Compensating the Compensating Variation

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Abstract. The message of this note is that in a general equilibrium setting the compensating variation is numéraire dependent. In contrast, the equivalent variation is neutral regarding the choice of value units. We illustrate with a simple example and propose an even simpler solution to overcome this bias in the compensating variation; all that is required to have a correct welfare estimate is to compensate the compensating variation by normalization with a price index. This type of correction is necessary to overcome the often blind implementation of welfare measures in numerical general equilibrium.

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1. The set up.

Welfare evaluation relies heavily in the use of the compensating and equivalent variation measures, CV and EV respectively, for comparing equilibrium states. They are exact welfare measures, have a clear cut interpretation in terms of income, give bounds for consumer surplus for normal goods, and solve the problems of directly using utility levels (Willig, 1976). Utilities are just ordinal and thus any literal interpretation beyond the sign of the change in utilities between two equilibrium states is not justified. Assume well behaved consumers so that all demand functions, Marshallian and Hicksian, have the standard properties. From the utility maximization problem we derive the indirect utility function $v(p,m)$, which gives maximal utility at prices p and income m. In turn, the solution of the expenditure minimization problem yields the expenditure function $e(p,u)$. It gives the minimal cost of achieving utility u at prices p. Both optimization problems will share, under standard conditions, the same optimal consumption bundle if we make $m = e(p, v(p,m))$ and $u = v(p, e(p,u))$ (Varian, 1992; Mas-Colell et al, 1995).

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From an agent's perspective, optimal consumption decisions only require knowledge of the vector of market prices p and that agent's income level m. Consider now two equilibrium states (0: original, 1: final) from this agent's viewpoint. All the relevant information for this individual decision making is contained in the pairs (p^0, m^0) and (p^1, m^1).

For this individual, welfare changes between the two equilibrium states, from u^0 to u^1 in utility terms, can be measured by the compensating and equivalent variations, CV and EV, with expressions:

$$\text{CV} = e(p^1, u^1) - e(p^1, u^0)$$

$$\text{EV} = e(p^0, u^1) - e(p^0, u^0)$$

Since we are aiming at a negative result, it is sufficient to show it under simplified conditions. For this, we assume that the utility function is linearly homogeneous. Under this condition (which includes all the standard CES functions) it is well-known that the expenditure function is multiplicatively separable: $e(p,u)=e(p,1)\cdot u$, with $e(p,1)$ being the expenditure necessary to achieve a unit of utility. Take the expression for CV and use this separability property to obtain:

$$\text{CV} = e(p^1,1)\cdot u^1 - e(p^1,1)\cdot u^0 = e(p^1,1)\cdot(u^1 - u^0) =$$

$$= e(p^1,1)\cdot u^1 \frac{(u^1 - u^0)}{u^1} = e(p^1,u^1)\cdot\frac{(u^1 - u^0)}{u^1} = m^1\cdot\frac{(u^1 - u^0)}{u^1}$$

Similarly, the equivalent variation can be seen to be:

$$\text{EV} = m^0\cdot\frac{(u^1 - u^0)}{u^0}$$

2. The general equilibrium issue.

Notice that the selection of a numéraire to represent the equilibrium price vector will not have any effect on the utility ratios that appear in the CV and EV expressions. Under general equilibrium, however, income levels are price dependent. We may represent this by writing $m(p^0)$ and $m(p^1)$. Not only this, the choice of numéraire will affect the expression of income in the initial and final equilibriums. However, in numerical general equilibrium applications this only matters for the final equilibrium state. The reason is that the implementation of the initial equilibrium requires the selection of special units so that all empirically observed value magnitudes can be separated into value units and physical units. The natural option is to select (fictitious) units in such a way that a physical fictitious unit has a worth of 1 dollar. These fictitious physical units are then used under any counterfactual equilibrium. With this calibration trick all goods have initial prices equal to 1, even the numéraire, whichever good it happens to be. This gives complete freedom in the initial selection of the numéraire. Notice too that thanks to the calibration procedure, that sets all prices equal to 1, EV is always well defined and has a constant value, regardless of the initially selected numéraire. When the equilibrium changes, however, the choice of numéraire becomes relevant to measure final income levels and, therefore, it has an effect on the expression of CV. A direct comparison of EV and CV might be misleading since they are evaluated at different equilibrium prices. More on this in the example below.

3. A simple example and a simpler solution.

Take a consumer in a two good exchange economy. This consumer owns endowments $\omega=(2,8)$ and is described by a Cobb-Douglas utility function $u(x_1,x_2)=(x_1x_2)^{0.5}$.

Initial equilibrium prices are $p^0=(1,1)$ and good 1 is the numéraire. The equilibrium consumption for this agent is $x^0=(5,5)$, his income is $m^0=10$ and his optimal utility is $u^0=5$. Some shock, say a tax policy, perturbs now the equilibrium and the new equilibrium price vector turns out to be $p^1=(1,2)$. For these prices, the consumer's equilibrium figures become $x^1=(9,4.5)$, $m^1=18$, $u^1=6.36$. Welfare effects using the expressions above give EV=2.73, CV=3.86 (with EV<CV)

Change now the numéraire to good 2. Since relative prices would not change, prices would take the expression $p^1=(0.5,1)$. The equilibrium consumption bundle and utility level would be unaffected, but income would now be $m^1=9$. The measure of the compensating variation would change to CV=1.93 (with EV>CV). Something is amiss when depending on the numéraire the relationship between CV and EV shifts. Notice that everything has been rescaled by a factor of ½, the same factor used in re-dimensioning the vector price. The question is how to report the CV in a way that is independent of the arbitrary and apparently innocuous choice of numéraire. In the example we have played with two possible numéraires but there are in fact infinitely many possibilities. Any unit of value that keeps the relative price p_1/p_2 in the final equilibrium equal to ½ would do. Although the welfare sign would always be correctly captured, the estimated volume would be misleading if not reported correctly. Inadvertently, applied general equilibrium practitioners have been using, reporting and ascribing literal value to the CV welfare measure paying no attention to the crucial role played by the numéraire.

The solution is straightforward and requires that CV is itself compensated to eliminate any nominal effects induced by the choice of numéraire. Any price index $\overline{p}^1$ based on equilibrium prices p^1 will make the calculation for the compensating variation independent of the selected numéraire with a constant value of $\mathrm{CV}/\overline{p}^1$, regardless of the selected good used as numéraire. Therefore all that remains is the selection of a price index. If we wish to measure the purchasing power of income, a consumers' price index would just do.

Another example of the same type of numéraire induced bias would occur when calculating the true index of cost of living (Konüs, 1939) in a general equilibrium model. This index is defined by $\lambda = e(p^1, u)/e(p^0, u)$. For a linearly homogeneous utility function the index simplifies to $\lambda = e(p^1, 1)/e(p^0, 1)$ and both the Laspeyres (utility reference u^0) and Paasche (utility reference u^1) versions coincide. For the same consumer of the example we calculate minimal expenditures for unitary utilities in both equilibrium states. For the initial equilibrium $p^0=(1,1)$ we would find $e(p^0,1)=2$. After the equilibrium is perturbed and good 1 is the numéraire, we calculate the minimal expenditure at prices $p^1=(1,2)$ and obtain $e(p^1,1)=2.83$ with the Konüs index being $\lambda=1.41$. We now change the numéraire to being good 2, the price vector is now $p^1=(0.5,1)$, with minimal expenditure reaching $e(p^1,1)=1.41$ and thus $\lambda=0.71$. Notice, as before, that everything has been rescaled by a factor of ½. The question becomes, once again, how to report the Konüs index in a way that is independent of the choice of numéraire. The answer is that, here too, we should rely on a price index $\overline{p}^1$ and then report $\lambda/\overline{p}^1$ as the corrected true index of cost living. This value is independent of the chosen unit of value in the general equilibrium model.

For utilities that are not linearly homogeneous the same problems occur, and the same type of solution applies. The only practical difference is that calculations cannot use the simplified expressions for EV and CV, and for λ as well, that have allowed us to visualize directly where and how the measurement problem manifests itself.

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