Measuring the true index of cost of living under general equilibrium

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Abstract

We show that the index of cost of living introduced by Konüs (1939) is numéraire dependent in a general equilibrium setting. This dependency gives rise to ambiguity situations for the interpretation of the index. To correct for this ambiguity we show that we need to neutralize the standard Konüs index using a price index. This correction eliminates the interpretational problem due to, and inherited from, the selection of the numéraire. We also provide a simplified expression of the index for the case of homothetic utilities. Two numerical examples show the discrepancies between the literal estimates, as they would be used under partial equilibrium, and the neutralized version. The discrepancies would give rise to erroneous assessments in the evaluation of the welfare effects resulting from the adoption of new policies.

Keywords: Konüs index, true cost of living, general equilibrium, welfare effects.

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1. Introduction

Under partial equilibrium, when prices of commodities are given in nominal terms, the index introduced by Konüs (1939), and known as the true cost of living index, is always well defined provided the utility function is known. When we consider a general equilibrium set up, however, the representation of the equilibrium price vector is unique but only up to a selection of units. The selection of numéraire will therefore have an effect on the numerical estimates of the Konüs index since such selection will directly affect the ratio of minimal expenditures of consumers when equilibrium prices change. The main purpose of this note is to provide to the empirically oriented general equilibrium researcher a practical method to correct and solve the problem of the dependency of the Konüs index on the choice of the numéraire. If left uncorrected, this dependency can in fact give rise to misleading welfare evaluations of enacted policy alternatives.

This note is organized as follows. In section 2 we introduce the concept of Konüs index in a partial equilibrium set up. We also consider the specification of the index under the general class of homothetic utilities. In section 3 we show by means of two examples the problems arising regarding the correct interpretation of the index when we are placed in a general equilibrium set up. We conclude this section with a practical result to make the Konüs index immune to changes in the measuring units of commodities or monetary values. Last section concludes with some general comments and remarks.

2. The Konüs index in partial equilibrium

The Konüs index is an index that aims at capturing the so-called *true* cost of living. Consider a standard consumer whose preference relation satisfies all the canonical properties of micro theory. For a given price vector p, and utility level u, the expenditure function $e(p, u)$ measures the minimal expenditure needed to attain utility level u. To be more precise,

$$e(p,u) = Min\left\{ px \mid u(x) = u \right\}$$

where x is a vector of consumption goods.

We consider two equilibrium situations with initial prices being p^0 and final prices being p^1. The true index of cost of living at u is defined by Konüs as the expenditures ratio:

$$\kappa = \frac{e(p^1,u)}{e(p^0,u)} \tag{1}$$

In other words, it informs of the change in the minimal expenditure needed to keep the consumer at a given utility level u. To have a numerical estimate of the Konüs index a level of utility must be chosen as reference.

There are two obvious options, initial utility u^0 or final utility u^1. When we choose the utility level at the initial equilibrium prices expression (1) becomes:

$$\kappa^0 = \frac{e(p^1,u^0)}{e(p^0,u^0)} \tag{2}$$

Initial utility comes from the solution of the consumer's problem at prices p^0. Let x^0 be the optimal solution vector so that $u^0=u(x^0)$. Recall that by definition:

$$e(p^1,u^0) = Min\left\{p^1x \mid u(x)=u^0\right\} \leq p^1x^0 \tag{3}$$

since $u^0=u(x^0)$ implies that x^0 is feasible for the minimization problem (3). From here and the fact that $e(p^0,u^0)=p^0x^0$ we can write (2) as:

$$\kappa^0 = \frac{e(p^1,u^0)}{e(p^0,u^0)} = \frac{Min\left\{p^1x \mid u(x)=u^0\right\}}{e(p^0,u^0)} \leq \frac{p^1x^0}{e(p^0,u^0)} = \frac{p^1x^0}{p^0x^0} \tag{4}$$

In conclusion, the Konüs index at initial utility is bounded above by a Laspeyres price index. This index is observable from available equilibrium data.

Similar considerations would show that when we choose final utility u^1 as the reference utility in expression (1), the Konüs index is then bounded from below by a Paasche price index. We omit the algebra here and just write the result:

$$\frac{p^1x^1}{p^0x^1} \leq \kappa^1 = \frac{e(p^1,u^1)}{e(p^0,u^1)} \tag{5}$$

So, even if the Konüs index is not directly observable –and for this to happen we would need all the information on preferences that would allow us to derive the expenditure function– we can conclude that it has bounds that are observable from equilibrium data on prices and quantities.

2.1 <u>The case of homothetic utilities</u>

For linearly homogeneous utility functions (or under constant returns to scale for technologies) the expenditure function takes the particularly convenient form $e(p,u)=e(p,1)u$ with $e(p,1)$ being the minimal expenditure necessary to achieve a unitary level of utility (Varian, 1992). Less known is the fact that for the more general class of homothetic utility functions (such as are all the CES utility functions) the expenditure function turns out to be multiplicatively separable, namely $e(p,u)=e(p,1)h(u)$, with h

being a monotone increasing function[1]. As it is known, a homothetic utility is a positive monotone transformation of a linearly homogenous utility function. The proof of this separability result is straightforward (see the Appendix).

Thanks to this separability property in the homothetic case, the Konüs index takes the form:

$$\kappa = \frac{e(p^1,u)}{e(p^0,u)} = \frac{e(p^1,1)h(u)}{e(p^0,1)h(u)} = \frac{e(p^1,1)}{e(p^0,1)} \qquad (6)$$

no matter what level of utility, initial u^0 or final u^1, we take as reference. Hence for homothetic functions, utility levels play no role in the index. Thus $\kappa^1 = \kappa^0$ and the common value for the index has Laspeyres and Paasche bounds—which are both observable with the appropriate prices and quantities equilibrium data:

$$\frac{p^1x^1}{p^0x^1} \le \kappa \le \frac{p^1x^0}{p^0x^0} \qquad (7)$$

The interpretation of (6) is that if $\kappa > 1$ the consumer needs to allocate more expenditure at equilibrium 1 than at equilibrium 0 to achieve the same level of unitary utility. Acquiring this basic utility level becomes more costly and, thanks to the homotheticity property, this extends to the actual equilibrium utility values. This has a negative interpretation in welfare terms. Similarly, $\kappa < 1$ indicates the opposite, namely, that attaining utility becomes more affordable at the new equilibrium and this is considered to be beneficial for the welfare of the consumer.

3. The Konüs index in general equilibrium

We consider first two numerical examples to illustrate what we may call the numéraire dependence problem for the calculation and interpretation of the Konüs index under general equilibrium. Our examples are relevant to illustrate what may happens in the usual practice that we encounter in the applied general equilibrium methodology, where all prices are equal to one in the initial equilibrium, and a numeraire is chosen (usually labor or capital services) for the simulated equilibria.

Example 1

Let us now consider the fictitious example in the Guerra-Sancho (2017) paper on the need to compensate the compensation variation from the influence of the numéraire.

[1] In fact, neither of the two leading graduate level microeconomics textbooks (Varian, 1992; Mas-Colell et al, 1995) mentions this property of the expenditure or cost function under homotheticity.

We take a consumer in a general equilibrium setting who owns endowments of two goods ω=(2,8) and is described by a standard symmetric Cobb-Douglas utility function:

$$u(x_1, x_2) = \sqrt{x_1 \cdot x_2} \tag{8}$$

We assume to begin with that initial equilibrium prices are p^0=(1,1) and good 1 is the numéraire. The benchmark equilibrium consumption for this agent is x^0=(5,5), his income (value of his endowment) is m^0=10 and his optimal utility is u^0=5. Consider now a shock, say a tax policy, that perturbs the equilibrium so that the new equilibrium price vector turns out to be p^1=(1,2) with good 1 still being the numéraire.

For the initial equilibrium p^0=(1,1) we would find $e(p^0,1)$=2. After the equilibrium is perturbed and good 1 is the numéraire, we calculate the minimal expenditure for unitary utility at prices p^1=(1,2) and obtain $e(p^1,1)$=2.83 with the Konüs index being κ=1.41 from expression (6). Notice that $\kappa>1$.

We now change the numéraire to good 2, the price vector is now p^1=(0.5,1), with minimal expenditure reaching $e(p^1,1)$= 1.41 and thus κ=0.71. Now we have $\kappa<1$. Nothing has changed in the real equilibrium but the Konüs index report two numbers with diametrically opposite meaning. This apparent contradiction depends directly on the selection of units. Notice that everything has been rescaled by a factor of ½. The question becomes, once again, how to report the Konüs index in a way that is independent of the choice of numéraire.

The answer is that we should rely on a price index to neutralize the choice of numéraire and then report the corrected true index of cost living. This value will be independent of the chosen unit of value in the general equilibrium model. Given a vector of prices, p, a price index $\overline{p}$ is defined by $\overline{p} = \sum_{j=1}^{n} \alpha_j p_j$ with $\alpha_j \geq 0$ and $\sum_{j=1}^{n} \alpha_j = 1$.

We will now define the corrected, up to the choice of numéraire, Konüs index by the expression:

$$\overline{\kappa} = \frac{e(p^1,u)/\overline{p}^1}{e(p^0,u)/\overline{p}^0} \tag{9}$$

For homothetic utilities this is clearly the same as:

$$\overline{\kappa} = \kappa \frac{\overline{p}^0}{\overline{p}^1} \tag{10}$$

Using the data in the example and taking as price index the standard mean of prices we would have, for good 1 being the numéraire:

$$e(p^1,1)/\overline{p}^1 = 2.83/1.5 = 1.88$$

For equilibrium 0 we would have:

$$e(p^0,1)/\overline{p}^0 = 2/1 = 2$$

Let us now change the numéraire to good 2 so that equilibrium prices are now (0.5,1). Recall that this selection does not have any effect on real equilibrium magnitudes. In this case:

$$e(p^1,1)/\overline{p}^1 = 1.41/0.75 = 1.88$$

$$e(p^0,1)/\overline{p}^0 = 2/1 = 2$$

In both cases the corrected Konüs index would be unaffected by the selection of numéraire:

$$\overline{\kappa} = \frac{e(p^1,1)/\overline{p}^1}{e(p^0,1)/\overline{p}^0} = \frac{1.88}{2} = 0.94$$

With no ambiguity regarding the numéraire we can now conclude that the (corrected) true index of cost of living has gone down between equilibrium states 0 and 1, $\overline{\kappa} < 1$. Notice that this example corresponds to the case where the underlying general equilibrium model is *calibrated* to empirical data so that all initial equilibrium prices are set equal to 1. In this calibrated situations, the selection of the numéraire leaves unaffected the denominator of the index since for any conceivable numéraire all initial price indices have unitary value. But as we have shown, the numerator is directly affected by the numéraire.

Example 2

We now use the well-known Shoven and Whalley general equilibrium model of 1984 in the Journal of Economic Literature to calculate the true index of cost of living. In this paper the authors do not calculate the Konüs index among their welfare indicators. Our computational implementation of the same SW model yields easily the results for this welfare indicator. Since there are two consumers ("poor" and "rich") in their model, we report the Konüs index for each consumer. A glance at the SW text shows it is an *uncalibrated* model so that in this case the selection of the numéraire will affect the denominator in the Konüs index as well. Indeed, their initial equilibrium prices are not equal to 1.

We report the results for two possible numéraire selections, labor and capital, in Table 1. In their paper they use the price of labor as the only numéraire. We run the same fiscal simulation they use to illustrate the power of numerical general equilibrium.

[Table 1 around here]

Reading this table, when only focusing on labor as numéraire and without the correction for the numéraire effect, we would conclude that the cost of living goes down for both consumers since the uncorrected Konüs index is κ<1. Once the effect of the numéraire is neutralized we observe that the "rich" consumer is worse off ($\bar{\kappa}$>1) in the sense that it is more costly for him to attain the basic unitary level of utility whereas the "poor" consumer is better off, for exactly the opposite reason ($\bar{\kappa}$<1).

When we select capital as the numéraire good, the second half of the Table shows that all the estimations are re-dimensioned. But now both consumers have uncorrected Konüs indices κ>1. Clearly this cannot be the case since nothing has changed in the economy except the numéraire. Once we perform the correction with the price index we recover the correct values and verify, as expected, that they coincide with the values for the previous numéraire.

3.1 A practical result

These examples show that the neutralization of the price effects due exclusively to the selection of units is critical in order to have consistent estimates. A question that arises from these results is if there is a particular price normalization that yields a corrected Konüs index $\bar{\kappa}$ equal to the definitional one κ. The answer is positive and immediate.

To simplify the reasoning and notation we focus first on the homothetic case of preferences because, in that case, the Konüs index has a unique value in equilibrium.

Proposition: *Let the numéraire be the price index* $\bar{p}$*, i.e.* $\bar{p}=1$ *. Then* $\bar{\kappa}=\kappa$*.*

Indeed, this selection obviously guarantees that equilibrium prices will take values such that $\bar{p}^1=\bar{p}^0=1$ and the conclusion follows trivially from equation (9).

A more general solution to our problem that goes beyond the Konüs index itself goes as follows. Recall that the expenditure function is homogeneous of degree 1 in prices *p*. Hence $e(p,u)/\bar{p}=e(p/\bar{p},u)=e(q,u)$ and, no matter what the initial selection of numéraire may be, the (normalized) price vector q with $q_j=p_j/\bar{p}$ satisfies that its price index is always equal to 1, the numéraire. From here, $\bar{\kappa}=\kappa=e(q^1,u)/e(q^0,u)$. In other words, the solution to the ambiguity problem in the welfare measure requires the use of a price index, once a certain numéraire is selected for whatever desirability reasons. For the non-homothetic case, the same reasoning of the neutralization solution would apply but we would have two different estimates of the true index of cost of living depending on the chosen reference utility level, initial or final.

As we have seen, alternatively to the use of a corrected Könus index, we might also opt for the numéraire itself being any particular price index and then equilibrium prices would be normalized using this price index. Under such normalization, the estimated Könus index will directly provide the correct welfare value. However, this type of normalization has not been the usual methodological way to procced in numerical general equilibrium simulation exercises. Most often modelers have chosen a certain commodity, labor in the majority of cases, as the numéraire yardstick. Hence the need to correct, in all these cases, the derived Konüs index with the corresponding price index.

4. Concluding remarks

Exercises in empirical general equilibrium allow us to compare equilibrium states under alternative policies. An appropriate estimate of the corresponding welfare effects is of course necessary if we want to provide sound advice to policy makers. We should do away with any level of ambiguity in the interpretation of results. The Konüs index for measuring the true cost of living, if taken literally as in its partial equilibrium definition, would yield undesirable ambiguity due to the apparently innocuous choice of numéraire. The correction of the Konüs index via a price index solves the problem in a general equilibrium setting. Surely, the definition of which price index is finally chosen matters for the numerical estimates of the true index of cost of living under general equilibrium. Different price indices will yield different estimates for the (corrected) Konüs index but these estimates will always be unaffected by the selection of numéraire. There are very many possibilities in terms of price indices, as we all well know. For instance, the measure of inflation will be different if we use a consumer's price index or any other alternate price index such as a retail price index or a producer's price index. Inflation is a phenomenon that can be measured with different metrics. The same happens with the (corrected) estimate of the Konüs index. However, the fact that the Konüs index aims at measuring the cost of living of consumers suggests that a consumer's price index that captures purchasing power would be the appropriate compensating index. In this case, moreover, the definitional Konüs index coincides with the one providing the correct welfare estimates as our result shows.

Table 1: Konüs index in the SW general equilibrium model

Numéraire: Labor w=1

Uncorrected:

Final unitary expenditure:	Rich consumer	Poor consumer
$e(p^1,1)$:	1.20376	1.13052
Initial unitary expenditure:	Rich consumer	Poor consumer
$e(p^0,1)$:	1.23197	1.17899
Konüs λ :	0.97710	0.95888

With price index corrections:

Final unitary expenditure:	Rich consumer	Poor consumer
$e(p^1,1)/\overline{p}^1$:	1.02341	0.96115
Initial unitary expenditure:	Rich consumer	Poor consumer
$e(p^0,1)/\overline{p}^0$:	1.02129	0.97737
Konüs $\overline{\lambda}$:	1.00208	0.98340

Numéraire: Capital r=1

Uncorrected:

Final unitary expenditure:	Rich consumer	Poor consumer
$e(p^1,1)$:	1.07750	1.00255
Initial unitary expenditure:	Rich consumer	Poor consumer
$e(p^0,1)$:	0.89697	0.85840
Konüs λ :	1.19011	1.16792

With price index corrections:

Final unitary expenditure:	Rich consumer	Poor consumer
$e(p^1,1)/\overline{p}^1$:	1.02341	0.96115
Initial unitary expenditure:	Rich consumer	Poor consumer
$e(p^0,1)/\overline{p}^0$:	1.02129	0.97737
Konüs $\overline{\lambda}$:	1.00208	0.98340

Appendix: Separability of the expenditure function for homothetic utilities

A homothetic utility is a positive monotone transformation of a linearly homogenous utility function. Let u be linearly homogenous and let g be a real valued function such that g'>0. Then the composite function $(g \circ u) = g(u)$ is a homothetic utility.

Let us now assume that preferences represented by $(g \circ u)$ satisfy all the "well-behaved" standard assumptions of micro-theory. We set up the expenditure minimization problem:

$$e(p,u) = \underset{x}{\text{Min}}\{px \mid g(u(x)) = u\}$$

By strict monotonicity of g:

$$\underset{x}{\text{Min}}\{px \mid g(u(x)) = u\} = \underset{x}{\text{Min}}\{px \mid u(x) = g^{-1}(u)\}$$

Since u is linearly homogeneous $u(x)/g^{-1}(u) = u(x/g^{-1}(u))$ and from here:

$$\underset{x}{\text{Min}}\{px \mid u(x) = g^{-1}(u)\} = \underset{x}{\text{Min}}\left\{px \mid u\left(\frac{x}{g^{-1}(u)}\right) = 1)\right\}$$

We now perform a change of variable so that $z = x/g^{-1}(u)$. Substituting and recalling that for given u the minimization solution is the same for z than for x:

$$\underset{x}{\text{Min}}\left\{px \mid u\left(\frac{x}{g^{-1}(u)}\right) = 1\right\} = \underset{x}{\text{Min}}\left\{px\frac{g^{-1}(u)}{g^{-1}(u)} \mid u\left(\frac{x}{g^{-1}(u)}\right) = 1\right\} =$$

$$= g^{-1}(u)\underset{x/g^{-1}(u)}{\text{Min}}\left\{p\frac{x}{g^{-1}(u)} \mid u\left(\frac{x}{g^{-1}(u)}\right) = 1\right\} =$$

$$= g^{-1}(u)\underset{z}{\text{Min}}\{pz \mid u(z) = 1\} = g^{-1}(u)e(p,1)$$

We therefore find that $e(p,u) = e(p,1)g^{-1}(u) = e(p,1)h(u)$ with $h(u) = g^{-1}(u)$ and $h' > 0$ if $g' > 0$. Thus minimal expenditure is strictly increasing in u.

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