR^3}.$$ 

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occur at time \( t \). Whenever creditors are not sure that their debt claim will be fully served, Proposition 2.1 applies and a bank run is the only Nash equilibrium in pure strategies if and only if

\[ \tau_A = 0. \]  

This is the case if \( \tilde{A} \leq \tau_A \). 

Proof of Lemma 3.1

If \( i_A \) is publicly observable, creditors observe the exact value of \( V_A \) at \( t = 1 \). Lemma 2.1 applies and a bank run is the only Nash equilibrium in pure strategies if and only if \((1 - \tau_A)V_A < D_A\).

Consider now the roll-over decision of an individual atomistic creditor if \( i_A \) is not observable. Whenever creditors are not sure that their debt claim will be fully served, they demand a risk premium. The maximum face value which banker \( A \) can promise at \( t = 1 \) to her creditors in case of roll-over is \((1 - \tau_A)V_A^r(\theta_A)\). Default on this claim must occur at \( t = 2 \) if banker \( A \) was unsuccessful in screening at \( t = 0 \) and \( i_A \neq \theta_A \). In this case, creditors receive \((1 - \tau_A)V_A^r(\Phi)\). Banker \( A \) is willing to promise any risk premium in order to avoid early liquidation.

Short-term debt claims are served sequentially. If bank \( A \) cannot fully serve all other creditors at \( t = 1 \) \((D_A > (1 - \tau_A)[V_A - \Phi])\), roll-over by one individual agent yields zero. This is the case if \( \tilde{D}_A > V_A - \Phi \), with \( \tilde{D}_A = \frac{D_A}{1 - \tau_A} \). If creditors do not roll-over but \( \tilde{D}_A \leq V_A - \Phi \), roll-over by one individual agent yields up to \((1 - \tau_A)V_A^r(\theta_A)\) (the bank can always serve an individual atomistic debt claim as long as \( \tilde{D}_A \leq V_A - \Phi \)). Running like everyone else yields \( D_A \) (if \( \tilde{D}_A \leq V_A - \Phi \)) or an expected amount of \((1 - \tau_A)[V_A - \Phi]\) (if \( \tilde{D}_A > V_A - \Phi \)). If all other creditors choose to roll-over, projects need not be liquidated prematurely. Roll-over yields \( \min\{D_A, (1 - \tau_A)V_A\} \). Running yields \( D_A \).

We can distinguish six cases:

1. \( V_A^r > V_A^r > \tilde{D}_A + \Phi > \tilde{D}_A \): If all other creditors run, both roll-over and running yield \( D_A \). If all creditors roll-over, both running and roll-over yield \( D_A \). By offering a debt claim which pays off a tiny bit more than \( D_A \) at \( t = 2 \), the bank can achieve roll-over as the only equilibrium.

\[ \frac{\partial E[V_A - I_{runA}]_A}{\partial \psi} = \frac{\partial E[V_A - I_{runA}]_A}{\partial i_A} - \Phi \left[ \frac{b}{2a} + \frac{Si_A - (Sk_A - \tilde{D})}{2aR} - \frac{\mu - a + b}{2a} \right] \]

\[ = (1 - \pi_A)\psi \frac{S}{2aR} \left( \frac{\pi_A S}{R^2} \right) - \Phi \left[ \frac{b}{2a} + \frac{Si_A - (Sk_A - \tilde{D})}{2aR} - \frac{\mu - a + b}{2a} \right] \]

\[ = -\Phi \left[ \frac{b}{2a} + \frac{Si_A - (1 - \pi_A)\psi \pi_A S}{2aR} - \Phi \right] (\text{if } \tilde{D}_A \leq V_A - \Phi). \]

Condition (A2) holds. In order for the derivative above to be positive, it must be true that:

\[(1 - \pi_A) \psi - (1 - \pi_A)\pi_A \psi < 0. \]

This is never the case for \( \pi_A, \psi \in [0,1] \). It follows that the expected net value of bank \( A \)'s portfolio is falling in \( \psi \).

\[ 31 \text{As before, I continue to assume that banker } A \text{ cannot credibly promise in the interim period to forgo some part of her claim } \tau_A[V_A - \Phi]. \]

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2. \( V_A^i > D_A + \Phi > V_A^u > D_A \): If all other creditors run, roll-over can yield at most an expected amount \( p(1 - \tau_A)V_A^i + (1 - p) \times 0 \). Running yields \( pD_A + (1 - p)(1 - \tau_A)[V_A^u - \Phi] \). An individual creditor prefers to roll-over in this situation if and only if \( V_A^i > D_A + \frac{1 - p}{p}[V_A^u - \Phi] \). If all creditors roll-over, both roll-over and running yield \( D_A \). Roll-over is a Nash equilibrium.

3. \( V_A^i > D_A + \Phi > D_A > V_A^u \): If all other creditors run, roll-over can yield at most an expected amount \( p(1 - \tau_A)V_A^i + (1 - p) \times 0 \). Running yields \( pD_A + (1 - p)(1 - \tau_A)[V_A^u - \Phi] \). An individual creditor prefers to roll-over in this situation if and only if \( V_A^i > D_A + \frac{1 - p}{p}[V_A^u - \Phi] \). If all creditors roll-over, running yields \( D_A \) while roll-over yields up to \( p(1 - \tau_A)V_A^i + (1 - p)(1 - \tau_A)V_A^u \). The bank needs to offer a risk premium to compensate creditors for the risk that \( i_A \neq \theta_A \) and \( V_A = V_A^u \). Roll-over is a Nash equilibrium if and only if \( \mathbb{E}[V_A|Q_1] > D_A \).

4. \( D_A + \Phi > V_A^i > V_A^u > D_A \): If all other creditors run, roll-over yields 0. Running like the others yields an expected payoff of \( (1 - \tau_A)[\mathbb{E}[V_A|Q_1] - \Phi] \). A bank run is an equilibrium. If all creditors roll-over, both roll-over and running yield \( D_A \). By offering a debt claim which pays off a tiny bit more than \( D_A \) at \( t = 2 \), the bank can achieve roll-over as a Nash equilibrium.

5. \( D_A + \Phi > V_A^i > D_A > V_A^u \): If all other creditors run, roll-over yields 0. Running like the others yields an expected payoff of \( (1 - \tau_A)[\mathbb{E}[V_A|Q_1] - \Phi] \). A bank run is an equilibrium. If all creditors roll-over, running yields \( D_A \) while roll-over can yield at most an expected amount \( (1 - \tau_A)\mathbb{E}[V_A|Q_1] \). The bank needs to offer a risk premium to compensate creditors for the risk that \( i_A \neq \theta_A \) and \( V_A = V_A^u \). Roll-over is a Nash equilibrium if and only if \( \mathbb{E}[V_A|Q_1] > D_A \).

6. \( D_A + \Phi > D_A > V_A^i > V_A^u \): If all other creditors run, roll-over yields 0. Running like the others yields an expected payoff of \( (1 - \tau_A)[\mathbb{E}[V_A|Q_1] - \Phi] \). A bank run is an equilibrium. If all creditors roll-over, running yields \( D_A \) while roll-over can yield at most an expected amount \( (1 - \tau_A)\mathbb{E}[V_A|Q_1] \). We know that \( (1 - \tau_A)\mathbb{E}[V_A|Q_1] < D_A \). No risk premium offered is high enough to avoid a bank run. A bank run is the only equilibrium.

**Proof of Lemma 3.2**

The observation of \( \theta_B \) is informative with respect to the common component \( \Theta \). Banker \( A \) optimally updates her belief about \( \Theta \) using Bayes’ rule:

\[
\Theta|\theta_B \sim U ( \max\{ \mu - b, \theta_B - a \}, \min\{ \mu + b, \theta_B + a \} ) .
\]

I continue to study the case of a relatively weak common component: (A1) \( a > b \). This implies that even conditional on observing \( \theta_B \), banker \( A \)’s posterior of \( \Theta \) will at times be identical to her prior. Only extreme observations of \( \theta_B \) change her posterior.

\[
\hat{\mu} = \mathbb{E}[\theta_A|\theta_B] = \mathbb{E}[\Theta|\theta_B] = \frac{1}{2} \max\{ \mu - b, \theta_B - a \} + \frac{1}{2} \min\{ \mu + b, \theta_B + a \} .
\]

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In addition to $\hat{\mu}$, we also define:

$$\hat{b} \equiv \frac{1}{2} \min\{\mu + b, \theta_B + a\} - \frac{1}{2} \max\{\mu - b, \theta_B - a\}.$$ 

Now the problems with and without knowledge of $\theta_B$ are identical up to the two parameters $\mu$ and $b$, or $\hat{\mu}$ and $\hat{b}$, respectively. It is straightforward to show that:

$$b \geq \hat{b}, \quad \text{and} \quad \mu + b \geq \hat{\mu} + \hat{b}.$$ 

Together, these two inequalities imply that conditions (A1)-(A3) are sufficient conditions for:

(a1) \quad a > \hat{b},

(a2) \quad \hat{\mu} - a + \hat{b} < \frac{1}{R} \left[ S \left( \hat{\mu} - a \left( \frac{2S}{R} - 1 \right) - \Phi \frac{S}{R^2} \right) - (Sk_A - \tilde{D}_A) \right], \quad \text{and}

(a3) \quad \hat{\mu} - a \left( \frac{2S}{R} - 1 \right) < \hat{\mu} + a - \hat{b}.

**Proof of Lemma 3.3**

There are three possible situations for banker $A$’s portfolio choice: (1.) screening by banker $A$ is successful, (2.) screening by banker $A$ fails, but screening by banker $B$ is successful and $i_B$ is observable, (3.) neither $\theta_A$ nor $\theta_B$ is known. Ex-ante, before screening takes place, the unconditional expected net value of banker $A$’s portfolio is equal to:

$$E[V_A - 1_{\text{run}_A} \Phi] =$$

$$p \left[ E[V_A | i_A = \theta_A] - (1 - \pi_A) [p\pi_B \Phi Pr_u(i_A^*) + (1 - p\pi_B) \Phi Pr_o(i_A^*)] \right]$$

$$+ (1 - p) p\pi_B \left[ E[V_A | i_A = i_A^*] - \pi_A \Phi Pr_o(i_A^*) - (1 - \pi_A) \Phi Pr_u(i_A^*) \right]$$

$$+ (1 - p) (1 - p\pi_B) \left[ E[V_A | i_A = i_A^*] - \pi_A \Phi Pr_o(i_A^*) - (1 - \pi_A) \Phi Pr_u(i_A^*) \right].$$

Even if screening was successful, a bank run may occur if $i_A$ is unobservable, $\theta_A$ is low, and creditors put a low probability weight on the event that screening was a success. If $i_B$ is observable and banker $B$ has screened successfully, creditors know that banker $A$ had the option to react to the observation of $\theta_B$. In this case, the probability of a bank run is given by:

$$Pr_u(i_A^*) = Pr \left\{ p \left[ Sk_A + (R-S) \theta_A \right] + (1-p) \left[ S (k_A - i_A^*) + R \min \{i_A^*, \theta_A\} \right] < \tilde{D}_A \right\}. $$
If $i_B$ is not observable or banker $B$ has failed at screening her risky project, creditors know that banker $A$ did not have the option to react to the observation of $\theta_B$. In this case, the unconditional probability of a bank run is given by:

$$\Pr_u(i_A^*) = \Pr \left\{ p \left[ Sk_A + \left( R - S \right) \theta_A \right] + (1-p) \left[ S \left( k_A - i_A^* \right) + R \min \left\{ i_A^*, \theta_A \right\} \right] < \tilde{D}_A \right\}.$$ 

These expressions are derived under the assumption that creditors have rational expectations about $i_A$.

1. First I evaluate the effect of $\pi_A$ on the gross value of banker $A$’s portfolio if screening has failed. Her portfolio choice $x$ depends on the observability of $\theta_B$.

$$\frac{\partial \mathbb{E}_{\theta_A} [V_A | i_A = x]}{\partial \pi_A} = \frac{\partial \mathbb{E}_{\theta_A} [V_A | i_A = x]}{\partial x} \frac{\partial x}{\partial \pi_A} = \left[ -S + R \frac{\mu + a - x}{2a} \right] \left( -\Phi \frac{S}{R^2} \right).$$

The last equality follows from conditions (A1),(A2), and (A3). If $\theta_B$ is unknown to banker $A$, $x = i_A^*$ and the derivative above is deterministic. If banker $A$ knows $\theta_B$, her portfolio choice $x = i_A^{**}$ is a function of the random variable $\theta_B$. We know that $i_A^* = \mathbb{E}[i_A^{**}]$. This gives:

$$\frac{\partial \mathbb{E} [V_A | i_A = i_A^*]}{\partial \pi_A} = \frac{\partial \mathbb{E} [V_A | i_A = i_A^{**}]}{\partial \pi_A} = -\frac{\pi_A \Phi^2 S^2}{2aR^3}.$$

The gross value of banker $A$’s portfolio is reduced to the extent that banker $A$ takes into account the possibility of a bank run.

2. I continue to consider the case that screening by banker $A$ has failed. If banker $A$’s portfolio choice is public information, the probability of a bank run depends on her portfolio choice $x$:

$$\Pr_x(x) = \Pr \left[ \theta_A < \frac{1}{R} \left[ Sx - (Sk_A - \tilde{D}_A) \right] \right] = \frac{b}{2a} + \frac{S\mathbb{E}[x] - (Sk_A - \tilde{D}_A)}{2aR} - \frac{\mu - a + b}{2a}.$$

The second equality follows from conditions (A1),(A2), and (A3). We know that $\mathbb{E}[i_A^{**}] = i_A^*$, which allows us to conclude:

$$\Pr_x(i_A^*) = \Pr_x(i_A^{**}) = \frac{1}{2} - \frac{1}{2aR} \left[ \mu(R - S) + a \left( \frac{2S^2}{R} - S \right) + \pi_A \Phi \frac{S^2}{R^2} + Sk_A - \tilde{D}_A \right].$$

The observation of $\theta_B$ does not decrease the unconditional probability of a bank run because it does not reduce the uncertainty stemming from $\varepsilon_A$. If conditions (A1), (A2), and (A3) hold, this is the only risk which matters for banker $A$’s
portfolio choice at the margin. It follows that:

$$\frac{\partial \Pr_u(i_A^*)}{\partial \pi_A} = -\frac{\Phi S^2}{2aR^3}.$$

3. If banker A’s portfolio choice remains hidden, the probability of a bank run depends on creditors’ expectation of her portfolio choice in case screening has failed:

$$\Pr_u(x) = \Pr \left[ \theta_A < \frac{1}{R - pS} \left( (1 - p)Sx - (Sk_A - \tilde{D}_A) \right) \right] = \frac{b}{2a} + \frac{(1 - p)S\mathbb{E}[x] - (Sk_A - \tilde{D}_A)}{2a(R - pS)} - \frac{\mu - a + b}{2a}.$$

The second equality uses conditions (A1), (A2’), and (A3). From $\mathbb{E}[i_A^{**}] = i_A^*$, it follows:

$$\Pr_u(i_A^*) = \Pr_u(i_A^{**}) = \frac{1}{2} \frac{1}{2a(R - pS)} \left[ \mu(R - S) + (1 - p)a \left( \frac{2S^2}{R} - S \right) + (1 - p)\pi_A \frac{S^2}{R^2} + Sk_A - \tilde{D}_A \right].$$

Accordingly:

$$\frac{\partial \Pr_u(i_A^*)}{\partial \pi_A} = -\frac{1}{2a} \frac{1 - p}{R - pS} \frac{S^2}{R^2}.$$

**Proof of Lemma 3.5**

For $p = 0$, we are back in the case considered in Section 2.3. $\Delta(\pi_A) > 0$ for any value $\pi_A \in [0, 1)$:

$$\Delta(\pi_A) = (1 - \pi_A) \Phi^2 \frac{S}{2aR^3} > 0.$$

Furthermore, the function $\Delta(\pi_A)$ is continuous in $p$ and in $\pi_A$.

**Proof of Lemma 3.6**

In general, given a portfolio choice $i_A$ independent of $\varepsilon_A$, taking the expectation of banker A’s portfolio value with respect to $\varepsilon_A$ gives:

$$\mathbb{E}_{\varepsilon_A} [V_A] = Sk + i_A(R - S) - \frac{R}{4a} (\mu + \eta - a - i_A)^2.$$

Define $e = \mathbb{E} \left[ \Theta | Q_{0}^{A} \right] - \mu - \eta$ as the forecast error of banker A in estimating $\Theta$ conditional on her end-of-period-0 information set $Q_{0}^{A}$:

$$\mu + \eta - a - i_A = -e - 2a \frac{R - S}{R} + \pi_A \Phi \frac{S}{R^2}.$$
It follows for the expected value of banker A’s portfolio:

$$\mathbb{E} [V_A] = Sk + i_A^*(R - S) - \frac{R}{4a} \left[ \mathbb{E} [e^2] + \left[ 2a \frac{R - S}{R} - \pi_A \Phi \frac{S}{R^2} \right]^2 \right].$$

This is true no matter how many portfolio choices of rivals banker A observes. The only element which depends on the number of signals observed is the error variance $\mathbb{E}[e^2]$. It follows:

$$\mathbb{E} [V_A | i_A = i_A^*] - \mathbb{E} [V_A | i_A = i_A^*] = \frac{R}{4a} \mathbb{E} \left( \left[ \mathbb{E} [\Theta | \varnothing] - \Theta \right]^2 - \left[ \mathbb{E} [\Theta | \theta_B] - \Theta \right]^2 \right).$$

The width of the support of banker A’s belief about the position of $\Theta$ decides on the error variance. If banker A does not observe $\theta_B$, we know that:

$$\left[ \mathbb{E} [\Theta | \varnothing] - \Theta \right]^2 = \frac{(2b)^2}{12} = \frac{b^2}{3}.$$

If banker A observes $\theta_B$, her posterior about $\Theta$ depends on the realization of $\theta_B$. Let $M$ denote the width of the support of banker A’s posterior for $\Theta$. The probability distribution of the random variable $M$ is:

$$\varphi(M) = \begin{cases} \frac{M}{2ab}, & \text{if } 0 < M < 2b, \\ \frac{a-b}{a}, & \text{if } M = 2b. \end{cases}$$

It follows:

$$\left[ \mathbb{E} [\Theta | \theta_B] - \Theta \right]^2 = \frac{a-b}{a} \times \frac{(2b)^2}{12} + \int_0^{2b} \frac{M}{2ab} \times \frac{(M)^2}{12} dM = \frac{b^2}{3} - \frac{b^3}{6a}.$$

**Proof of Proposition 3.7**

The proof follows along the lines of the proof to Proposition 2.6. The only difference is in Step (3.) which demonstrates that there is a unique solution for $\pi_A^*$. To see that, consider the first derivative of banker A’s objective with respect to $\pi_A$:

$$\frac{\partial \tau_A X_A}{\partial \pi_A} = \frac{\partial X_A}{\partial \pi_A} - (1 - \tau_B) \frac{k_A}{w - k_A} \frac{\partial X_B}{\partial \pi_A} + \frac{d k_A}{d \pi_A} \frac{d \tau_A X_A}{d \pi_A}.$$

If $\tau_A$ is chosen optimally, this becomes:

$$\frac{\partial \tau_A X_A}{\partial \pi_A} = \frac{\partial X_A}{\partial \pi_A} - (1 - \tau_B) \frac{k_A}{w - k_A} \frac{\partial X_B}{\partial \pi_A}.$$
The second derivative with respect to $\pi_A$ is:

$$\frac{\partial^2 \tau_A X_A}{\partial \pi_A^2} = \frac{\partial^2 X_A}{\partial \pi_A^2} - (1 - \tau_B) \frac{k_A}{w - k_A} \frac{\partial^2 X_B}{\partial \pi_A^2} + \frac{dk_A}{d\pi_A} \frac{d}{dk_A} \left[ \frac{\partial \tau_A X_A}{\partial \pi_A} \right].$$

We know that:

$$\frac{d}{dk_A} \left[ \frac{\partial \tau_A X_A}{\partial \pi_A} \right] = \frac{d}{d\pi_A} \left[ \frac{\partial \tau_A X_A}{\partial k_A} \right].$$

In equilibrium $\tau_A$ (and therefore $k_A$) is chosen optimally and the term above is equal to zero. Furthermore, we know from the analysis above that the value of a bank is concave in transparency: $\frac{\partial^2 X_A}{\partial \pi_A^2} < 0$. We also know that the value of the rival bank is linear in $\pi_A$: $\frac{\partial^2 X_B}{\partial \pi_A^2} = 0$. It follows that banker A’s objective is strictly concave in $\pi_A$. There is a unique solution $\pi_A^*$. 

**Proof of Proposition 3.10**

The proof follows along the lines of the proof to Proposition 2.6. Step (3.) is as in the proof to Proposition 3.7. The only innovation with respect to the previous two proofs in step (2.) which demonstrates that there is a unique solution for $\tau_A^*$, and in step (5.) which pins down $\tau_A^*$. Consider the expected payoff of banker A used in the proof to Proposition 2.6. From $r_A = r_B$, it follows:

$$\tau_A \mathbb{E} [V_A - 1_{\text{run}_A} \Phi] = \mathbb{E} [V_A - 1_{\text{run}_A} \Phi] - \frac{k_A}{w - k_A} (1 - \tau_B) \mathbb{E} [V_B - 1_{\text{run}_B} \Phi].$$

The first order condition of banker A for an optimal choice of $\tau_A$ becomes:

$$S - (1 - \tau_B) \left( (N - 1) \frac{w X_B}{(w - k_A)^2} - S \frac{k_A}{w - k_A} \right) = 0.$$

Proceeding to step (5.), we use symmetry: $k_A = \frac{w - k_A}{N - 1} = \frac{w}{N}$. Banker A’s first order condition for an optimal choice of $\tau_A$ becomes:

$$S - (1 - \tau_B) \left( \frac{N}{N - 1} \frac{X_B}{k_A} - S \frac{X_B}{N - 1} \right) = 0.$$

It follows:

$$\tau_A^* = \frac{\rho - S}{\rho - \frac{S}{N}},$$

with: $\rho = \frac{\mathbb{E} [V_A - 1_{\text{run}_A} \Phi]}{k_A}$.

**Proof of Lemma 3.11**

Consider the difference of these two terms. With probability $1 - p \pi_N^*$, having $j + 1$ instead of $j$ banks does not matter for banker B’s information set $Q^B_0$ because the additional banker fails at screening or her portfolio choice is not public information. The difference
of the two terms is zero in this case. With probability \( p\pi^*_N \), the value of \( \theta_N \) is revealed and the information set \( Q^B_0 \) contains one more element because of the additional bank. The difference of the two terms is non-negative in this case. On expectation, it is equal to:

\[
\frac{\partial \mathbb{E}[V_B - 1_{run_B}\Phi]}{\partial \pi^*_A}
\bigg|_{N=j} - \frac{\partial \mathbb{E}[V_B - 1_{run_B}\Phi]}{\partial \pi^*_A}
\bigg|_{N=j+1} = p\pi^*_N(1-p) p \frac{R}{4a} \sum_{j=0}^{N-2} \binom{N-2}{j} (1 - \pi^*_N p)^{N-2-j}(\pi^*_N p)^j
\]

\[
\left[ \mathbb{E}[e(j)^2] - \mathbb{E}[e(j+1)^2] - \left( \mathbb{E}[e(j+1)^2] - \mathbb{E}[e(j+2)^2] \right) \right].
\]

What exactly is the value of: \( \mathbb{E}[e(j)^2] - \mathbb{E}[e(j+1)^2] \)? By defining \( 2m_j \) as the width of the banker’s posterior for \( \Theta \) after observing \( j \) realizations of \( \theta_l \), we can write:

\[
\mathbb{E}[e(j)^2] - \mathbb{E}[e(j+1)^2] = \mathbb{E}\left[\frac{(2m_j)^2}{12} - \frac{(2m_{j+1})^2}{12}\right] = \frac{1}{3} \mathbb{E}[m^2_j - m^2_{j+1}].
\]

Unfortunately, we cannot derive a tractable expression for \( \mathbb{E}[m^2_j] \). Therefore, we must take the value of \( m_j \) as given here. Conditional on some number \( m_j \leq b \), what is the distribution of \( m_{j+1} \)? We know about \( M \equiv 2m_{j+1} \):

\[
\varphi(M) = \begin{cases} 
\frac{M}{2am_j}, & \text{if } 0 < M < 2m_j, \\
\frac{a-2m_j}{a}, & \text{if } M = 2m_j.
\end{cases}
\]

It follows:

\[
\mathbb{E}[m^2_{j+1}] = \frac{1}{4} \mathbb{E}[M^2] = \frac{1}{4} \left[ \frac{a-2m_j}{a} (2m_j)^2 + \int_0^{2m_j} \frac{M}{2am_j} M^2 dM \right] = m^2_j - \frac{m^3_j}{2a}.
\]

The expected reduction of the error variance becomes:

\[
\mathbb{E}[e(j)^2] - \mathbb{E}[e(j+1)^2] = \frac{1}{3} \left[ m^2_j - m^2_j + \frac{m^2_j}{2a} \right] = \frac{m^3_j}{2a}.
\]

What about the corresponding expression after one more observation? To answer this question, we need to know the probability distribution after one more observation. To answer this question, we need to know the probability distribution after one more observation. To answer this question, we need to know the probability distribution after one more observation.
It is straightforward to see that:

$$
\varphi(M' = 2m_j) = \left(\frac{a - m_j}{a}\right)^2.
$$

For all realizations of $M'$ with $0 \leq M' < 2m_j$, we calculate:

$$
\varphi(M' = y) = \frac{a - m_j}{a} \times \frac{y}{2am_j} + \frac{a - \frac{y}{2}}{a} + \int_{y}^{2m_j} \frac{M}{2am_j} \frac{y}{aM} dM
$$

$$
= \frac{2ay + m_jy - \frac{3}{2}y^2}{2a^2m_j}.
$$

It follows:

$$
E \left[ m^2_{j+2} \right] - \frac{1}{4} E \left[ M'^2 \right] = \frac{1}{4} \left[ \left(\frac{a - m_j}{a}\right)^2 \left(2m_j\right)^2 + \int_{0}^{2m_j} \frac{2ay + m_jy - \frac{3}{2}y^2}{2a^2m_j} y^2 dy \right]
$$

$$
= m_j^2 - \frac{m_j^3}{a} + \frac{3}{10} \frac{m_j^4}{a^2}.
$$

The expected reduction of the error variance becomes:

$$
E \left[ e(j + 1)^2 \right] - E \left[ e(j + 2)^2 \right] = \frac{1}{3} \left[ m^2_{j+1} - m^2_{j+2} \right] = m_j^2 \left(1 - \frac{3m_j}{5a}\right).
$$

Finally it follows:

$$
E \left[ e(j)^2 \right] - E \left[ e(j + 1)^2 \right] - (E \left[ e(j + 1)^2 \right] - E \left[ e(j + 2)^2 \right]) = \frac{1}{10} \frac{m_j^4}{a^2}.
$$

On expectation, this term is positive for any finite number $N$.

**Proof of Proposition 3.12**

Consider the equilibrium allocation described by Proposition 3.10 for the case of $N$ banks. Denote $\tau_A^*(N)$ as the corresponding equilibrium price and define the fall in this price caused by adding one bank as: $\Delta \tau_A^* = \tau_A^*(N) - \tau_A^*(N + 1)$. Similarly, we define the size of the information externality as a function of $N$:

$$
B(N) \equiv (1 - p) \frac{R}{4a} \sum_{j=0}^{N-2} \binom{N - 2}{j} (1 - \pi_N^* p)^{N-2-j} (\pi_N^* p)^j \left[ E \left[ e(j)^2 \right] - E \left[ e(j + 1)^2 \right] \right],
$$

and $\Delta_B \equiv B(N) - B(N + 1)$ as the fall in the size of the information externality caused by adding one additional bank. The equilibrium level of transparency for the case of $N$
banks is smaller than for the case of $N + 1$ banks if and only if:

$$[1 - \tau^*_A(N)] B(N) > [1 - \tau^*_A(N + 1)] B(N + 1)$$
$$\Leftrightarrow [1 - \tau^*_A(N)] B(N) > [1 - \tau^*_A(N) + \Delta_\tau] [B(N) - \Delta_B]$$
$$\Leftrightarrow [1 - \tau^*_A(N)] \Delta_B > \Delta_\tau [B(N) - \Delta_B].$$

From Proposition 3.10 we know that:

$$\tau^*_A(N) = \frac{\rho - S}{\rho - \frac{S}{N}}$$
and:

$$\Delta_\tau = \frac{\rho - S}{(N \rho - S)(\rho(N + 1) - S)}.$$

If $R$ is sufficiently close to $S$, $\rho$ is close to $S$ and $\Delta_\tau$ close to zero for all values of $N$. Furthermore, we know from Lemma 3.11 that $\Delta_B$ is strictly positive for any finite number $N$.

The probability of a run on bank $A$ is convex in $\pi_A$. For any number $N$, we know from Corollary 3.9 that this probability is falling in response to a marginal increase in $\pi_A$ from its equilibrium level (if $\pi^*_A > 0$).

**Proof of Proposition 4.1**

Subtracting the left hand side of the first order condition for $\pi^*_A$ in Proposition 3.7 from the first order condition for $\pi^{SB}_A$ yields:

$$(1 - p) p \frac{Rb^3}{24a^2} + (1 - \tau^*_B) (1 - p) p \frac{Rb^3}{24a^2}.$$

If $R$ is close to $S$, $\tau^*_B$ is close to zero for all parameter values. In this case, the term above is monotonically increasing in $b$ and $R$, and falling in $a$.

**Proof of Proposition 4.2**

If $R$ is close to $S$, $\tau^*_B$ is close to zero for all values of $\pi_A$. The left hand side of the first order conditions for $\pi^*_A$ and for $\pi^{SB}_A$ is (almost) linear in $\pi_A$ in this case. Define $\pi^{MAX}_A$ as the value of $\pi_A$ which maximizes the expected net value of banker $A$’s portfolio. If $R$ is sufficiently close to $S$, then we have:

$$\pi^{MAX}_A \approx \frac{1}{2} [\pi^*_A + \pi^{SB}_A].$$

The probability of a bank run is convex in $\pi_A$. The value $\pi^{MAX}_A$ is the equilibrium choice for $N = \infty$. We know from Corollary 3.9 that the probability of a bank run is falling at $\pi^{MAX}_A$ (if $\pi^{MAX}_A > 0$). The second derivative of the probability of a bank run with respect to $\pi_A$ is constant. It follows that the probability of a bank run is lower for $\pi^{SB}_A$ than for $\pi^*_A$. 

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Proof of Proposition 4.3
Creditors take into account the possibility that screening by bank A was successful. If $\theta_A \geq \theta$ and the policy maker does not reveal $i_A$, creditors roll-over. The policy maker strictly prefers not to disclose $i_A$ in this case, because disclosure might reveal that bank A had failed at screening which would trigger a run. The policy maker’s decision not to disclose does not reveal anything about $i_A$ because the policy maker does not know $i_A$. If $\theta_A < \theta$ and the policy maker does not reveal $i_A$, creditors choose to run on bank A. The policy maker strictly prefers to disclose now, because a run is avoided if bank A had been successful at screening and $i_A = \theta_A$. This happens with probability $p$.

Proof of Proposition 4.4
There are two types of policy makers. They differ with respect to what they know about $i_A$. Type 1 knows that bank A has screened successfully: $i_A = \theta_A$. In case that creditors roll-over even if they do not observe $i_A$, the policy maker is indifferent whether to disclose this information. If creditors run if they do not observe $i_A$, disclosure is strictly better. Type 2 knows that bank A has failed at screening: $i_A \neq \theta_A$. In case that creditors roll-over even if they do not observe $i_A$, the policy maker strictly prefers no-disclosure. If creditors run if they do not observe $i_A$, she is indifferent.

1. Partial Revelation Equilibrium: Both type 1 and type 2 policy makers play disclosure in case no-disclosure triggers a run ($\theta_A < \theta$), and no-disclosure in case no-disclosure implies roll-over ($\theta_A > \theta$). Since both types play the same strategy, creditors cannot infer anything about $i_A$ from the policy maker’s action. Partial revelation is an equilibrium just as in the case of an uninformed policy maker.

2. Full Revelation Equilibrium: Type 1 always plays disclosure irrespective of $\theta_A$, while type 2 always plays no-disclosure. Creditors respond by running whenever $i_A$ is not disclosed. They roll-over whenever $i_A$ is disclosed and $i_A = \theta_A$. Full revelation is an equilibrium.
References


