Abstract

What determines the optimal monetary trade-off between internal objectives (inflation, and output gap) and external objectives (competitiveness and trade imbalances) when inefficient capital flows cause exchange rate misalignment and distort current account positions? We characterize this trade-off analytically, using the workhorse model of modern monetary theory in open economies under incomplete markets—where inefficient capital flows and exchange rate misalignments can arise independently of nominal distortions. We derive a quadratic approximation of the utility-based global policy loss function under fairly general assumptions on preferences and openness, and solve for the optimal targeting rules under cooperation. We show that, in economies with a low degree of exchange rate pass-through, the optimal response to inefficient capital inflows associated with real appreciation is contractionary, above and beyond the natural rate: the optimal policy curbs excessive demand at the cost of exacerbating currency overvaluation. In contrast, a high degree of pass-through, and/or low trade elasticities, warrants expansionary policies that lean against exchange rate appreciation and competitive losses, at the cost of inefficient inflation.

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Better macro performance comes from a monetary rule that recognizes how an external deficit raises the natural real rate of interest.” Obstfeld and Rogoff [2010] p. 34.

1 Introduction

External deficits and large swings in exchange rates associated with cross-border capital flows confront monetary authorities with complex trade-offs between price stability, growth, trade imbalances and exchange rate competitiveness. In the actual experience of policy-making, the trade-offs between these “internal” and “external” objectives are often resolved in different ways. Drawing on the experience of industrialized countries, a case in point is the monetary policy response to capital inflows and widening external deficits in Germany and the United States in the 1990s. In the early 1990s, the Bundesbank responded to external deficits in the aftermath of German unification by adopting a contractionary monetary stance for a long time period, during which capital inflows into the country translated into a steep appreciation of the D-mark in nominal and real terms. In contrast, in the second half of the 1990s, when the global “Saving Glut” envisioned by Bernanke [2005] started to to cause the dollar exchange rate to appreciate and created a significant deterioration of the US current account, the US monetary authorities kept their monetary stance persistently accommodative. The issue is again coming into consideration at the time of writing, since central banks on the recovery path from the Great Recession are lifting rates from the zero lower bound. In this process, the United States is experiencing increasing capital inflows, while the euro area is experiencing sizeable current account surpluses. How should these different external developments be factored into an efficient stabilization policy?

The question we address in this paper concerns what determines the optimal monetary trade-off between internal objectives (inflation, and output gap) and external objectives (competitiveness and trade imbalances) in the face of inefficient capital flows that cause exchange rate misalignment and distort current account positions. We provide an answer using the workhorse open economy monetary model with incomplete markets—the two-country New Keynesian model in which the only internationally traded asset is an non-contingent bond (as in the seminal contribution by Obstfeld and Rogoff [1995]; see also Costinot et al. [2015] and Davila and Korinek [2017]). Our point of departure is the conventional wisdom that net inflows of capital raise the natural rate of interest, hence should be matched by a tighter monetary stance—synthesized by Obstfeld and Rogoff [2010] in the quote opening this paper. In the presence of financial market imperfections, however, the natural rate allocation is not necessarily a desirable compass for stabilization policy. Because shocks are not fully insurable in a bond economy, pecuniary externalities make the real exchange rate misaligned, independently of nominal rigidities – the same core financial market distortion analyzed in Costinot et al. [2015]. As a result, the valuation of current and future national outputs is distorted, and so are the incentives to borrow and lend across borders. Similarly to their valuation effects on outstanding foreign assets and liabilities

1 See Eichengreen and Wyplosz [1993] and Buiter et al. [1998]. Systematic evidence on the monetary response to capital inflows is scarce. Kruger and Pasricha [2016] and Pasricha [2017] document that a neutral or an expansionary stance is actually as frequent as a contractionary one. An early study stressing procyclicality in monetary policy is Kaminsky, Reinhart and Vegh [2004].
stressed by the literature (see, e.g., Gourinchas and Rey [2014]), exchange rate movements drive
differences in national wealth by affecting the present discounted value of a country’s output
(namely, the natural borrowing constraint in a bond economy). With financial market frictions,
real exchange rate misalignments thus induce an inefficient wealth wedge across countries.

Our main result consists of showing analytically that the optimal monetary response to
inefficient capital inflows is not necessarily contractionary, and does not follow the natural rate.
Instead, it depends on structural features that are inherent to open economies: the degree of
exchange rate pass-through and the elasticity of output to the real exchange rate. The role
played by these features in shaping the optimal monetary stance squares with basic economic
intuition. In response to excessive capital inflows that over-appreciate the Home currency, it is
optimal to tighten the Home monetary stance when exchange rate misalignment has little impact
on competitiveness and growth (owing to low exchange rate pass-through). The optimal stance,
however, is more contractionary than required by the natural rate compass, up to causing
a fall in consumer price index (CPI) inflation. Conversely, it is optimal to pursue a Home
monetary expansion that mitigates the real exchange rate appreciation, despite its inflationary
effect on aggregate demand, when competitiveness and growth are sensitive to exchange rate
misalignment (due to a high exchange rate pass-through, and/or a low trade elasticity). The
direction of adjustment in this case lends theoretical support to views that are critical of the
natural rate prescriptions (see, e.g., Eichengreen [2011]).

Our analysis has significant implications for exchange rate volatility and external imbalances.
In our characterization of the optimal policy, low pass-through strengthens the welfare incentives
to stabilize aggregate demand, even if this means exacerbating currency movements. Our results
suggest that, with incomplete markets, optimal stabilization will tend to raise exchange rate
volatility in economies where import and export prices do not fully adjust to exchange rate
movements, relative to economies characterized by a high degree of exchange rate pass-through.
We also show that, by leaning against appreciation, an expansionary stance discourages capital
inflows—with the notable exception of economies with a trade elasticity well below unity, where
the optimal policy boosts demand and, irrespective of the degree of pass-through, domestic
borrowing.

In deriving these results, our paper makes at least two novel contributions to the literature.
First, it provides a second-order accurate approximation of the global welfare function and op-
timal targeting rules under cooperation for the standard New Keynesian two-country model
with incomplete markets. The welfare function encompasses different models of exchange rate
pass-through (ERPT)—with export prices being sticky either in the currency of the producers
(producer currency pricing or PCP, whereas ERPT is complete) or in the currency of the desti-
nation market (local currency pricing or LCP, whereas ERPT is incomplete).\(^2\) The derivation
of this function does not rely on specific forms of market incompleteness (e.g., bond economies
and financial autarky obtain as special cases), nor on restrictive assumptions about preferences
(e.g., it is not restricted to the case of unitary trade elasticity). We show that in addition to
output gaps and inflation rates, the arguments of the welfare function include real exchange
rate misalignment and relative demand misallocation, themselves a function of inefficient capital

\(^2\)We focus here on the the two symmetric cases of ERPT, leaving the analysis of the asymmetric case, the
dominant currency pricing (DCP) recently emphasized by Gopinath [2016], to future work.
flows. Combined, these arguments define a “wealth gap” that turns out to play a key role in optimal policy design. Based on our general loss function, we characterize and discuss optimal targeting rules under both PCP and LCP for economies that trade only non-contingent bonds across borders. These rules hold for a wide range of shocks (including preferences, productivity, markups, etc., current or anticipated), but, unlike the welfare function, are specific to bond economies.\(^3\)

Second, the paper derives a transparent analytical characterization of macroeconomic dynamics under the optimal monetary policy in economies in which inefficient capital flows are due to anticipated or “news shocks.” The news shocks may stem from political risk (i.e., anticipation of capital controls; see, e.g., Acharya and Bengui [2015]), the strength of financial frictions (see, e.g., Gabaix and Maggiori [2015] and Cavallino [2016]), technology or preferences impinging on savings—without loss of generality, we focus on the latter. In the first-best (complete market and flex-price) allocation, even though households are forward looking, relative prices and quantities depend only on the current-period (exogenous) fundamentals, not on their expected realizations in the future—in line with the well-known results in Barro and King [1984].\(^4\) Relative to this benchmark, under incomplete markets, the entire cross-border flow of capital that responds to news shocks is inefficient. Remarkably, we show that in model specifications that are standard in the literature, capital flows in response to news shocks are exogenous to monetary policy and macroeconomic adjustment. We can thus bring our study to bear directly on a case often debated in policy circles, where monetary policy can only mitigate the effects of inefficient capital flows on domestic macroeconomic dynamics, but cannot curb their size.\(^5\)

Two specific results are worth stressing. First, from the vantage point of monetary policymaking, inefficient capital inflows open gaps that act much like endogenous “markup” shocks—they raise trade-offs between inflation and the output gap. However, while the exogenous markup shocks typically assumed in the monetary literature create aggregate global distortions, we show that the inefficiencies from capital inflows have opposing effects on different economies, that cancel out in the aggregate. A key implication is that, under the optimal policy, the Home and Foreign monetary stance will be symmetric but with the opposite sign.\(^6\) Second, our analysis identifies structural features (i.e., shocks and ranges of trade elasticities) such that capital inflows may actually depreciate the domestic currency and depress domestic demand. In these cases, the optimal monetary response is unambiguously expansionary, to sustain domestic activity, in both LCP and PCP economies. We show that the theoretical core of this result rests on the classical controversy on the transfer problem—originally debated by Keynes and Ohlin concerning the war reparation imposed on Germany after World War I, and more recently reconsidered in the debate on current account rebalancing (see, e.g., Obstfeld and Rogoff [2005]).

To characterize the optimal policy as transparently as possible, we initially focus on a baseline specification that we dub the Cole and Obstfeld (CO) economy, following Cole and

\(^3\)For an analysis of optimal policy under financial autarky and PCP, see Corsetti Dedola and Leduc [2010].

\(^4\)Recall that in the workhorse monetary model we use in our analysis, preferences are time separable and there is no capital accumulation. See Devereux and Engel [2006, 2009] for an analysis of the optimal monetary response to news shocks under complete markets.

\(^5\)In addition, a specification after Cole and Obstfeld [1991] enhances comparability with seminal contributions to the literature (e.g., Clarida et al. [2002]) that impose similar restrictions on parameters.

\(^6\)This is in contrast with the optimal response to the exogenous markup shocks commonly assumed by the monetary literature, which may be symmetric across borders, in particular under LCP (see e.g. Corsetti et al. [2010] page 902-904).
Obstfeld [1991]. This baseline specification sets a unitary trade elasticity, complemented by the assumption of log consumption utility and a linear disutility of labor (the latter is relevant for tractability in the LCP case, as importantly shown by Engel [2011]). We then generalize our results to the case of non-unitary trade elasticity, showing that the analytical characterization of the optimal monetary stance in the CO economies still provides tight guidance for policy analysis for sufficiently large trade elasticities.

**Literature** Our analysis builds on a vast body of work that, over the last two decades, has redefined open economy macroeconomics (see Benigno and Benigno [2003]; Clarida, Gali and Gertler [2002]; Corsetti and Pesenti [2005]; Devereux and Engel [2003]; Engel [2011]; Ferrero, Gertler, and Svensson [2008]; and Gali and Monacelli [2005], among others, as discussed in Corsetti, Dedola, and Leduc [2010]). It is nonetheless useful to emphasize two strands of this literature that help highlight our contribution.

The first is the literature epitomized by Engel [2011], who studies optimal policy under deviations from the law of one price via LCP into the otherwise canonical open economy New Keynesian model developed by Clarida, Gali and Gertler [2002]. In this literature, risk sharing is perfect so that inefficient capital flows, demand misallocation and real exchange rate misalignment are the products of nominal rigidities only. A key result stressed by Engel [2011] for LCP economies is that monetary policy can support a constrained-optimal allocation with CPI-price stability and no exchange rate misalignment—under complete markets, this also closes any cross-country demand gaps (as defined in Section 3.1 below). Our paper complements this literature by stressing that the simplest form of financial market imperfections, incomplete financial markets, rules out this possibility. With incomplete risk sharing, there is always a trade-off between price stability and misalignment, as real exchange rate misalignment and cross-country demand gaps are no longer proportional to each other. This is so independently of whether ERPT is complete (PCP) or incomplete (LCP). As shown in our analysis, however, ERPT is crucial in determining the optimal monetary stance and the extent to which exchange rate misalignment is stabilized.

The second strand of the literature includes a small number of contributions that, like ours, provide analytical characterizations of the optimal monetary policy in two-country models with incomplete financial markets. Obstfeld and Rogoff [2002] and Devereux [2004] examine static frameworks without capital flows, and in which prices are set one period in advance—therefore, necessarily abstracting from the welfare implications of current account dynamics and inflation. Devereux and Sutherland [2007] study a dynamic setting similar to ours, but in which markets are effectively complete under flexible prices so that price stability also attains the efficient natural rate allocation. Under PCP, Benigno [2009] emphasizes deviations from price stability,
in economies in which net foreign asset holdings are asymmetrical in the nonstochastic steady state. However, the focus is on economies in which deviations from both purchasing power parity (PPP) and the law of one price are assumed away, in contrast with the analysis of real exchange rate misalignment at the core of optimal policy design analyzed in our paper.

Monetary policy with incomplete financial markets is the focus of recent numerical analyses by Rabitsch [2012], who revisits the benefits from international cooperation, and Senay and Sutherland [2016], who study the properties of optimal rules in a incomplete markets model with bonds and equities. Unlike our analysis, in their setting monetary policy also operates by manipulating the risk-sharing properties of assets—essentially, via their impact on the exchange rate (and prices)—monetary policy moves the ex post return on assets contingent on shocks. In our analysis, however, we focus on the more conventional channel of monetary transmission, operating exclusively by affecting current and anticipated real rates.\(^\text{12}\)

Our study is naturally related to recent literature that emphasizes the role of pecuniary externalities under collateral constraints, financial accelerator (balance-sheet) effects and over-and underborrowing relative to the [constrained-) efficient allocation [see Benigno et al. [2010]; Bianchi [2011]; Bianchi and Mendoza [2010]; Costinot et al. [2015]; Davila and Korinek [2017]; Fahri and Werning [2015]; and Lorenzoni [2008], among others].\(^\text{13}\) Devereux and Yu [2016] characterize optimal monetary policy under discretion in a small open economy with occasionally binding borrowing constraints. Relative to these papers, our contribution considers a standard framework with natural borrowing constraints on short-term debt, which, in equilibrium, respond to both exogenous shocks and the endogenous real appreciation created by inefficient capital inflows—indeed an appreciation relaxes the natural borrowing constraint, by raising the international value of (present discounted) domestic output. A distinct feature is our specific focus on monetary policy in a global equilibrium characterized by overborrowing (and obviously underborrowing in the other country) with respect to both the first-best and the constrained-efficient allocation.

Last, but not least, our results are in line with Woodford [2009], showing that financial integration does not compromise monetary control, i.e., the ability of the central bank to pursue a desired monetary stance. Yet, as stressed by Rey [2013] and Fahri and Werning [2015], inefficient capital flows may create adverse trade-offs across policy goals, hampering a central bank’s ability to maintain the economy on an efficient path. Our main contribution is to inspect the monetary policy trade-offs created by capital flows, and characterize the optimal monetary response that can provide a fungible first-line defence in the absence of other readily implementable measures, or complement other policy instruments—ranging from macroprudential policy to capital controls—when these are in place.

The rest of the paper is organized as follows. The next section synthetically goes over the standard two-good, two-country, New Keynesian model that we take as the framework for our analysis. Section 3 derives the global loss function, discussing each of its arguments in some

\(^{12}\)A number of other papers numerically solve open economy models under incomplete markets, and examine optimal policy often using ad hoc loss functions. See, for example, Kollmann [2003] and Bergin and Tchakarov [2003].

\(^{13}\)Cavallino [2016] examines foreign exchange interventions as a second instrument (in addition to conventional interest rate policy) available to the central bank to redress inefficient capital flows in an economy with borrowing constraints similar to those of Gabaix and Maggiori [2015].
detail, and characterizes the cooperative optimal targeting rules under PCP and LCP. Section 4 studies optimal monetary stabilization policy under PCP and LCP in CO economies whereas capital flows are independent of policy. In section 5, we generalize our results for the case of non unitary trade elasticity. Section 6 concludes.

2 The model economy

The analysis builds on the standard open economy version of the workhorse model in monetary economics (see, e.g., Clarida, Galí and Gertler [2002] and Engel [2011]), with well-known characteristics. The world economy consists of two countries of equal size, $H$ and $F$. Each country specializes in one type of tradable good, produced in a number of varieties or brands defined over a continuum of unit mass. Brands of tradable goods are indexed by $h \in [0, 1]$ in the Home country and $f \in [0, 1]$ in the Foreign country. Firms producing the goods are monopolistic supplier of one brand only and use labor as the only input to production. These firms set prices either in local or producer currency units and in a staggered fashion as in Calvo [1983]. Asset markets are complete at the national level, but incomplete internationally.

In what follows, we describe our set-up focusing on the Home country, with the understanding that similar expressions also characterize the Foreign economy—variables referring to Foreign firms and households are marked with an asterisk.

2.1 The household’s problem

2.1.1 Preferences

We consider a cashless economy in which the representative Home agent maximizes the expected value of her lifetime utility, where instantaneous utility $U$ is a function of a consumption index, $C$, and (negatively) of labor effort $L$, specialized as follows:

$$U[C_t, L_t] = \zeta_{C,t} \frac{C_t^{1-\sigma}}{1-\sigma} - \kappa \frac{L_t^{1+\eta}}{1+\eta}, \quad \sigma, \eta > 0 \quad (1)$$

whereas the model also allows for shocks to marginal utilities of consumption $\zeta_{C,t}$. Foreign agents’ preferences are symmetrically defined. Households consume both domestically produced and imported goods. We define $C_t(h)$ as the Home agent’s consumption as of time $t$ of the Home good $h$; similarly, $C_t(f)$ is the Home agent’s consumption of the imported good $f$. We assume that each good $h$ (or $f$) is an an imperfect substitute for all other goods’ varieties, with constant elasticity of substitution $\theta > 1$:

$$C_{H,t} \equiv \left[ \int_0^1 C_t(h) \frac{\theta-1}{\phi} dh \right]^{\frac{\phi}{\theta-1}}, \quad C_{F,t} \equiv \left[ \int_0^1 C_t(f) \frac{\theta-1}{\phi} df \right]^{\frac{\phi}{\theta-1}}. \quad (2)$$

The full consumption basket, $C_t$, in each country, aggregates Home and Foreign goods according to the following standard CES function:

$$C_t \equiv a_{H}^{1/\phi} C_{H,t}^{\phi-1/\phi} + a_{F}^{1/\phi} C_{F,t}^{\phi-1/\phi} \frac{\phi}{\phi-1}, \quad \phi > 0, \quad (3)$$
where \( a_H \) and \( a_F \) are the weights on the consumption of Home and Foreign traded goods, respectively, and \( \phi \) is the constant (trade) elasticity of substitution between \( C_{H,t} \) and \( C_{F,t} \).

### 2.1.2 Price indexes

The price index of the Home goods is given by:

\[
P_{H,t} = \left[ \int_0^1 P_t(h)^{1-\theta} dh \right]^{1\over 1-\theta},
\]

and the price index associated with the consumption basket, \( C_t \), is:

\[
\mathbb{P}_t = \left[ a_H P_{H,t}^{1-\phi} + a_F P_{F,t}^{1-\phi} \right]^{1\over 1-\phi}.
\]

Let \( E_t \) denote the Home nominal exchange rate, expressed in units of Home currency per unit of Foreign currency. The real exchange rate (RER) is customarily defined as the ratio of CPIs expressed in the same currency, i.e., \( Q_t = E_t^H P^* \). The terms of trade (TOT) are instead defined as the relative price of domestic imports in terms of exports: \( T_t = P_F E_t P_H^* \) if firms set prices in local currency and \( E_t P_F^* \) under producer currency pricing.

### 2.1.3 Budget constraints

Home and Foreign agents trade an international bond, \( B_H \), which pays in units of Home currency and is zero in net supply. Households derive income from working, \( w_t L_t \), from domestic firms’ profits, \( \Pi(h) \), lump-sum transfers \( T_t \), and from interest payments, \( (1 + i_t)B_{H,t} \), where \( i_t \) is the nominal bond’s yield, paid at the beginning of period \( t \) but known at time \( t - 1 \). Households use their disposable income to consume and invest in state-contingent assets. The individual flow budget constraint for the representative agent \( j \) in the Home country is therefore:

\[
P_{H,t} C_{H,t} + P_{F,t} C_{F,t} + B_{H,t+1} \leq w_t L_t + (1 + i_{t-1})B_{H,t} + \int_0^1 \Pi(h)dh + T_t.
\]

The household’s problem thus consists of maximizing lifetime utility, defined by (1), subject to the constraint (6).

### 2.2 Firms

Firms employ domestic labor to produce a differentiated product \( h \) according to the following linear production function:

\[
Y(h) = \zeta_Y L(h),
\]

where \( L(h) \) is the demand for labor by the producer of the good \( h \) and \( \zeta_Y \) is a technology shock common to all producers in the Home country, which follows a statistical process to be specified below.

Firms are subject to nominal rigidities à la Calvo so that, at any time \( t \), they keep their price fixed with probability \( \alpha \). We assume that when firms update their prices, they do so
simultaneously in the Home and Foreign markets. Following the literature, we consider two models of nominal price distortions in the export markets. According to the first model, firms can set prices in local currencies — this is the LCP hypothesis. The maximization problem is then as follows:

$$
\max_{P_t(h), P_t^*}(h) \quad E_t \left\{ \sum_{k=0}^{\infty} p_{bt,t+k} \alpha^k \left( \left[ P_t(h) D_{t+k}(h) + E_t P_t^*(h) D_{t+k}^*(h) \right] - MC_{t+k}(h) \left[ D_{t+k}(h) + D_{t+k}^*(h) \right] \right) \right\}
$$

(8)

where $p_{bt,t+k}$ is the firm’s stochastic nominal discount factor between $t$ and $t+k$, and the firm’s demand at Home and abroad is given by:

$$
D_t(h) = \int \left( \frac{P_t(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t} dh
$$

$$
D_t^*(h) = \int \left( \frac{P_t^*(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^* dh
$$

In these expressions, $P_{H,t}$ and $P_{H,t}^*$ denote the price index of Home goods in the Home and Foreign countries — the latter expressed in Foreign currency.

By the first-order condition of the producer’s problem, the optimal price $P_t(h)$ in domestic currency charged to domestic customers is:

$$
P_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D_{t+k}(h) MC_{t+k}(h)}{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D_{t+k}(h)} ;
$$

(9)

while the price (in foreign currency) charged to customers in the Foreign country is:

$$
P_t^*(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D_{t+k}^*(h) MC_{t+k}(h)}{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D_{t+k}^*(h)} .
$$

(10)

According to the alternative model, we posit that firms set prices in the producer currency — this is the PCP hypothesis. In this case, exchange rate pass-through is complete. Given that demand elasticities are the same across markets, in domestic currency the price charged to foreign consumers is the same as the optimal price charged at Home: the law of one price holds: $P_t^*(h) = P_t(h)/E_t$. The optimal price is similar to (9), whereas Home demand is replaced by global demand.

Since all the producers that can choose their price set it to the same value, we obtain the following equations for $P_{H,t}$ and $P_{H,t}^*$

$$
P_{H,t}^{1-\theta} = \alpha P_{H,t-1}^{1-\theta} + (1 - \alpha) P_t(h)^{1-\theta} ,
$$

(11)

$$
P_{H,t}^{1-\theta} = \alpha P_{H,t-1}^{1-\theta} + (1 - \alpha) P_t^*(h)^{1-\theta} .
$$

Similar relations hold for the Foreign firms.
2.3 Asset markets and exchange rate determination

In specifying the asset market structure, we restrict trade to one financial instrument only, a safe nominal bond. While capturing the notion that international financial markets do not provide efficient risk insurance against all shocks, intertemporal trade still implies forward-looking exchange rate determination, as a byproduct of equilibrium in financial markets. Namely, by combining the Euler equations for the Home households

\[
\frac{U_C(C_t, \zeta_{C,t})}{P_t} = (1 + i_t) E_t \left[ \frac{\beta U_C(C_{t+1}, \zeta_{C,t+1})}{P_{t+1}} \right]
\]

and the Foreign households:

\[
\frac{U_C(C^*_t, \zeta^*_{C,t})}{P^*_t} = (1 + i^*_t) E_t \left[ \frac{\beta U_C(C^*_{t+1}, \zeta^*_{C,t+1})}{P^*_{t+1}} \right],
\]

efficient trade in the international bond will imply the following uncovered interest parity condition, which equates the nominal stochastic discount rates in expectations:

\[
E_t \left[ \beta \frac{U_C(C_{t+1}, \zeta_{C,t+1})}{U_C(C_t, \zeta_{C,t})} \frac{P_t}{P_{t+1}} \right] = E_t \left[ \beta \frac{U_C(C^*_{t+1}, \zeta^*_{C,t+1})}{U_C(C^*_t, \zeta^*_{C,t})} \frac{P^*_t}{P^*_{t+1}} \right].
\]

Solved forward, this equation pins down the equilibrium exchange rate.

Under complete markets, the condition (12) holds state-by-state, rather than in expectations, since agents trade in contingent assets up to the point when, at the margin, the valuation of an extra unit of currency is equalized across borders. When countries are symmetric, this implies that the relative utility value of wealth, denoted by \(W_t\),

\[
W_t = \frac{U_C(C_t, \zeta_{C,t})}{U_C(C_t, \zeta_{C,t})} \frac{1}{Q_t} = \frac{U_C(C^*_t, \zeta^*_{C,t})}{U_C(C^*_t, \zeta^*_{C,t})} \frac{1}{Q_t}
\]

is identically equal to one (see, e.g., Gravelle and Rees [1992], Backus and Smith [1993] and Obstfeld and Rogo [2001]). Note that the marginal utility of consumption across borders is adjusted for the respective prices of the consumption basket.

Under incomplete markets, however, the equilibrium condition (12) only holds in expectations: any shocks will induce a wedge in the (ex post) relative value of wealth across borders, so that in general \(W_t \neq 1\). As shown below, \(W_t\) defines a theoretically grounded and efficient measure to account for asset markets imperfections in the policy problem—in line with the approach by Woodford [2010], who studies monetary trade-offs under financial frictions in a closed economy setting.

2.4 Log-linearized equilibrium

Throughout the paper, the model’s equilibrium conditions and constraints will be written out in log-deviations from the non-stochastic steady state—we will assume a symmetric steady-state
in which the net foreign asset position is zero and the markup distortion is eliminated, with appropriate subsidies. Details on the log-linearized model equations are given in appendix.

Notation-wise: denoting with upper-bar steady-state values, \( \hat{x}_t = \ln \frac{x_t}{\overline{x}} \) will represent deviations under sticky prices. In general, we will not denote the same variable differently across alternative specifications of the model—PCP vs. LCP, unitary trade elasticity vs. generic trade elasticity—as each model will be discussed in a distinct section or subsection. We make two exceptions to this rule. First, we will use the superscript \( f_b \) to denote variables in the unique “first-best” allocation, corresponding to the case of complete asset markets, flexible prices and no markup distortions. Second, in Sections 4 and 5, we will use the superscript \( n_a \) to denote variables in the “natural” (flex-price) allocation when the trade elasticity is set to one or left unconstrained.

Before proceeding, it is useful to single out two properties of the log-linearized equilibrium in our bond economy. First, the uncovered interest parity condition (12) implies

\[
E_t \hat{W}_{t+1} = \hat{W}_t.
\]

Because of incomplete risk sharing, shocks will generally result in a unit root in the relative value of wealth across borders—corresponding to a unit root in net foreign assets. While we will carry out our analysis using a specification of the model in which \( \hat{W}_t \) (and net foreign wealth) is not stationary, in the appendix we show that nonstationarity does not play any substantive role in our result. Second, under a symmetric steady state with zero net foreign wealth, up to first order, net foreign assets (and thus \( \hat{W}_t \)) do not respond to the ex post returns on internationally traded assets. In other words, real net foreign assets are capitalized at the steady-state real interest rate \( \beta^{-1} \). This feature has important implications for optimal monetary policy; namely, starting from a symmetric steady state with zero net foreign wealth, monetary policy cannot correct misallocations in demand and misalignment by manipulating the ex post return on existing assets to affect the wealth distribution (as in, e.g., Devereux and Sutherland [2008] and Benigno [2009]). Instead, it will operate via relative prices, output allocation and net foreign assets accumulation.

3 Monetary policy trade-offs in open economies with incomplete markets

Our main objective is to examine the monetary policy trade-offs brought about by inefficient capital flows in economies where asset markets are incomplete. In this section, we first define and discuss the welfare-relevant gaps shaping policy trade-offs in open economies. We then derive a general quadratic policy loss function obtained from a second-order approximation of agents’ utility for generic incomplete markets (i.e., without specifying the form of market incompleteness). Finally, we characterize the optimal cooperative policy under commitment, in terms of optimal targeting rules. To complete our analysis of monetary policy under incomplete markets, in an appendix we also reconsider how imperfect risk sharing affects the monetary transmission to macroeconomic variables.
3.1 Welfare-relevant gaps in an open economy

As is customary in monetary stabilization analysis, we will write policy objectives and targeting rules in terms of welfare-relevant gaps (all denoted with a tilde), expressing relevant variables as deviations from their first-best allocation values.

3.1.1 The first-best allocation benchmark

Under the assumption that real net foreign assets are zero in steady state \((B = 0)\), the first-best allocation can be characterized as follows. The first-best output in the Home and Foreign country, \(\tilde{Y}^{fb}_{H,t}\) and \(\tilde{Y}^{fb}_{F,t}\), are, respectively:

\[
\tilde{Y}^{fb}_{H,t} = \frac{2a_H(1-a_H)(\sigma \phi - 1)\left(\tilde{C}_{C,t} - \tilde{C}_{C,t}^*\right) + \tilde{C}_{C,t}^* + (1+\eta)\tilde{C}_{Y,t}}{\eta + \sigma},
\]

\[
\tilde{Y}^{fb}_{F,t} = \frac{2a_H(1-a_H)(\sigma \phi - 1)\left(-\tilde{Y}^{fb}_{H,t}\right) + (1-a_H)\left(\tilde{C}_{C,t} - \tilde{C}_{C,t}^*\right) + \tilde{C}_{C,t}^* + (1+\eta)\tilde{C}_{Y,t}}{\eta + \sigma}.
\]

The terms of trade and the real exchange rate are:

\[
\tilde{T}^{fb}_{t} = \frac{\sigma \left(\tilde{Y}^{fb}_{H,t} - \tilde{Y}^{fb}_{F,t}\right) - (2a_H - 1)\left(\tilde{C}_{C,t} - \tilde{C}_{C,t}^*\right)}{4(1-a_H)\alpha_H \left(\sigma \phi - 1\right) + 1},
\]

\[
\tilde{Q}^{fb}_{t} = (2a_H - 1)\tilde{T}^{fb}_{t} = \sigma \left(\tilde{C}_{t}^{fb} - \tilde{C}_{t}^{*fb}\right).
\]

The cross-border financial flows, characterized up to first order, are:

\[
\tilde{B}^{fb}_{t} - \beta^{-1}\tilde{B}^{fb}_{t-1} = (1-a_H)\sigma^{-1}\left[(2a_H(\sigma \phi - 1) + 1 - \sigma)\tilde{T}^{fb}_{t} - \left(\tilde{C}_{C,t} - \tilde{C}_{C,t}^*\right)\right]
\]

where, with slight abuse of notation, \(\tilde{B}^{fb}_{t}\) refers to “notional” real net foreign assets in the first best, and real net foreign assets, \(B_t = \frac{B_{H,t+1}}{P_t}\), are scaled with steady-state output, so that \(\tilde{B}^{fb}_{t} \simeq \frac{B^{fb}_{t} - B}{Y^{fb}}\).

For the purpose of our analysis, the key property of the first-best allocation is that financial and trade flows, as well as relative prices, only respond to shocks affecting contemporaneous (not future, anticipated) productivity and preferences.\(^{14}\) A notable implication is that neither the short-term real interest rate (given by the growth rates in marginal utilities), nor the long-term interest rate (equal to current consumption) moves in response to anticipated shocks.

3.1.2 Misalignment: real exchange rate gaps

A recurrent theme in policy debates concerns the possibility that international relative prices are misaligned and cross-border borrowing/lending is too high or too low—corresponding to either excessive or insufficient demand in different countries. Drawing on previous work of ours (Corsetti et al. [2010]), we now define gaps to account for these policy concerns, using the same logic underlying the definition of the welfare-relevant output gap.

\[^{14}\text{This is so because the model economy abstracts from capital accumulation and other sources of sluggish adjustment, such as habits or adjustment costs. Introducing these features would change the results to follow mainly quantitatively.}\]
Exchange rates are misaligned when they deviate from the value they would take in the efficient allocation.\(^\text{15}\) Since there are different measures of international relative prices, there are different (complementary) measures of misalignment. For the relative price of consumption across countries, the welfare-relevant gap is \(\hat{RER}_t\):

\[
\hat{RER}_t = \hat{Q}_t = \hat{Q}_t - \hat{Q}_t^{fb}.
\]  

(18)

Analogously, for the relative price of tradables, the terms-of-trade gap is \(\hat{TOT}_t\):

\[
\hat{TOT}_t = \tilde{T}_t = \tilde{T}_t - \tilde{T}_t^{fb}.
\]  

(19)

Finally, misalignments can also occur when nominal rigidities in local currency translates into cross-border deviations from the law of one price (henceforth LOOP). In this case, identical goods are inefficiently traded at different prices at Home and abroad. These price differences define another dimension of misalignment, which, measured on average for the basket of Home goods, is:

\[
\Delta_{H,t} = (\bar{E}_t + \bar{P}_{H,t}^* - \bar{P}_{H,t})
\]  

(20)

where \(\Delta_{H,t}\) is equal to zero when the LOOP holds. Note that, to the extent that \(P_{H,t}^*\) and \(P_{H,t}\) are sticky, the law of one price is violated with any movement in the exchange rate. Specifically, domestic currency depreciation tends to increase the Home firms’ receipts in Home currency from selling goods abroad, relative to the Home market: Home currency depreciation raises \(\Delta_{H,t}\). Similar considerations apply to \(\Delta_{F,t}\).  

### 3.1.3 Demand misallocation and the wealth gap

Inefficient external positions could be captured by tracing capital flows in excess of the financial flows in an efficient allocation, i.e., \(\hat{B}_t - \hat{B}_t^{fb}\).\(^\text{16}\) However, there is a better, more direct measure of policy-relevant distortions associated with cross-border misallocations. This is the “relative demand gap,” denoted by \(\tilde{D}_t\), and defined as the cross-country difference in private (consumption) demand relative to the first best:

\[
\tilde{D}_t = \tilde{C}_t - \tilde{C}_t^*.
\]

One key advantage of \(\tilde{D}_t\) is that, combined with the real exchange rate gap, \(\hat{Q}_t\), it adds up to the “wealth” gap, \(\tilde{W}_t\), defined as follows:

\[
\tilde{W}_t = \sigma \tilde{D}_t - \hat{Q}_t,
\]  

(21)

---

\(^{15}\)We stress that, conceptually, the first-best exchange rate is not necessarily (and in general will not be) identical to the “equilibrium exchange rate,” traditionally studied by international and public institutions, as a guide to policy-making. “Equilibrium exchange rates” typically refer to some notion of long-term external balance, against which to assess short-run movements in currency values possibly reflecting nominal rigidities and all kinds of real and financial frictions. On the contrary, the efficient exchange rate is theoretically and conceptually defined, at any time horizon, in relation to a hypothetical economy in which all prices are flexible and markets are complete. In fact, our measure of misalignment (as the difference between current exchange rates and the efficient one) is constructed, in strict analogy to the notion of a welfare-relevant output gap, as the difference between current output and the efficient level of output, which does not coincide with the natural rate (i.e., the level of output with flexible prices).

\(^{16}\)It is worth stressing that this measure would be well defined also under financial autarky, whereas \(\hat{B}_t = 0\).
where \( \tilde{W}_t \) is equal to log-deviations in the relative value of wealth (13). If markets are complete, \( \tilde{W}_t = 0 \) always, even when the overall allocation is not efficient because of nominal rigidities or other distortions. If markets are incomplete, instead, \( \tilde{W}_t \) will generally not be zero, and can have either sign, with a straightforward interpretation. A positive gap \( \tilde{W}_t > 0 \) means that, given the relative price of consumption, the consumption of the Home (national representative) individual is inefficiently high vis-à-vis foreign consumption. While consumption smoothing is optimal from an individual-agent perspective in response to anticipated shocks, from a global welfare perspective relative Home wealth would be too high.\(^{17}\) Conversely, a negative gap suggests that relative Home demand is inefficiently low given the exchange rate, and/or, for a given \( \tilde{D}_t \), the shock causes inefficient real depreciation (relative to first best).

### 3.2 Why and how do incomplete markets affect monetary policy?

The wealth gap defined in the previous subsection fully captures the implications of imperfect financial markets for the policy trade-offs faced by policy-makers in the design of optimal stabilization rules. Under complete markets, \( \tilde{W}_t = \sigma \tilde{D}_t - \tilde{Q}_t = 0 \). The demand gap \( \tilde{D}_t \) and real exchange rate misalignment \( \tilde{Q}_t \) can each be different from zero—depending on the effect of nominal rigidities or other distortions (e.g., taxes or markup shocks). Yet, as a consequence of perfect risk sharing, they will always remain proportional to each other: closing \( \tilde{Q}_t \) will be tantamount to closing \( \tilde{D}_t \), and vice versa. Under incomplete markets, instead, since \( \tilde{W}_t \) will generally deviate from zero, \( \tilde{D}_t \) and \( \tilde{Q}_t \) are no longer proportional to each other. In general, the optimal monetary rule will not close any of these gaps completely, but will have to trade off minimizing these gaps with inflation and output gaps.

In some notable cases (which we analyze in detail in Section 4), capital flows and the corresponding wealth gap \( \tilde{W}_t \) will be exogenous to policy: this means that the monetary authorities will not be able to affect the combined inefficiencies arising from both the misallocation in demand and the real exchange rate misalignment. As shown in the appendix, under both LCP and PCP, when \( \tilde{W}_t \) is exogenous, monetary policy affects the demand gap and misalignment in the same direction. Namely, a Home monetary tightening (easing) narrows (widens) the former while making the real exchange more overvalued (undervalued). Yet, as we show in Section 4, monetary policy will still be able to determine in a constrained-efficient way how to spread the welfare costs of macroeconomic adjustment across the different gaps, including the two components of \( \tilde{W}_t \).

When instead monetary policy impacts capital flows and \( \tilde{W}_t \), the effect of a Home monetary tightening depends on structural features such as risk aversion \( \sigma \), the trade elasticity \( \phi \), the degree of openness and price rigidities. As shown in the appendix, under PCP, a contractionary monetary policy always increases capital inflows but narrows \( \tilde{W}_t \) for \( \phi > 1 + \frac{2\mu - 1}{2\sigma} \); the opposite occurs — i.e., capital inflows fall and \( \tilde{W}_t \) increases after a tightening — for \( \phi < 1 + \frac{2\mu - 1}{2\sigma} - 1 \). Under LCP, a contractionary monetary policy always increases capital inflows but narrows \( \tilde{W}_t \).

\(^{17}\)With incomplete markets, price movements are not efficient. An appreciation of the real exchange rate associated with a Home consumption boom is a leading example of a pecuniary externality. While fully rational from an individual perspective, agents’s decisions to borrow and lend move international relative prices inefficiently. These are no longer correct indicators of relative scarcity: consumption is higher where the price of the consumption bundle is also higher; see Geanakoplos and Polemarchakis [1986].
for \( \sigma > 1 \) and \( \phi > 1 \); the opposite occurs for \( \sigma < 1 \) and \( 0 \leq \phi < 1 - \frac{2a_H - 1 + \frac{\alpha}{2}}{2a_H \frac{1}{1 - \phi}} < 1 \).\(^{18}\) We analyze the optimal monetary policy response to capital flows in this case in Section 5.

The wealth gap \( \tilde{W}_t \) affects all gaps in the economy. It first enters relative price misalignments as follows

\[
\tilde{T}_t + \tilde{\Delta}_t = \frac{\sigma \left( Y_{H,t} - \tilde{Y}_{F,t} \right) - (2a_H - 1) \left( \tilde{W}_t + \tilde{\Delta}_t \right)}{4a_H \left( 1 - a_H \right) \left( \sigma \phi - 1 \right) + 1},
\]

\[
\tilde{Q}_t = (2a_H - 1) \left( \tilde{T}_t + \tilde{\Delta}_t \right) + \tilde{\Delta}_t
\]

where we used the fact that, under symmetry, \( \tilde{\Delta}_{H,t} = \tilde{\Delta}_{F,t} = \tilde{\Delta}_t \) (see Engel [2011]). Note that, while \( \tilde{Q}_t \) and \( \tilde{T}_t \) are a function of each other, they can move differently in response to shocks because of home bias in preferences and deviations from the law of one price. Taking the difference in budget constraints, we obtain:

\[
\sigma \tilde{D}_t = \sigma \left[ -2\beta^{-1} \left( \tilde{B}_t - \beta \tilde{B}_{t-1} \right) + \tilde{Y}_{H,t} - \tilde{Y}_{F,t} \right] - 2 \left( 1 - a_H \right) \sigma \tilde{T}_t + (1 - a_H) \left[ 4a_H \left( 1 - a_H \right) \left( \sigma \phi - 1 \right) - 1 \right] \sigma^{-1} \left[ \left( 2a_H \left( \sigma \phi - 1 \right) + 1 - \sigma \right) \tilde{T}_t + \tilde{\Delta}_t \right].
\]

Everything else equal, capital inflows (\( \tilde{B}_t < 0 \)) cause the demand gap to turn positive, \( \tilde{D}_t > 0 \).

The wealth gap affects inflation dynamics, directly and indirectly (via relative price misalignment). The Phillips Curves in our model (four under LCP, collapsing into two under PCP) are written below:

\[
\pi_{H,t} - \beta E_t \pi_{H,t+1} = \frac{(1 - \alpha \beta) (1 - \alpha)}{\alpha} \left[ \frac{\left( \sigma + \eta \right) Y_{H,t} + \tilde{\mu}_t^*}{(1 - a_H) \left[ 2a_H \left( \sigma \phi - 1 \right) \left( \tilde{T}_t + \tilde{\Delta}_t \right) - \tilde{\Delta}_t - \tilde{W}_t \right]} \right],
\]

\[
\pi_{F,t} - \beta E_t \pi_{F,t+1} = \frac{(1 - \alpha \beta) (1 - \alpha)}{\alpha} \left[ \frac{(\sigma + \eta) Y_{F,t} + \tilde{\mu}_t^*}{(1 - a_H) \left[ 2a_H \left( \sigma \phi - 1 \right) \left( \tilde{T}_t + \tilde{\Delta}_t \right) - \tilde{\Delta}_t - \tilde{W}_t \right]} \right],
\]

where \( \tilde{\mu}_t \) and \( \tilde{\mu}_t^* \) denote markup shocks. As emphasized by CDL [2010], these expressions make it apparent that the wealth gap, as well as price misalignments, are akin to inefficient (but exogenous) markup shocks. Indeed, when markets are incomplete, the distinction between “efficient” and “inefficient” shocks becomes less useful for the purpose of policy design. Also, shocks to tastes and technology endogenously open a wealth gap and create misalignments—and thus raise meaningful policy trade-offs between output and inflation under both LCP and PCP.

\(^{18}\)See Section 4 for a definition of \( \nu_1 \) and \( \nu_2 \), which are functions of the degree of price rigidities \( \alpha \).
3.3 A general (quadratic) global policy loss function

From the model, we derive a second-order approximation of the equally weighted sum of the utility of the Home and Foreign national representative agents—written in terms of the gaps defined above, all in quadratic forms. The policy loss functions include not only “internal” objectives (inflation and output gaps), but also “external” ones (relative price misalignments and the relative demand gap).

Specifically, under the assumption of appropriate subsidies offsetting firms’ markup to deliver an efficient, non-distorted steady state, the period-by-period quadratic welfare function for incomplete market economies is as follows:

\[ L_t^W - (L_t^W)^{1b} \propto \]

\[ -\frac{1}{2} \left\{ \frac{(\sigma + \eta) \left( \tilde{Y}_{H,t}^2 + \tilde{Y}_{F,t}^2 \right) + \frac{\alpha}{(1 - \alpha \beta) (1 - \alpha \theta)} \left( \pi_t^2 + \pi_t^*^2 \right)}{2a_H (1 - a_H) (\sigma \phi - 1) + 1} \left[ (\sigma \phi - 1) \sigma \left( \tilde{Y}_{H,t} - \tilde{Y}_{F,t} \right)^2 - \phi \left( \tilde{\Delta}_t + \tilde{W}_t \right)^2 \right] \right\} \]

\[ + t.i.p., \]

where for convenience we have substituted out terms-of-trade misalignments using their equilibrium relation with output gaps, deviations from the law of one price, and relative demand gaps. While this loss function is written for an LCP economy, its PCP counterpart can be readily obtained by setting the LOOP deviations to zero \((\tilde{\Delta}_t = 0)\), and using the fact that, under the law of one price, the inflation term \(\pi_t \equiv a_H \pi_{H,t}^2 + (1 - a_H) \pi_{F,t}^2\) and \(\pi_t^* \equiv a_H \pi_{H,t}^*^2 + (1 - a_H) \pi_{F,t}^*^2\) reduces to \(\pi_t \equiv \pi_{H,t}^2\) and \(\pi_t^* \equiv \pi_{F,t}^2\). It can be shown that expression (24) encompasses the cases of financial autarky (no asset is traded internationally), international trade in one bond, as well as international trade in any number of assets, including complete markets.\(^{19}\) In this sense, the above function generalizes and complements the ones derived in previous work of ours (CDL [2010]) for the case of autarky and complete markets.\(^{20}\)

3.4 Optimal targeting rules in bond economies

To characterize the optimal cooperative policy under commitment, we maximize the present discounted value of the sum of (24) over time, subject to the log-linearized equilibrium conditions and constraints characterizing the competitive equilibrium allocation in bond economies. In the interest of transparency and tractability, we adopt a timeless perspective (see, e.g., Woodford [2010]), and focus on the (widely studied) case of economies whereas non-contingent bonds are the only assets traded across borders. The derivation is in the appendix.

Following a standard practice in international economics, the optimal cooperative policy can be written in terms of two targeting rules: a global rule summing up inflation and output gaps across countries, and a cross-country rule, expressed in terms of differences in gaps across countries. The optimal policy is obtained by solving the optimization problem above subject to the log-linearized equilibrium conditions and subject to the log-linearized equilibrium conditions and constraints characterizing the competitive equilibrium allocation in incomplete market economies. In the interest of transparency and tractability, we adopt a timeless perspective (see, e.g., Woodford [2010]), and focus on the (widely studied) case of economies whereas non-contingent bonds are the only assets traded across borders. The derivation is in the appendix.

\(\text{footnote}{^{19}}\) With \(\tilde{W}_t = 0\), and \(\phi = 1\), our loss function is comparable to the corresponding expressions in Engel [2009], who carries out the analysis under the assumption of complete markets and focuses on the case of a unitary elasticity.

\(\text{footnote}{^{20}}\) Gaps (other than output gaps and inflation) similar to the ones we use in our analysis identify policy objectives arising from heterogeneity among sectors and agents in economies distorted by financial imperfections, in addition to nominal rigidities (see, e.g., Curdia and Woodford [2009] for an analysis in a closed economy.)
countries. From a global perspective, the optimal targeting rule is

\[ 0 = \left( \bar{Y}_{H,t} - \bar{Y}_{H,t-1} \right) + \left( \bar{Y}_{F,t} - \bar{Y}_{F,t-1} \right) + \theta \left[ a_H \pi_{H,t} + (1 - a_H) \pi_{F,t} + a_H \pi_{H,t}^* + (1 - a_H) \pi_{H,t}^* \right] ; \]

where in the case of a PCP economy the inflation term becomes \( \pi_{H,t} + \pi_{F,t}^* \) — as noted above under PCP, world CPI and PPI inflation rates coincide. From a global perspective, the optimal cooperative monetary policy stabilizes output gaps and inflation at the global level. To the extent that world inflation is zero (in the absence of exogenous markup shocks), the sum of output gaps and consumption deviations is also zero. An important implication is that the optimal monetary stance will have the opposite sign across countries. Another implication is that we can write:

\[ \tilde{W}_t \equiv 2\sigma \tilde{C}_t - \tilde{Q}_t. \]

These results also hold in the natural rate allocation.

In contrast, deriving cross-country or country-specific rules involves solving a system of difference equations in the different Lagrange multipliers from the optimal policy problem, making it possible to obtain tractable general expressions—comparable to the global rule—only under some parameter restrictions. We will analyze the LCP and PCP economies in turn.

### 3.4.1 Low pass-through (LCP) economies

In the LCP case, a tractable rule is derived by Engel [2011] under the assumptions that markets are complete and \( \eta = 0 \) (infinite labor elasticity). A first important result in our paper is that, as long as labor elasticity is infinite, it is possible to derive a tractable cross-country targeting rule also under incomplete markets. This is given by the following expression:

\[ 0 = \theta (\pi_t - \pi_t^*) + \tilde{D}_t - \tilde{D}_{t-1} \]

\[ + \frac{4a_H (1 - a_H) \phi}{2a_H (\phi - 1) + 1} \frac{\sigma - 1}{\sigma} \left( [\tilde{W}_t - \tilde{W}_{t-1}] + [\tilde{\Delta}_t - \tilde{\Delta}_{t-1}] \right). \]

Without the last term on the right-hand side, this expression defines the targeting rule under complete markets, where the cross-country targeting criterion involves only CPI inflation and consumption differentials. The last term, in the wealth gap and deviations from the law of one price, is specific to incomplete markets economies.

Under perfect risk sharing, Engel [2011] derives the important result that, as long as \( \eta = 0 \), the relative prices \( \tilde{T}_t + \tilde{\Delta}_t \) are exogenous with respect to monetary policy—for any value of \( \sigma \). In the appendix, we establish that the same result also holds under incomplete markets, if we restrict agents to have log-utility, i.e., \( \sigma = 1 \). Thus, we are able to show that, in LCP economies with \( \eta = 0 \) and \( \sigma = 1 \), monetary policy cannot affect \( \tilde{T}_t + \tilde{\Delta}_t \) — and since in this case cross-border capital flows are solely a function of \( \tilde{T}_t + \tilde{\Delta}_t \), they are independent of monetary policy for any value of the trade elasticity. This will have notable implications for the analysis in the sections to follow.

Observe that the last term on the right-hand side of the optimal rule (25) drops out when \( \sigma = 1 \): the expression for the cross-country rule (25) is the same under both complete and
incomplete markets. However, it does not follow that monetary policy is the same in the two cases. To illustrate the difference, we combine the above expression with the definition of \( \tilde{W}_t \) to rewrite the optimal (cooperative) policy in the form of a country-specific rule for the Home economy (and a symmetric one for Foreign country). Abstracting from markup shocks, this reads:

\[
0 = \theta\pi_t + 1/2 \cdot \left[ (\tilde{W}_t - \tilde{W}_{t-1}) + (\tilde{Q}_t - \tilde{Q}_{t-1}) \right]
= \theta\pi_t + (\tilde{C}_t - \tilde{C}_{t-1}).
\]

When markets are complete (\( \tilde{W}_t = 0 \)), the above reduces to the expression derived by Engel [2011]: with perfect risk insurance, provided that shocks are “efficient” (i.e., they affect tastes and/or technology only, while \( \tilde{\mu}_t = \tilde{\mu}_t^* = 0 \)), the optimal policy sets CPI inflation rates to zero. A zero inflation policy closes the consumption gap and eliminates real exchange rate misalignments at once—reflecting the fact that these gaps are proportional to (exogenous) relative prices \( \tilde{T}_t + \tilde{\Delta}_t \). This is not possible when markets are incomplete (\( \tilde{W}_t \neq 0 \)).

It may be worth stressing that under LCP closing the real exchange rate gap (i.e., setting \( \tilde{Q}_t = 0 \)) does not necessarily eliminate deviations from the law of one price—nor prevent inefficient deviations from the law of one price \( \tilde{\Delta}_t \) from mapping into output gap fluctuations. This is apparent from the following expression:

\[
\tilde{Q}_t = (2a_H - 1) \left( \tilde{T}_t + \tilde{\Delta}_t \right) + \tilde{\Delta}_t = (2a_H - 1) \frac{\sigma (\tilde{Y}_{F,t} - \tilde{Y}_{H,t}) - (2a_H - 1) (\tilde{W}_t + \tilde{\Delta}_t)}{4a_H (1-a_H) (\sigma\phi - 1) + 1} + \tilde{\Delta}_t.
\]

Because of nominal distortions in import and export pricing in local currency, the optimal policy allocation cannot be first best, whether or not risk sharing is perfect.

### 3.4.2 High pass-through (PCP) economies

The analytics of the cross-country targeting rule under PCP stands in sharp contrast to the LCP case above. No parameter restriction is required to derive a compact expression for the following cross-country targeting rule in a bond economy:

\[
0 = E_t \left( \tilde{Y}_{H,t+1} - \tilde{Y}_{H,t} \right) - E_t \left( \tilde{Y}_{F,t+1} - \tilde{Y}_{F,t} \right) + \theta \left( E_t \pi_{H,t+1} - E_t \pi_{F,t+1}^* \right). \tag{26}
\]

Notably, this bond-economy optimal rule is a “forward-looking version” of the cross-country targeting rule under complete markets (see Engel [2011] and CDL [2010]), shown hereafter as:

\[
0 = \left( \tilde{Y}_{H,t} - \tilde{Y}_{H,t-1} \right) - \left( \tilde{Y}_{F,t} - \tilde{Y}_{F,t-1} \right) + \theta \left( \pi_{H,t} - \pi_{F,t}^* \right). \tag{27}
\]


Combining once again the global and cross-country rules for bond economies, and abstracting
from markup shocks, we can write a country-specific (cooperative) rule for the Home economy:\footnote{\textsuperscript{21}Under financial autarky, the cross-country rule already derived in Corsetti et al. (2010) can be written as follows:}

\[ 0 = \left( \tilde{Y}_{H,t} - \tilde{Y}_{H,t-1} + \theta \pi_{H,t} \right) + \frac{2a_H (1-a_H) \phi}{\sigma + \eta (4a_H (1-a_H) (\sigma \phi - 1) + 1)} \frac{2a_H (\sigma \phi - 1) + 1 - \sigma}{2a_H (\phi - 1) + 1} \left( \tilde{W}_t - \tilde{W}_{t-1} \right). \]

Note that, if either markets are complete ($\tilde{W}_t = 0$) or $\sigma = \phi = 1$, the above expression becomes a function of purely domestic objectives:

\[ \tilde{Y}_{H,t} - \tilde{Y}_{H,t-1} + \theta \pi_{H,t} = 0. \] (28)

Each country would stabilize its own output gap and GDP-deflator inflation—a result that identifies an important case of “isomorphism” of optimal policy in closed and open economies.

To gain insight on how incomplete markets impinge on key policy trade-offs under PCP, we subtract the two Phillips Curves from each other using the equilibrium expression for the terms of trade:

\[ \pi_{H,t} - \pi_{F,t}^* = \beta \left( E_t \pi_{H,t+1} - E_t \pi_{F,t+1}^* \right) + \begin{cases} 
(\eta + \sigma) \left( \tilde{Y}_{H,t} - \tilde{Y}_{F,t} \right) + \tilde{\mu}_t - \tilde{\mu}_t^* \\
-2 (1 - a_H) \cdot \left[ 2a_H (\sigma \phi - 1) \left( \frac{\sigma}{4a_H (1-a_H) (\sigma \phi - 1) + 1} \frac{\tilde{Y}_{H,t} - \tilde{Y}_{F,t}}{\tilde{W}_t} - \tilde{W}_t \right) \right] 
\end{cases} \]

Under complete markets ($\tilde{W}_t = 0$), the terms-of-trade gap actually becomes strictly proportional to the differences in output gaps. It is then easy to verify that, consistent with the cross-country targeting rule (28), monetary authorities face no significant trade-off between inflation and output gap stabilization, except in the presence of exogenous markup shocks $\tilde{\mu}_t$ or $\tilde{\mu}_t^*$. In other words, as long as (current and anticipated) shocks affect both output and its first-best counterpart, the optimal policy consists of keeping national output gaps closed and inflation exactly equal to zero. Under complete markets and PCP, this is indeed the optimal response to ("efficient") shocks to productivity and preferences.

In the presence of financial imperfections, however, the terms-of-trade gap and output gaps are not proportional to each other. Any shock, including efficient shocks to tastes and technology, results in a wealth gap $\tilde{W}_t \neq 0$, forcing monetary authorities to trade off inflation and output gaps. This is true also when $\sigma = \phi = 1$, in which case the targeting rules are formally identical across complete and incomplete markets, yet domestic welfare-relevant output gaps do not behave identically.
4 Optimal trade-offs varying exchange rate pass-through

In this and the next section, we bring our analysis to bear on the optimal conduct of monetary policy in economies that experience inefficient capital flows and study the macroeconomic dynamics that result from the implementation of the optimal targeting rules. Throughout our analysis, we will specifically focus on shocks in the form of “news,” indicating anticipated changes in preference parameters. As emphasized by Devereux and Engel [2006, 2009], an important reason for analyzing “news shocks” is that they highlight the forward-looking nature of exchange rate determination. From our perspective, a key additional reason is that, as shown in subsection 3.1.1, in the first-best allocation the current values of macro variables do not respond at all to news foreshadowing changes in fundamentals in the future: the response of “gaps” (in anticipation of future changes in technology and preferences) thus coincides with the response in the equilibrium allocation until the anticipated shock materializes—with obvious gains in tractability and analytical transparency.

For expository clarity, in this section we focus our analysis on a bond economy with a unitary trade elasticity, log-consumption utility and linear disutility of labor—a specification we dub a “Cole and Obstfeld” or CO economy. In the next section, we generalize our results to the case of non-unitary trade elasticity. The key advantage of beginning our study by assuming a unitary elasticity is that, in this case, news shocks generate capital flows that are exogenous to policy and macroeconomic adjustments and are thus identical under both LCP and PCP. Thanks to this property, we can compare optimal monetary policy across different economic and policy environments, holding these flows (and thus the underlying shocks) constant.

4.1 A “Cole and Obstfeld” economy with capital flows exogenous to policy

As is well known since Cole and Obstfeld [1991] and subsequent work, in an environment with a Cobb-Douglas aggregator of domestic and imported goods (\( \phi = 1 \)), log consumption utility (\( \sigma = 1 \)) and symmetric home bias, productivity risk is efficiently shared via endogenous terms-of-trade movements, regardless of whether financial markets are complete or incomplete. However, full risk sharing is not granted in the presence of other sources of risk directly affecting net foreign assets, ranging from political risk (i.e., anticipation of capital controls; see, e.g., Acharya and Bengui [2015]), to financial shocks (see, e.g., Gabaix and Maggiori [2015] and Cavallino [2016]), and/or preference shocks impinging on savings. As many of these shocks have broadly similar analytical representations, there is little or no loss of generality in focusing on shocks to preferences that affect the intertemporal valuation of consumption, thus resulting in a motive to save and lend across borders, and generating cross-country capital flows.

Furthermore, throughout the analysis we will posit a linear disutility of labor, \( \eta = 0 \), as this last restriction is necessary for tractability in the LCP economies. We start by discussing the first-best and competitive allocations, then turn to the optimal policy.
4.1.1 Cross-border (notional) flows in the first-best allocation

As we have already shown, in the first-best allocation, no macro variable (but the long-term interest rate) responds to news shocks. With \( \sigma = \phi = 1 \), (17) simplifies to

\[
\hat{B}_t^{fb} - \beta^{-1}\hat{B}_{t-1}^{fb} = -(1 - a_H) \left( \hat{\zeta}_{C,t} - \hat{\zeta}^*_C,t \right).
\]

A surge of (efficient) financial inflows \( \hat{B}_t^{fb} < 0 \) can only be driven by contemporaneous relative preference shocks in the Home country \( \left( \hat{\zeta}_{C,t} - \hat{\zeta}^*_C,t > 0 \right) \). In Table 1, we write the corresponding allocation. Note that in this table we do not yet impose \( \eta = 0 \), as a way to gain insight into the implications of such assumption.

### Table 1. The first-best allocation in the CO economy

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<tr>
<td>( \hat{Y}^{fb}<em>{H,t} = \frac{1}{1 + \eta} \left[ a_H \hat{\zeta}</em>{C,t} + (1 - a_H) \hat{\zeta}^*_C,t \right] )</td>
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<tr>
<td>( \hat{Y}^{fb}<em>{F,t} = \frac{1}{1 + \eta} \left[ (1 - a_H) \hat{\zeta}</em>{C,t} + a_H \hat{\zeta}^*_C,t \right] )</td>
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<tr>
<td>( \hat{Q}^{fb}<em>t = (2a_H - 1)^2 \frac{\eta}{1 + \eta} (2a_H - 1) \left( \hat{\zeta}</em>{C,t} - \hat{\zeta}^*_C,t \right) )</td>
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<tr>
<td>( \hat{D}^{fb}<em>t = \left( \hat{\zeta}</em>{C,t} - \hat{\zeta}^*_C,t \right) - \hat{Q}^{fb}_t )</td>
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If \( \eta > 0 \), provided consumption preferences are biased towards Home goods \( 2a_H > 1 \), capital inflows from Home preference shocks in favor of current consumption would result in a Home currency real appreciation \( \hat{Q}^{fb}_t < 0 \). But with a linear disutility of labor, \( \eta = 0 \), the first-best real exchange rate remains constant \( \hat{Q}^{fb}_t = 0 \). The only effect of the shock is to raise output in both countries, in proportion to the consumption Home bias. Given that the exchange rate is unresponsive, the consumption differential rises efficiently, one-to-one with the contemporaneous relative preference shock: \( \hat{D}^{fb}_t = \left( \hat{\zeta}_{C,t} - \hat{\zeta}^*_C,t \right) \).

4.1.2 Financial flows in bond economies

In contrast to the first-best allocation, when the only traded assets are non-contingent bonds, financial flows respond not only to contemporaneous fundamentals, but also to expectations of future fundamentals:

\[
\hat{B}_t = \hat{B}_{t-1} + (1 - a_H) \beta \sum_{j=0}^{\infty} \beta^j E_t \left[ \left( \hat{\zeta}_{C,t+1+j} - \hat{\zeta}^*_{C,t+1+j} \right) - \left( \hat{\zeta}_{C,t+j} - \hat{\zeta}^*_{C,t+j} \right) \right].
\]

An anticipated future fall in the relative degree of impatience \( \left( \hat{\zeta}_{C,t+1+j} - \hat{\zeta}^*_{C,t+1+j} < 0 \right) \) causes capital to flow into the Home country—recall that a negative \( \hat{B}_t \) denotes inflows into the Home country. These flows are inefficient: while trade in bonds is welfare maximizing from an individual household’s perspective (at Home and abroad), the entire capital account deficit is excessive relative to the first-best allocation—since in response to anticipated shocks no (notional) capital would flow across borders on impact under perfect risk sharing and flexible prices (i.e., \( \hat{B}^{fb}_t = 0 \)). Note that the size of the inefficient inflows is increasing in openness (decreasing in home bias \( a_H \)).
Inefficient capital flows open a wealth gap:

\[
(1 - a_H) \tilde{W}_t = -\left( \hat{B}_t - \beta^{-1}\hat{B}_{t-1} \right) - (1 - a_H) \left( \tilde{\zeta}_{C,t} - \tilde{\zeta}_{C,t}^* \right).
\]  

(30)

Two key results can be derived from the expressions above. First, as in the CO economy, both \( \hat{B}_t \) and \( \tilde{W}_t \) are a function of the exogenous preference shocks only, and therefore independent of nominal rigidities and monetary policy regimes. Second, a capital inflow (\( \hat{B}_t < 0 \)) driven by news shocks will invariably lead to a positive wealth gap. From a global welfare perspective, as the Home economy accommodates a higher desire to save among Foreign residents, the relative Home demand \( \hat{D}_t \) grows excessive, and/or the real exchange rate becomes misaligned.

Observe nonetheless that the wealth gap is not necessarily positive in the presence of capital inflows. The above expression suggests that, even if \( b_{B,t} < 0 \), \( f_{W,t} \) can be negative in response to contemporaneous (as opposed to “news”) taste shocks, raising the utility of current Home consumption (and associated with a relative increase in efficient output, \( b_{Y,t} < 0 \)). In this case, although capital flows into the Home country, domestic consumption is inefficiently low relative to the foreign one. The implications for optimal monetary policy will be discussed in detail in the next section.

4.1.3 The natural rate allocation with news shocks

With imperfect insurance, inefficient capital flows result in misallocation independent of price stickiness. This is apparent from inspecting the flexible price or natural rate allocation in response to news shocks, shown in Table 2. In the table, all variables are expressed as deviations from the efficient allocations—defining gaps denoted with a superscript “na”—, and, since \( \hat{Q}_t^b = 0 \), \( \tilde{Q}_t^{na} = \hat{Q}_t \).

<table>
<thead>
<tr>
<th>Table 2. The natural rate allocation in the CO economy</th>
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<tbody>
<tr>
<td>( \hat{Y}_H,t^{na} = -\hat{Y}_F,t^{na} = - (1 - a_H) \tilde{W}_t )</td>
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<tr>
<td>( \hat{Q}_t^{na} = - (2a_H - 1) \tilde{W}_t )</td>
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<tr>
<td>( \hat{T}_t^{na} = -\tilde{W}_t )</td>
</tr>
<tr>
<td>( \hat{D}_t^{na} = 2 (1 - a_H) \tilde{W}_t )</td>
</tr>
<tr>
<td>( \hat{C}_t^{na} = -\hat{C}_t^{*na} = \frac{1}{2} \tilde{D}_t^{na} = (1 - a_H) \tilde{W}_t )</td>
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</table>

In the natural rate allocation of our CO economies, output gaps, exchange rate misalignment and relative demand gap are all proportional to the (exogenous) gap \( \tilde{W}_t \) in response to news shocks. When \( \tilde{W}_t > 0 \), capital inflows result in a negative welfare-relevant output gap, an overvalued real exchange rate and an excessive level of domestic consumption, both in absolute terms and relative to Foreigners. Since news shocks bring about purely redistributive inefficiencies, the Foreign economy just mirrors the Home responses.

In response to news shocks, all gaps jump on impact, reflecting the wealth gap opened by the news. Afterwards, since \( E_t \tilde{W}_{t+1} = \tilde{W}_t \), they remain constant until the change in the

\[22\text{By using (29), you can also write this expression as (1 - a_H) E_t \tilde{W}_{t+1} = - (1 - a_H) \left[ \beta \sum_{j=0}^{\infty} \beta^j E_t \left( \tilde{\zeta}_{C,t+j} - \tilde{\zeta}_{C,t+j}^* \right) \right] + \frac{1-\beta}{\beta} \hat{B}_{t-1}. \]
fundamentals materializes. Note that, in the intervening period between the news and future changes in fundamentals, the short-term natural rate of interest (equal to the growth rate of consumption under flexible prices) is not affected at all by the news shocks.

It can be shown that, in our CO economies, the natural allocation in Table 2 above not only coincides with PPI price stability under PCP (this is a well known result), but also with CPI price stability under LCP (but for relative prices \( \tilde{T}_t + \tilde{\Delta}_t \) and the output gap). This result will be quite useful in the policy analysis to follow.

4.2 Domestic demand stabilization with low pass-through (LCP economies)

For given inefficient capital flows (29) and the associated wealth gap (30)) in response to news shocks, Table 3 shows the constrained-efficient allocation under LCP, when monetary authorities implement the optimal targeting rules, rewritten below for convenience:

\[
\theta \pi_t + \frac{1}{2} (\tilde{Q}_t - \tilde{Q}_{t-1}) = -\frac{1}{2} (\tilde{W}_t - \tilde{W}_{t-1}),
\]

Table 3: Constrained-efficient allocation under LCP with news shocks

| \( \tilde{Y}_{H,t} \) | \( 2a_H (1 - a_H) (\tilde{T}_t + \tilde{\Delta}_t) + 1/2 \cdot (2a_H - 1) \tilde{D}_t \) |
| \( \pi_t \) | \( -\frac{1}{2} (1 - a_H) (\beta \nu_2 - 1) \tilde{W}_t - \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \tilde{W}_{t-1} + (1 - \nu_1) \tilde{Q}_{t-1} \) |
| \( \tilde{T}_t + \tilde{\Delta}_t \) | \( \nu_1 (\tilde{T}_{t-1} + \tilde{\Delta}_{t-1}) - \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \tilde{W}_t \) |
| \( \tilde{Q}_t \) | \( -(2a_H - 1) \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \tilde{W}_t - \frac{1}{\beta \nu_2} \tilde{W}_{t-1} + \nu_1 \tilde{Q}_{t-1} \) |
| \( \tilde{D}_t \) | \( 2 (1 - a_H) \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \tilde{W}_t + \frac{1}{\beta \nu_2} \tilde{W}_{t-1} + \nu_1 \tilde{Q}_{t-1} \) |

In the table, the variables \( \nu_1, \nu_2 \) and \( \alpha_2 \) represent eigenvalues—where \( \nu_{1,2} \) differs from \( \alpha_{1,2} \) only in that they does not depend on \( \theta \). For future reference, it is worth noting that

\[ \nu_{1,2} = \frac{1 + \beta + (1 - \alpha \beta)(1 - \alpha)}{\alpha} \theta \pm \sqrt{\left[ 1 + \beta + \frac{(1 - \alpha \beta)(1 - \alpha)}{\alpha} \theta \right]^2 - 4 \beta} \]

and \( \nu_{1,2} \) differ from the above only in that the term \( \frac{(1 - \alpha \beta)(1 - \alpha)}{\alpha} \) is not multiplied by \( \theta \). It is worth noting that the eigenvalues \( \alpha_2 \) and \( \nu_2 \) determine the discounted weight attributed to expectations of future fundamentals in driving the dynamics of the real exchange rate and of relative prices \( \tilde{T}_t + \tilde{\Delta}_t \). Note that the higher the degree of price stickiness \( \alpha \), the larger the stable eigenvalues \( \alpha_1 \) and \( \nu_1 \), the lower the speed of adjustment of gaps under the optimal policy. Correspondingly, the lower the unstable eigenvalues \( \alpha_2 \) and \( \nu_2 \), the less expected future fundamentals are discounted in determining the gaps.
these eigenvalues are related as follows

\[ 0 < \nu_1 < 1 < \beta^{-1} + \frac{(1 - \alpha \beta) (1 - \alpha)}{\alpha \beta} \theta < \nu_2, \]

\[ 0 < \nu_1 < 1 < \beta^{-1} + \frac{(1 - \alpha \beta) (1 - \alpha)}{\alpha \beta} < \nu_2, \]

\[ \nu_2 < \nu_2 < \nu_2 < \beta^{-1} \]

so that \( 0 < \frac{(\beta \nu_2 - 1)}{\beta \nu_2} < 1, 0 < \frac{(\beta \nu_2 - 1)}{\beta \nu_2} < 1, \) and \( \frac{(\beta \nu_2 - 1)}{\beta \nu_2} < \frac{(\beta \nu_2 - 1)}{\beta \nu_2}. \)

Consider the world-economy response to news shocks at time \( t_0, \) resulting in capital inflows and a positive wedge gap \( \tilde{W}_{t_0} > 0. \) According to Table 3, implementing the targeting rules leads to a fall in Home CPI inflation on impact, given by the following expression:

\[ \pi_{t_0} = - (1 - \alpha_H) \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \tilde{W}_{t_0} \leq 0. \quad (31) \]

The (constrained-) optimal contractionary stance at Home does contain the inefficient surge in Home consumption relative to the Foreign one. The relative demand gap is positive

\[ \tilde{D}_{t_0} = 2 (1 - \alpha_H) \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \tilde{W}_{t_0} > 0 \quad (32) \]

but smaller than under CPI price stability (compare with Table 2, whereas \( \frac{(\beta \nu_2 - 1)}{\beta \nu_2} < 1 \)). The Home real exchange rate correspondingly appreciates on impact:

\[ \tilde{Q}_{t_0} = - \left[ (2 \alpha_H - 1) \frac{(\beta \nu_2 - 1)}{\beta \nu_2} + \frac{1}{\beta \nu_2} \right] \tilde{W}_{t_0} < 0. \quad (33) \]

by more than under price stability (since the expression in square brackets is greater than one).\(^{26}\)

Recall that, since \( \tilde{Q}_{f_t}^b = 0 \) in response to news shocks, the welfare relevant gap and the real exchange rate move one-to-one: \( \tilde{Q}_t = \tilde{Q}_t. \)

Because the optimal stance is relatively contractionary at Home, the output gap is always smaller than under CPI price stability.\(^{27}\) Yet, under the optimal policy, the welfare-relevant

\(^{26}\)Observe that, dynamically, the optimal stance induces a predictable exchange rate dynamic, where Home real appreciation is followed by depreciation. To illustrate this dynamic, one can use the expression for \( \tilde{Q}_t \) in Table 3 to decompose the movement of the exchange rate into a long-run permanent appreciation component and a component driven by the expected cumulated real interest rate differential across countries. Comparing the two, what determines this dynamic is the following inequality:

\[ \frac{1}{\beta \nu_2} > (2 \alpha_H - 1) \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \frac{\nu_1}{(1 - \nu_1)} = (2 \alpha_H - 1) \frac{1}{\beta \nu_2}. \quad (34) \]

The expected appreciation in the long run reflects the permanent wealth effects associated with the capital inflow under incomplete markets.

\(^{27}\)Namely:

\[ \tilde{Y}_{H,t_0}^{CPI} = - (1 - \alpha_H) \left[ 1 - 2 \alpha_H \left( 1 - \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \right) \right] \tilde{W}_{t_0}, \]

where now gaps under strict CPI stability are denoted with a \( CPI \) superscript.
output gap is not necessarily negative on impact:

\[
\tilde{Y}_{H,t_0} = 2a_H \left(1 - a_H \right) (\tilde{T}_t + \tilde{\Delta}_t) + 1/2 \cdot (2a_H - 1) \tilde{D}_t
\]

\[
= - (1 - a_H) \left[ \frac{(\beta x_2 - 1)}{\beta x_2} - 2a_H \left( \frac{(\beta x_2 - 1)}{\beta x_2} - \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \right) \right] \tilde{W}_{t_0} \leq 0.
\]

Specifically, it is possible that the positive impact of the capital inflow on the relative demand gap, \(\tilde{D}_t\) outweighs the negative effect of the terms-of-trade gap and deviations from the LOOP, \(\tilde{T}_t + \tilde{\Delta}_t\).\(^{28}\) It is easy to see that, on impact, the output gap is negative if the following condition is satisfied:

\[
\frac{\beta x_2 - 1}{\beta \nu_2 - 1} > 2a_H.
\]

This condition is more likely to hold in economies that are very open (i.e., economies with a low home bias \(a_H\))—intuitively, openness increases the relative weight of \((\tilde{T}_t + \tilde{\Delta}_t)\) and decreases that of \(\tilde{D}_t\) in the output gap expression above. Furthermore, the condition always holds (for any degree of openness), in limit cases where prices are either very sticky \((x_2 \simeq \nu_2 \to 1/\beta)\) or very flexible \((x_2 \simeq \nu_2 \to \infty)\).

Together, these results show that, in our CO economies under LCP, the monetary authorities optimally trade off stabilization of domestic demand with a larger real exchange rate gap. Using our expressions, we can now dig deeper, and analyze how this trade-off, i.e., the extent to which monetary policy pursues one objective over the other, varies with the degree of nominal rigidities (thus exchange rate pass-through) and openness.

Concerning nominal rigidities and pass-through, note that, for \(\alpha \to 0\) (prices are flexible in the limit), \(\frac{(\beta x_2 - 1)}{\beta x_2} \to 1\) and \(\frac{1}{\beta x_2} \to 0\) : in absolute value, the coefficient of \(\tilde{W}_{t+s}\) declines in (33), but rises in (32). For higher degrees of price flexibility, optimizing policymakers tolerate a larger misallocation of demand, as they pay more attention to the inefficient real exchange rate appreciation. This is quite intuitive: as import prices become less sticky, exchange rate pass-through is higher. Competitiveness progressively becomes a stronger policy concern relative to aggregate demand stabilization (the more flexible prices are, the closer \(x_2\) is to \(\nu_2\), and the smaller the output gap is in absolute value). Remarkably, as prices become less sticky, a milder Home monetary contraction causes the equilibrium rate of inflation (31) to fall by more (since with less nominal rigidities prices react more strongly).

Similar considerations apply to openness: as the economy becomes more open, i.e., for \(a_H \to 1/2\) (the case of no home bias), the optimal policy pays more attention to real exchange rate misalignment. Indeed, for any given degree of price stickiness, when the economy becomes more open, a tight domestic monetary policy becomes progressively less effective in dealing with a demand boom fueled by capital inflows.\(^{29}\)

\(^{28}\)Recall that since \(\nu_2 < x_2\), the expression in square brackets can have either sign.

\(^{29}\)When \(\tilde{W}_{t_0} < 0\)—the case associated with an increase in the efficient level of current output—Home monetary policy is relatively expansionary to stimulate the inefficiently low domestic consumption. Relative to the above, the response of optimal monetary policy is the opposite, because capital inflows are now inefficiently low. The real exchange rate depreciates and is undervalued. However, undervaluation is lower with a high degree of pass-through and openness.
4.3 Exchange rate stabilization and competitiveness with high pass-through (PCP economies)

A comparison of our results across LCP and PCP economies is particularly suitable in our Cole-and-Obstfeld specification, since in response to identical news shocks, the sign and size of the ensuing capital flows and wealth gap—that is, the expressions for $\tilde{B}_t$ and $\tilde{W}_t$ in (30) and (29)—are exactly the same. Conditional on a given $\tilde{B}_t < 0$ and the associated $\tilde{W}_t > 0$, Table 4 presents the allocation under the optimal cooperative monetary policy in the PCP economy.

Table 4: Constrained-efficient allocation under PCP with news shocks

$$
\tilde{Y}_{H,t} = x_1 \tilde{Y}_{H,t-1} - (1 - a_H) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \tilde{W}_t
$$

$$
\theta \pi_{H,t} = - \left( \tilde{Y}_{H,t} - \tilde{Y}_{H,t-1} \right)
$$

$$
\tilde{T}_t = - \left( 1 - \frac{2(1 - a_H)}{\beta \kappa_2} \right) \tilde{W}_t + 2x_1 \tilde{Y}_{H,t-1}
$$

$$
\tilde{Q}_t = (2a_H - 1) \left[ - \left( 1 - \frac{2(1 - a_H)}{\beta \kappa_2} \right) \tilde{W}_t + 2x_1 \tilde{Y}_{H,t-1} \right]
$$

$$
\tilde{D}_t = 2(1 - a_H) \left[ 1 + \frac{(2a_H - 1)}{\beta \kappa_2} \right] \tilde{W}_t + 2(2a_H - 1) x_1 \tilde{Y}_{H,t-1}
$$

The Home optimal monetary response to the capital inflows is the opposite relative to the LCP case: when exchange rate pass-through is complete, Home monetary authorities pursue a monetary expansion. Compared with the natural rate allocation in Table 2, they tolerate some short-run (GDP deflator) inflation:

$$
\pi_{t_0} = (1 - a_H) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \tilde{W}_{t_0} > 0
$$

and lean on the appreciation of the real exchange rate

$$
\tilde{Q}_{t_0} = - (2a_H - 1) \left( 1 - \frac{2(1 - a_H)}{\beta \kappa_2} \right) \tilde{W}_{t_0} < 0.
$$

so as to contain competitiveness losses. Relative to the allocation in Table 2, the expansionary stance mitigates the negative output gap

$$
\tilde{Y}_{H,t_0} = - (1 - a_H) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \tilde{W}_{t_0} < 0,
$$

(this is so because $\frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} < 1$), at the cost of increasing the relative demand gap

$$
\tilde{D}_{t_0} = 2(1 - a_H) \left[ 1 + \frac{(2a_H - 1)}{\beta \kappa_2} \right] \tilde{W}_{t_0} > 0.
$$

The optimal degree of monetary expansion again depends on whether the economy is more or less open, and the degree of price stickiness.
4.4 Exchange rate volatility, inflation and output gaps in CO economies: LCP vs PCP

For our CO economies, the macroeconomic response to shocks under the optimal policy is illustrated by Figure 1. The figure plots the impulse responses of the relevant gaps to a shock anticipated to occur 20 quarters in the future (outside the time scale of the graph), causing an inflow of capital in the Home economy.\(^{30}\) The shock is normalized to produce an initial capital inflow as high as 1 percent of Home GDP. As shown by the first graph in the upper left corner, the stock of foreign debt increases exogenously along the optimal adjustment path. The size of capital flows is excessive: the wealth gap (shown in the graph in the upper right corner) jumps to a positive value and remains constant, according to (14). Both the capital inflows and the wealth gap are exogenous to macroeconomic adjustment and policy and, hence independent of LCP and PCP.

The remaining graphs in the figure distinguish between LCP economies (continuous lines) and PCP economies (dashed lines). The price response (lower left corner) shows that the monetary stance is relatively expansionary under PCP (GDP-deflator inflation is positive), contractionary under LCP (CPI inflation is negative).

Comparing the two economies highlights an important result. Under the optimal policy, the real exchange rate is always less volatile under PCP (where monetary authorities lean against appreciation) than under LCP (where monetary authorities exacerbate misalignment). Analytically, this follows from observing that under strict inflation targeting, the real exchange rate response under LCP (CPI targeting) is the same as under PCP (GDP deflator targeting), and thus equal to the natural rate allocation \( \hat{Q}^{na}_t = -(2a_H - 1) \hat{W}_t \). Relative to this natural rate allocation, we have shown that the optimal policy makes the real exchange rate less volatile under PCP, and more volatile under LCP. Correspondingly, the real exchange rate always undershoots its long-run value under PCP—and overshoots under LCP. Because of the expenditure-switching effects of the exchange rate, however, the output gap is more negative under PCP.\(^{31}\)

5 Optimal trade-offs varying trade elasticities

In this section, we generalize our analysis by relaxing the assumption of a unitary trade elasticity. This allows us to extend the analysis in at least three directions. First, unlike in the CO economies, cross-border flows also respond to shocks to productivity—in addition to shocks to preferences for saving and/or changes in taxes or capital controls. We can thus consider different types of business cycle disturbances. Second, the wealth gap \( \hat{W}_t \) associated with capital inflows (excessive relative to the first-best allocation) that is triggered by news shocks can also be negative—a negative \( \hat{W}_t \) implies that, in spite of inefficiently large capital inflows, Home relative wealth is inefficiently low. Policy prescriptions will thus need to take this into account,\(^ {30}\) The parameter values are as follows: \( \phi = \sigma = 1, a_H = .75, \beta = .99, \alpha = .75 \).

\(^{31}\) Analytically, this follows from comparing the expression for the output gaps under PCP, the natural allocation and LCP, whereas, since \( \nu_2 < \chi_2 \),

\[
(1 - a_H) \left( \frac{\beta \nu_2 - 1}{\beta \chi_2} \right) > (1 - a_H) \left[ 2a_H \left( \frac{\beta \nu_2 - 1}{\beta \nu_2} - \frac{\beta \chi_2 - 1}{\beta \chi_2} \right) \right] > (1 - a_H) \left[ 1 - 2a_H \left( 1 - \frac{\beta \nu_2 - 1}{\beta \nu_2} \right) \right].
\]
in comparison with the CO economies. Finally, capital flows may no longer be exogenous to monetary policy. We can thus characterize how optimal monetary policy affects the size of inefficient cross-border borrowing and lending.

To keep the analytical complexity at a minimum, we restrict our attention to “news shocks” only—no contemporaneous shock will appear in the equations to follow. For news shocks, when $\sigma = 1$, $\eta = 0$ but $\phi \neq 1$, the natural rate allocation differs from Table 2 only in the following, crucial, dimension:

$$\tilde{Y}_{H,t}^{na} = - (1 - a_H) [2 a_H (\phi - 1) + 1] \tilde{W}_{t+s}^{na} = \tilde{B}_{t}^{na}. \quad (35)$$

Thus, while a capital inflow ($\tilde{B}_{t}^{na} < 0$) will invariably lead to a negative output gap, the associated wealth gap, $\tilde{W}_{t}^{na}$, may be positive or negative, depending on the value of the trade elasticity. Specifically, given $\tilde{B}_{t}^{na} < 0$,

$$\tilde{W}_{t}^{na} > 0 \quad \text{if} \quad \phi > \frac{2 a_H - 1}{2 a_H} \leq 1/2. \quad (36)$$

Given $\tilde{W}_{t}^{na}$, however, all the other gaps behave exactly the same as in Table 2—they therefore depend on the elasticity $\phi$ through the response of the wealth gap. As shown above, for $\tilde{W}_{t}^{na} > 0$, capital inflows appreciate the exchange rate, the Home currency is overvalued and Home domestic demand is excessive. The opposite is true when, for elasticities below the threshold above, $\tilde{W}_{t}^{na} < 0$: capital inflows are associated with real depreciation and the Home real exchange rate is undervalued; Home demand is too low.

5.1 Monetary policy and the transfer problem

The implications of capital inflows for relative prices and the exchange rate are best understood in light of the “transfer problem,” the classical controversy in open economy macro originated by the debate between Keynes and Ohlin about the effects of war reparation payments on the terms of trade of a country (see Keynes [1929a,b,c] and Ohlin [1929a,b]). Under incomplete markets, capital inflows into Home are effectively a transfer from Foreign. From a global perspective, because of home bias in demand, if relative prices did not adjust, higher savings by Foreign residents and higher dissaving by Home residents would translate into an excess supply of Foreign goods. Equilibrium unavoidably requires a relative price adjustment—as John Williamson would put it, there is no “immaculate transfer” (see Krugman [2007]).

In an equilibrium in which $\tilde{W}_{t} > 0$, substitution effects from the real exchange rate are stronger than income effects: equilibrium adjustment to a transfer occurs via Home real appreciation, redirecting world demand towards Foreign goods. Because of the fall in the relative price of Foreign output, Foreign incomes fall and Home incomes rise by more than the size of the transfer at constant prices—the problem stressed by Keynes.

However, when $\tilde{W}_{t} < 0$—corresponding to the case in which Home and Foreign goods

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32Contemporaneous shocks mainly affect the relation between capital flows and the sign of the wealth gap; nevertheless, given the latter, the optimal monetary policy response is the same for both contemporaneous and anticipated shocks.

33However, for $\phi \neq 1$, the natural rate allocation is not equivalent any longer to that under CPI stability and LCP.
are strong complement—the income effects from relative price adjustment are stronger than substitution effects. In response to Home capital inflows, there is no equilibrium with Home appreciation/Foreign depreciation, because this would drive Foreign demand too low for the goods markets to clear at global level. Equilibrium requires Foreign appreciation/Home depreciation (see Corsetti et al. [2008a]).

The relative strength of income versus substitution effects has crucial implications for monetary policy design. As we will see below, in economies where the trade elasticity is sufficiently low that \( \hat{B}_t < 0 \) and \( \hat{W}_t < 0 \), sustaining domestic demand and output in response to capital inflows and currency undervaluation becomes the overriding concern of monetary policy: the optimal monetary stance is expansionary—for any degree of exchange rate pass-through and openness.\(^{34}\)

5.2 Low pass-through (LCP) economies

The equilibrium relation between capital flows and the wealth gap in LCP economies is shown in Table 5, together with the full solution for the dynamics of capital flows under the optimal policy. The two expressions in the table depend only on exogenous shocks, and (through the t.i.p. term)\(^{35}\) on the current and anticipated future evolution of relative prices in the first-best allocation, unaffected by policy. Thus, a first remarkable and arguably surprising result is that, in LCP economies, as long as \( \eta = 0 \) and \( \sigma = 1 \) cross-border capital flows and the wealth gap remain independent of policy even if the trade elasticity is different from unity (the case of CO economies).

Table 5: Capital flows under LCP and with news shocks, for \( \phi \neq 1 \)

\[
(1 - a_H) \left[ 1 + 2a_H (\phi - 1) \left( \frac{\beta \nu_2}{\beta \nu_2 - 1} \right) \right] \hat{W}_t = - \left( \hat{B}_t - \beta^{-1} \hat{B}_{t-1} \right) + \nonumber \\
2a_H (1 - a_H) (\phi - 1) \sum_{j=0}^{\infty} \nu_2^{-1} E_t \left( \left( \hat{T}^f_{t+j+1} - \hat{T}^f_{t+j} \right) - \beta^{-1} \left( \hat{T}^f_{t+j} - \hat{T}^f_{t+j-1} \right) \right) \\
\hat{B}_t - \hat{B}_{t-1} = \frac{2a_H (\phi - 1) \left( \frac{\beta \nu_2}{\beta \nu_2 - 1} \right)}{1 + 2a_H (\phi - 1) \left( \frac{\beta \nu_2}{\beta \nu_2 - 1} \right)} \left( \beta^{-1} \hat{B}_{t-1} - \hat{B}_{t-1} \right) + \text{t.i.p} 
\]

From the table, it is apparent that the trade elasticity \( \phi \) nonetheless matters for \( \hat{B}_t \) and \( \hat{W}_t \) in two crucial respects. It determines, first, whether a given “news shock” translates into inefficient borrowing or lending; and, second, whether \( \hat{B}_t \) and \( \hat{W}_t \) have the same or the opposite sign. Differently from the natural allocation above (generally unfeasible in LCP economies),

\(^{34}\)It is worth stressing that no such effect would materialize were markets complete: perfect risk diversification would eliminate any adverse income effects from shocks and exchange rate movements. Recall from Section 3.3.1 that in the first-best allocation news shocks would not trigger any financial flow across borders, so that in a bond economy all capital inflows in the natural allocation are invariably excessive, irrespective of the sign of \( \hat{W}_t^{\infty} \).

\(^{35}\)The terms independent of policy (t.i.p.) in the table are:

\[ \quad \text{t.i.p. = } \frac{1 + 2a_H (\phi - 1) \left( \frac{\beta \nu_2}{\beta \nu_2 - 1} \right)}{1 + 2a_H (\phi - 1) \left( \frac{\beta \nu_2}{\beta \nu_2 - 1} \right)} \beta \sum_{j=0}^{\infty} \beta^j E_t \left[ \left( \zeta_{C,t+j+1} - \tilde{\zeta}_{C,t+j+1} \right) - \left( \zeta_{C,t+j} - \tilde{\zeta}_{C,t+j} \right) \right] \\
+ 2a_H (1 - a_H) (\phi - 1) \sum_{j=0}^{\infty} \nu_2^{-1} E_t \left( \left( \hat{T}^f_{t+j+1} - \hat{T}^f_{t+j} \right) - \beta^{-1} \left( \hat{T}^f_{t+j} - \hat{T}^f_{t+j-1} \right) \right) \\
- 2a_H (1 - a_H) (\phi - 1) \frac{1 + 2a_H (\phi - 1) \left( \frac{\beta \nu_2}{\beta \nu_2 - 1} \right)}{1 + 2a_H (\phi - 1) \left( \frac{\beta \nu_2}{\beta \nu_2 - 1} \right)} \beta \sum_{j=0}^{\infty} \beta^j E_t \left[ \left( \hat{T}^f_{t+j+1} - \hat{T}^f_{t+j} \right) - \beta^{-1} \left( \hat{T}^f_{t+j} - \hat{T}^f_{t+j-1} \right) \right] + \\
\sum_{j=0}^{\infty} \beta^j \left[ (1 - \nu_1) \beta^j \sum_{j=0}^{\infty} \nu_2^{-1} E_t \left( \left( \hat{T}^f_{t+j+1} - \hat{T}^f_{t+j} \right) - \beta^{-1} \left( \hat{T}^f_{t+j} - \hat{T}^f_{t+j-1} \right) \right) \right] \right] \]
the threshold value of the trade elasticity below which $\hat{B}_t$ and $\tilde{\nu}_t$ have the same sign no longer depends exclusively on the home bias parameter, and differs depending on the shocks hitting the economy. For the case of taste shocks, the threshold is:\footnote{As shown above, with $\sigma = 1$ and $\eta = 0$, the terms-of-trade response to (current or anticipated) taste shocks in the first-best allocation is $\hat{T}_t^{fb} = 0$. So, the expressions in Table 5 simplify as follows:}

$$\phi < \frac{2a_H - \frac{\beta \nu_2}{(\beta \nu_2 - 1)}}{2a_H} < 1.$$\footnote{This result is apparent from the fact that $\hat{B}_t < 0$ necessarily implies that the sum of the last two lines in the second expression in the Table 3 have the opposite sign and are larger in absolute value than the third line in the same expression, which also appears in the equation for $\tilde{\nu}_t$. The threshold is sufficient for the term $1+2a_H(\phi -1)^{1-2(1-\beta \nu_2)/(\beta \nu_2 - 1)}$ to be non negative.}

This threshold is smaller, the more open the economies ($a_H \rightarrow 1/2$, $\phi \geq 0$) and the higher the degree of price stickiness ($\nu_2 \rightarrow 1/\beta$, so that $\frac{\beta \nu_2}{(\beta \nu_2 - 1)} \rightarrow 1$, $\phi \geq 0$)—resulting in a lower degree of pass-through. For the case of anticipated productivity shocks, the threshold is:

$$\phi < \frac{2a_H - \frac{\beta \nu_2}{(\beta \nu_2 - 1)}(1 - \beta \nu_1)}{2a_H} < 1.$$\footnote{This result is apparent from the fact that $\hat{B}_t < 0$ necessarily implies that the sum of the last two lines in the second expression in the Table 3 have the opposite sign and are larger in absolute value than the third line in the same expression, which also appears in the equation for $\tilde{\nu}_t$. The threshold is sufficient for the term $1+2a_H(\phi -1)^{1-2(1-\beta \nu_2)/(\beta \nu_2 - 1)}$ to be non negative.}

Since $\nu_1 \leq 1 < \beta^{-1} \leq \nu_2$, this expression is unambiguously above the one derived for the case of taste shocks.\footnote{This result is apparent from the fact that $\hat{B}_t < 0$ necessarily implies that the sum of the last two lines in the second expression in the Table 3 have the opposite sign and are larger in absolute value than the third line in the same expression, which also appears in the equation for $\tilde{\nu}_t$. The threshold is sufficient for the term $1+2a_H(\phi -1)^{1-2(1-\beta \nu_2)/(\beta \nu_2 - 1)}$ to be non negative.}

A second remarkable result in our LCP economies is that, given the sign and paths of $\hat{B}_t$ and $\tilde{\nu}_t$ in response to shocks (from Table 5), the trade elasticity $\phi$ does not enter directly the expressions for the response of inflation, demand gaps and the real exchange rate. Indeed, provided contemporaneous shocks are excluded from the analysis, these expressions are exactly the same as in Table 3, derived for the CO economies with a unitary elasticity.

From Table 3, we know that, in response to shocks that cause a capital inflow, $\hat{B}_t < 0$ associated with a positive wealth gap, $\tilde{\nu}_t > 0$, the Home monetary authorities will optimally let inflation decline, at the cost of exacerbating the Home real exchange rate appreciation (and overshooting) in the short run—they will implement a monetary tightening. Similar to our earlier analysis, the extent to which the optimal policy response translates into a fall in relative consumption will depend on the degrees of openness and stickiness of import prices, the latter in turn determining the degree of exchange rate pass-through. A variable for which the trade elasticity parameter $\phi$ makes a difference, however, is the welfare-relevant output gap. Its impact response to the optimal contractionary stance is:

$$\hat{Y}_{H,t} = - (1-a_H) \left[ \frac{(\beta \nu_2 - 1)}{\beta \nu_2} - 2a_H \left( \frac{(\beta \nu_2 - 1)}{\beta \nu_2} - \phi \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \right) \right] \tilde{\nu}_t < 0. \quad (37)$$
It is easy to show that the response of this gap to the optimal monetary contraction is unambiguously negative for values of φ sufficiently above 1.

The optimal policy response to excessive inflows is quite different in economies where domestic and foreign goods are highly complementary and the effects of the “transfer” change sign, for values of the trade elasticity below the thresholds above. With \( \hat{B}_t < 0 \) and \( \hat{W}_t < 0 \), the (exogenous) capital inflows are associated with inefficiently low domestic demand: monetary authorities optimally focus on domestic demand stabilization (see Table 3). The optimal stance is relatively expansionary (rather than contractionary) at Home, up to the point of bringing the Home output gap into positive territory (as follows from assessing (37) for \( \phi \rightarrow 0 \)). Relative to strict inflation targeting, Home aggregate demand and economic activity will be stronger, while the real exchange rate will clearly be weaker—i.e., even more undervalued.\(^{38}\)

### 5.3 High pass-through (PCP) economies

The allocation under the optimal policy in PCP economies is shown in Table 6, once again abstracting from contemporaneous shocks. Different from our results under LPC, it is apparent that capital flows are no longer independent of the macroeconomic allocation and therefore of policy. The optimal monetary stance may also affect the size of the inflows, even for \( \sigma = 1.\(^{39}\)

\[
\begin{align*}
\hat{W}_t &= A\beta \sum_{j=0}^{\infty} \beta^j \left[ \frac{2a_H (\phi - 1)}{4a_H (1-a_H)(\phi-1)+1} \right] \frac{1-\sigma}{1-\beta H_1} \beta^{\sigma H_1} \hat{Y}_H,t-1 \\
\hat{B}_t &= -B\beta \sum_{j=0}^{\infty} \beta^j \left[ -2a_H (\phi - 1) E_t \left( \hat{Y}_H,t+1 - \hat{Y}_{F,t+1} \right) \right] \\
\hat{Y}_H,t &= \chi_1 \hat{Y}_H,t-1 - (1-a_H) \left\{ \left[ 2a_H (\phi - 1) + 1 \right] \frac{1}{\beta^{\sigma H_2}} \hat{W}_t + \frac{2a_H (\phi - 1) + 1}{\beta^{\sigma H_2}} \hat{W}_t - \hat{W}_{t-1} \right\} \\
\theta_{\pi H,t} &= (1-\chi_1) \hat{Y}_H,t-1 - (1-a_H) \left\{ \left[ 2a_H (\phi - 1) + 1 \right] \hat{W}_t + \frac{2a_H (\phi - 1) + 1}{\beta^{\sigma H_2}} \hat{W}_t - \hat{W}_{t-1} \right\} \\
\hat{F}_t &= \frac{2\hat{Y}_{H,t} - (2a_H - 1)\hat{W}_t}{4a_H (1-a_H)(\phi - 1) + 1}
\end{align*}
\]

At least three results are worth stressing. First, if exchange rate pass-through is complete, the threshold for the trade elasticity at which the wealth gap \( \hat{W}_t \) switches sign under the optimal policy is the same as the one derived for the natural rate allocation (36), and thus identical for both (anticipated) taste and productivity shocks.\(^{40}\) Together with the results from the previous

\(^{38}\)With \( \sigma \neq 1 \), capital flows and wealth gaps respond to monetary policy. Yet, under reasonably general conditions, the results discussed in this subsection will go through: the sign of monetary policy is not determined by capital flow stabilization. Moreover, as shown in the appendix, for \( \sigma > 1 \) and \( \phi > 1 \), expansionary monetary policy always reduces the capital inflow; the opposite happens for \( \sigma < 1 \) and \( 0 < \phi < 1 - 1 - \frac{2\sigma - 1}{2\sigma - 1} < 1.\)

\(^{39}\)Under PCP it is possible to derive analytically tractable results for any \( \eta \geq 0 \) and \( \sigma \geq 1 \). However for comparability with the LCP case the table reports the solution for the case \( \eta = 0 \) and \( \sigma > 0 \).

\(^{40}\)This is so because, in the expression for \( \hat{B}_t \), first, the coefficient \( B \) of the term in the shocks is always non-positive for any value of \( \phi \), namely:
subsection, this establishes that the elasticity threshold below which a capital inflow causes a negative wealth gap is never larger (whether under LCP or PCP) than in the natural rate allocation. It is bounded above by $\frac{2a_H - 1}{2a_H}$, which is decreasing in openness (and goes to zero for $a_H \to 1/2$, the case of no home bias in consumption).

Second, under the optimal policy, the impact response of inflation to capital inflows is always positive, for any value of the elasticity $\phi$, i.e., whether $\tilde{W}_t$ is positive or negative:

$$\theta_{\pi_{H,t}} = (1 - a_H) \frac{\beta x_2 - 1}{\beta x_2} \left[ 4a_H (1 - a_H) (\phi - 1) + 1 \right] \frac{\tilde{W}_t}{2a_H (\phi - 1) + 1} > 0.$$  

Hence, the optimal monetary policy is always expansionary on impact (note that, unlike the case of LCP, there is no switch in the sign of the monetary policy). The trade elasticity, however, impinges on the inflationary impact of the optimal monetary expansion.

For elasticities above the threshold (36), the optimal policy is similar to the one derived in the CO economy. Capital inflows associated with inefficiently high Home demand ($\tilde{W}_t > 0$) call for easier monetary policy at Home (see the inflation expression above), to resist exchange rate overvaluation. The welfare-relevant output gap nonetheless remains negative, despite the fact that such a stance stokes inflationary pressures.

For elasticities below the threshold (36) such that $\tilde{B}_t$ and $\tilde{W}_t$ are both negative, the expansionary Home policy stance may even bring the welfare-relevant output gap into positive territory. As shown in the appendix, for low enough trade elasticities, the optimal Home monetary policy stance is increasingly driven by the need to prop up an inefficiently low domestic demand. In response to the capital inflow, Home residents’ consumption actually falls in relative terms for values of $\phi$ below the following threshold:

$$\phi \leq \frac{(2a_H - 1) (\beta x_2 - 1)}{2a_H} \leq \frac{(2a_H - 1)}{2a_H}.$$  

It is for this region of elasticities that the optimal monetary boost turns the output gap positive.

Our third result is that, unlike the CO economy and the LCP case, the monetary expansion now affects the size of capital inflows. In general, there are two channels to consider, working in opposite directions. By leaning against real appreciation, an expansionary monetary policy discourages capital inflows; by sustaining domestic demand, it raises domestic borrowing. It can be shown that, under the optimal policy, the first channel prevails for $\phi > 1$: relative to the benchmark of the natural rate allocation, capital inflows are smaller in absolute value. Yet, for elasticity values in the range $1 > \phi > \frac{2a_H - 1}{2a_H}$, it is the second channel that prevails: the optimal

\[
B = \frac{\{1-a_H\}}{4a_H (1-a_H)(\phi-1)+1} \\
\left[ 1 - \frac{1-x_2}{x_2} \frac{4a_H (1-a_H)(\phi-1)+1}{[2a_H(\phi-1)+1]^2} \frac{4a_H (1-a_H)(\phi-1)+1}{[2a_H(\phi-1)+1]^2} \frac{4a_H (1-a_H)}{[2a_H(\phi-1)+1]^2} \frac{(1-\beta)}{\beta(\phi+1)} \right] \leq 0.
\]

Second, the sign of the coefficient $A$ multiplying the same term in the expression for $\tilde{W}_t$, given by

$$A = \frac{[2a_H (\phi - 1) + 1]^{-1}}{4a_H (1-a_H) (\phi - 1) + 1 + 4a_H (1-a_H) \phi \frac{4a_H (\phi - 1)}{2a_H (\phi - 1) + 1}} \frac{(1-\beta)}{\beta x_2 (1-\beta x_1)} \leq 0$$

depends instead on whether $\phi$ is above or below the threshold (36).

\[\text{To see why, note that, conditional on } \tilde{B}_t < 0, \text{ the term on the right-hand side of the expression in the text is always positive (} [2a_H (\phi - 1) + 1]^{-1} \tilde{W}_t \text{ always has the opposite sign of } \tilde{B}_t).\]
monetary stance magnifies net borrowing relative to the natural rate allocation.\textsuperscript{42}

6 Conclusions

As a consequence of the 2008 global financial crisis, much research has been devoted to reconsider the set of policy tools and measures that can be activated to insulate national economies from the ebb and flows of cross-border capital flows. In this paper, we have taken the perspective of monetary policy decision making, and analyzed what monetary instruments can deliver when additional tools are not readily available and/or are of limited effectiveness. Our main question is how monetary policy can manage the effects of inefficient capital flows on domestic macroeconomic dynamic and welfare, by optimally trading off domestic and external objectives.

Our study provides key analytical insight into the efficient resolution of this trade-off. When international capital markets fall short of delivering a high degree of risk sharing (so that capital flows are associated with currency misalignment), the design of optimal monetary rules hinges on recognizing the direct and indirect relevance of competitiveness for domestic stabilization and welfare. With a high pass-through, indeed, the optimal response to inefficient capital inflows is directed to contain misalignment and overappreciation, tolerating a temporary surge in domestic inflation. However, the stabilization of aggregate demand is a priority—at the costs of missing out on external objectives and temporary below-target inflation—when imperfect pass-through mutes the price competitiveness effects of exchange rate appreciation, and trade elasticities are not too low.

Our results, derived under commitment, can be brought to bear on the case of cooperation under discretion, where policymakers are not able to improve the short-run trade-offs among competing goals by credibly guiding expectations of future policy rates and inflation. As is well known, in the closed economy counterpart of our model, or in its version under complete markets, optimal targeting rules derived under discretion will include all variables (a part of inflation) in levels, rather than in growth rates. Namely, under discretion monetary authorities cannot credibly pursue a nominal anchor in level. This will also be the case in the specification of our bond economy where capital flows are exogenous to monetary policy. In either our CO economies, or our LCP economies with \( \sigma = 1 \) and \( \alpha = 0 \), the targeting rules under discretion can be easily derived from our analysis in Section 3—crossing out lagged terms. Economic dynamics can be readily derived from our analysis in Sections 4 and 5. In more general specifications of the model, however, the accumulation of net foreign assets and liabilities will change the state of the economy over time: targeting rules derived under discretion will generally include a term capturing the optimal policy response to foreign debt accumulation, complicating their analytical characterization.

An important consequence of the inability of discretionary policymakers to credibly pursue a nominal anchor is that the nominal price level and the exchange rate will have a unit root: any rise of inflation above target will not be offset by credible policies pursuing a fall in inflation below target in the future. For our baseline specifications with exogenous capital flows, then, it is easy to verify that, with the optimal policy in place, inflation and exchange rate volatility will be higher under discretion than under commitment.

\textsuperscript{42}As shown in the appendix, an expansionary monetary shock always decreases capital inflows for \( \phi > 1 \).
Among possible directions for future research, an important one concerns the analysis of strategic interactions among policymakers. Numerical analyses of the Nash equilibrium under incomplete markets and PCP suggest that, although policymakers have an incentive to manipulate the terms of trade of the country in their own national interests, incomplete markets increase the weight attached to stabilization of domestic incomes (see e.g. Rabitsch [2012] and, for a small open economy, De Paoli [2009]). Based on our results, we can further observe that inefficient capital flows have strong redistributive effects across border. We have seen that cooperative policies attempt to redress these effects: in our analysis, when the optimal monetary policy at Home is either a contraction or an expansion, the Foreign monetary stance has the opposite sign. Without cooperation, however, these redistributive effects of capital inflows inherently create room for conflicts and strategic behavior.

Relatedly, while in this paper we focus on the benchmark cases of PCP and LCP, the evidence on the importance of pricing in vehicle currencies strongly motivates further work exploring the case of asymmetric pass-through, or Dominant Currency Pricing (DCP, see Gopinath [2016] and Casas et al. [2016]). An important question is which direction, when facing a capital inflow with currency overvaluation, monetary policy will take in the country which issues the dominant currency.
7 Appendix

7.1 The transmission of monetary policy with imperfect capital markets

In this appendix, we analyze how monetary policy impacts the welfare-relevant gaps defined in the main text. As is well known, there are notable differences in the transmission of monetary decisions across LCP and PCP economies. Specifically, a monetary expansion causing nominal depreciation weakens the terms of trade under PCP but tends to strengthen the terms of trade under LCP. Here, our specific interest is to understand how monetary transmission is affected by financial distortions.

Starting with the LCP model, consider for simplicity a Home monetary shock such that CPI inflation follows an autoregressive process, \( a_H \pi_{Ht+s} + (1 - a_H) \pi_{Ft+s} = \rho_s \pi > 0 \), \( s \geq 0 \)—assuming that the Foreign monetary authority responds by keeping CPI price stability, i.e., \( a_H \pi_{Ft+s} + (1 - a_H) \pi_{Ht+s} = 0 \), \( s \geq 0 \). For the reasons explained in the text, we focus on the case \( \eta = 0 \), when the LCP model is relatively straightforward to solve. With \( \eta = 0 \), the responses of the key variables are given in Table A1. In the table, since an expansionary Home monetary policy shock is obviously inefficient (all first-best deviations are equal to zero), the responses of welfare-relevant gaps coincide with the response of macro variables.

**Table A1: The effect of a monetary policy shock under LCP**

\[
\tilde{\bar{\pi}}_{t+s} = \tilde{\bar{\pi}}_t = \left(\frac{1}{1 - a_H}\right) \left[ a_H \pi_{Ht+s} + (1 - a_H) \pi_{Ft+s} \right] \pi
\]

\[
\tilde{\bar{B}}_t = (1 - a_H) \left\{ 2a_H (\phi - 1) \frac{\beta_\nu (\beta_\nu - 1)}{\beta_\nu^2} \bar{\pi} + \frac{1}{1 - \nu_1^a} \left[ \frac{1}{1 - \nu_1^a} \right] \rho_s \pi \right\}
\]

\[
\tilde{\bar{T}}_{t+s} + \tilde{\Delta}_{t+s} = \frac{1 - \nu_1^a \beta \nu_2}{1 - \nu_1^a \beta \nu_2} \tilde{\bar{\pi}}_{t+s}
\]

\[
\tilde{\bar{Q}}_{t+s} = \frac{1 - \nu_1^a \beta \nu_2}{1 - \nu_1^a \beta \nu_2} \rho_s \pi - (2a_H - 1) \left[ 1 - \frac{1 - \nu_1^a \beta \nu_2}{1 - \nu_1^a \beta \nu_2} \right] \tilde{\bar{\pi}}_{t+s}
\]

\[
\tilde{\bar{Y}}_{H,t+s} = a_H \frac{1 - \rho_\beta}{1 - \rho_\beta \rho_\pi} \rho_s \pi - (1 - a_H) \left[ 1 + 2a_H \frac{\phi_1 \nu_1^a \beta \nu_2}{1 - \nu_1^a \beta \nu_2} \right] \tilde{\bar{\pi}}_{t+s}
\]

\[
\tilde{\bar{Y}}_{F,t+s} = (1 - a_H) \frac{1 - \rho_\beta}{1 - \rho_\beta \rho_\pi} \rho_s \pi + (1 - a_H) \left[ 1 + 2a_H \frac{\phi_1 \nu_1^a \beta \nu_2}{1 - \nu_1^a \beta \nu_2} \right] \tilde{\bar{\pi}}_{t+s}
\]

\[
\tilde{\bar{D}}_{t+s} = \frac{1 - \rho_\beta}{1 - \rho_\beta \rho_\pi} \rho_s \pi + 2(1 - a_H) \tilde{\bar{\pi}}_{t+s}
\]

When markets are incomplete, a monetary shock generally causes the wealth gap \( \tilde{\bar{\pi}}_t \) to deviate from zero (recall that in the bond economy \( E_t \pi_{t+1} = \tilde{\bar{\pi}}_t \))—implying that the effects of a monetary policy shock under incomplete markets are generally different than those under complete markets. In a few notable special cases, however, the effects of monetary policy are the same as in economies with complete markets. One such case is \( \sigma = 1 \) (log consumption utility), whereas \( \tilde{\bar{\pi}}_t = 0 \), and neither capital flows \( \tilde{\bar{B}}_t \), nor the relative price misalignment, \( \tilde{\bar{T}}_{t+s} + \tilde{\bar{\Delta}}_{t+s} \), are affected by monetary policy. In this special case, a monetary easing unambiguously results in positive domestic and foreign output gaps, a positive real exchange rate gap, and a higher relative demand gap. Relative to this benchmark, if the gap \( \tilde{\bar{\pi}}_t \) is positive, the effects of monetary policy on the domestic output and the real exchange rate gaps are smaller, while the foreign output and the relative demand gaps react more. These differences reflect the fact that the misalignment \( \tilde{\bar{T}}_{t+s} + \tilde{\bar{\Delta}}_{t+s} \) is negative when \( \tilde{\bar{\pi}}_t > 0 \), implying “expenditure switching” in favor of Foreign exports. The opposite is true if the wedge is negative: the domestic output and real
exchange rate gaps react by more, while the transmission abroad is muted.

A monetary expansion can open a wealth gap in different directions, depending on elasticities. To see this, consider the following threshold expressed in terms of the trade elasticity:

$$\phi > 1 - \frac{2\alpha_H - 1 + \frac{2(1-\alpha_H)}{\beta}}{2\alpha_H \frac{\sigma \phi}{1+\sigma}} \geq 0.$$

A monetary easing brings about a positive gap \(\tilde{W}_t\) either when \(\sigma > 1\) and \(\phi\) is above the threshold shown above; or when \(\sigma < 1\) and \(\phi\) is below the threshold.

By the same token, a monetary expansion can lead to either an external surplus or an external deficit. From the second equation in the table, a sufficient condition for a monetary easing to lead to an inefficient capital outflow \(\tilde{B}_t = \tilde{B}_t > 0\) is that both \(\sigma > 1\) and \(\phi > 1\)—recall that \(\tilde{B}_t^b = 0\). A sufficient condition for inflows, \(\tilde{B}_t < 0\), instead, is \(\sigma < 1\) and \(\phi\) below the threshold. It follows that, depending on parameter values, a positive gap \(\tilde{W}_t > 0\) brought about by a monetary expansion may be associated with either outflows or inflows of capital. These in turn would attenuate (or amplify) the effects of monetary policy on domestic output and the real exchange rate (domestic consumption and foreign output).

The transmission of monetary policy under PCP is shown in Table A2. Relative to the previous table, monetary easing is now modelled as an increase in domestic PPI inflation \(\pi_{Ht+s} = \rho^s \pi > 0\), \(s \geq 0\), again under the assumption that the foreign monetary authority responds by keeping PPI price stability, i.e., \(\pi^s_{Ft+s} = 0\), \(s \geq 0\) and \(\eta = 0\).

<table>
<thead>
<tr>
<th>Table A2: The effect of a monetary policy shock under PCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tilde{W}_{t+s} = \tilde{W}_t = \frac{(2\alpha_H \phi - 1)\sigma - (2\alpha_H - 1)}{1 - (2\alpha_H \phi - 1)\sigma - (2\alpha_H - 1)} \tilde{W}_t)</td>
</tr>
<tr>
<td>(\tilde{B}_t = (1 - a_H) \frac{(2\alpha_H \phi - 1)\sigma - (2\alpha_H - 1)}{1 - (2\alpha_H \phi - 1)\sigma - (2\alpha_H - 1)} \beta \pi)</td>
</tr>
<tr>
<td>(\tilde{Q}<em>{t+s} = (2\alpha_H - 1) \tilde{Q}</em>{t+s} = (2\alpha_H - 1) \left[ \frac{1}{1 - (2\alpha_H \phi - 1)\sigma - (2\alpha_H - 1)} \rho^s \pi - \tilde{W}_{t+s} \right])</td>
</tr>
<tr>
<td>(\sigma \tilde{Y}<em>{H,t+s} = \left[ 1 + 2\alpha_H (1 - a_H) (\sigma \phi - 1) \right] \frac{1}{1 - (2\alpha_H \phi - 1)\sigma - (2\alpha_H - 1)} \rho^s \pi - (1 - a_H) \frac{2\alpha_H (\sigma \phi - 1) + 1}{\sigma} \tilde{W}</em>{t+s})</td>
</tr>
<tr>
<td>(\sigma \tilde{Y}<em>{F,t+s} = -2\alpha_H (1 - a_H) (\sigma \phi - 1) \frac{1}{1 - (2\alpha_H \phi - 1)\sigma - (2\alpha_H - 1)} \rho^s \pi + (1 - a_H) \frac{2\alpha_H (\sigma \phi - 1) + 1}{\sigma} \tilde{W}</em>{t+s})</td>
</tr>
<tr>
<td>(\sigma \tilde{D}<em>{t+s} = \frac{(2\alpha_H - 1)}{(1 - \alpha_H)(1 - \sigma \phi)} \rho^s \pi + 2(1 - a_H) \tilde{W}</em>{t+s}).</td>
</tr>
</tbody>
</table>

An expansionary Home monetary policy shock also causes the gap \(\tilde{W}_t\) to deviate from zero under PCP: under incomplete markets, the effects of a monetary policy shock do not coincide with those under complete markets. Again there are a few notable exceptions: under PCP, the special case in which monetary policy affects neither \(\tilde{W}_t = 0\) nor capital flows arises when \(\phi = \frac{1 + 2\alpha_H^{-1}}{2\alpha_H}\); if \(\sigma = 1\), then, this requires \(\phi = 1\)—a Cobb-Douglas consumption aggregator. In this special case, just like under complete markets, a monetary easing unambiguously results in a higher domestic output, relative demand and real exchange rate gaps. However, foreign output is affected only when \(\sigma \phi \neq 1\), and increases if \(\sigma \phi < 1\), namely, when goods are Edgeworth-complement. Relative to the benchmark with \(\phi = \frac{1 + 2\alpha_H^{-1}}{2\alpha_H}\), similar to LCP, a positive (negative) wealth gap means that the effects of monetary policy on domestic output and the real exchange rate are smaller (larger) than under complete markets, while domestic consumption and foreign output react more (less). These effects reflect the fact that the response of the terms of trade,
\( T_t \), is also smaller (larger), implying a weaker (stronger) expenditure switching in favor of Home goods. Specifically, the wealth gap is positive when the following conditions hold:

\[
\phi > \frac{1 + \frac{2\mu - 1}{\sigma}}{2a_H}, \quad \phi < \frac{1 - \frac{2(1-\mu)}{\sigma}}{2a_H}.
\]

From the second equation in the table, it is apparent that, for a monetary easing to lead to an inefficient capital outflow on impact, \( \tilde{B}_t > 0 \), it must be the case that \( \phi > \frac{1 + \frac{2\mu - 1}{\sigma}}{2a_H} \). Otherwise, it leads to capital inflows. Therefore, also under PCP a positive \( \tilde{W}_t > 0 \) may be associated with either outflows or inflows of capital, in turn attenuating or amplifying the effects of monetary policy on domestic output and the real exchange rate (domestic consumption and foreign output).

### 7.2 Costly intermediation and stationarity of net foreign assets

Our results so far have been derived in a specification of the model in which both \( B_t \) and \( C_t \) are not stationary. In this subsection, we show that nonstationarity does not play any substantive role. In the literature, a standard approach to ensure that \( B_t \) is stationary in bond economies is to assume that its changes are subject to some (portfolio) adjustment costs; Gabaix and Maggiori [2015] have recently shown that this sluggish adjustment can result from costly intermediation of cross-border flows when financial intermediaries operate under borrowing constraints. In our framework, a simple way to capture the same idea is to posit deviations from the uncovered interest rate parity condition that are proportional to net foreign assets:

\[
E_t C_{t+1} - C_t = -\delta B_t.
\]

With this modification, the solutions for \( B_t \) and \( C_t \) in the CO economy become:

\[
\tilde{B}_t = \gamma_1 B_t + (1 - a_H) \sum_{j=0}^{\infty} \gamma_2^{-j-1} E_t \left[ (\tilde{C}_{t+1+j} - \tilde{C}_{t+1+j}^*) - (\tilde{C}_{C,t+1+j} - \tilde{C}_{C,t+1+j}^*) \right],
\]

\[
\tilde{W}_t = \left( \frac{\tilde{B}_{t-1} - \beta \tilde{B}_t}{(1 - a_H) \beta} \right) - (\tilde{C}_{t} - \tilde{C}_{C,t})
\]

\[
= - \left[ (\tilde{C}_{C,t} - \tilde{C}_{C,t}^*) + \sum_{j=0}^{\infty} \gamma_2^{-j-1} E_t \left[ (\tilde{C}_{t+1+j} - \tilde{C}_{t+1+j}^*) - (\tilde{C}_{C,t+1+j} - \tilde{C}_{C,t+1+j}^*) \right] \right] - \frac{\gamma_1 - \beta}{(1 - a_H) \beta} \tilde{B}_{t-1}.
\]

where \( \beta < \gamma_1 < 1 < \gamma_2 \) are the roots of the characteristic equation associated with the above second-order difference equation:

\[
\beta \gamma^2 - (1 + \beta + \beta \delta) \gamma + 1 = 0.
\]

Both \( \tilde{W}_t \) and \( \tilde{B}_t \) are now stationary, but still functions of exogenous shocks only, so the optimal targeting rules are the same as those derived above under both LCP and PCP. Therefore, optimal monetary policy will react in the same way to a capital inflow, by tightening under LCP and easing under PCP (although of course with a different strength). Clearly, setting
δ = 0 in the last expression leads to \( \gamma_1 = 1 \) and \( \gamma_2 = 1/\beta \), which yields expressions (29) and (30) above.

### 7.3 Determinants of the optimal Home monetary stance under PCP with a non-unitary elasticity

To prove the result in Subsection 5.2 in the text, we start noting that, for low enough trade elasticities, the impact response of relative consumption to capital inflows,

\[
\tilde{C}_t = \frac{2(1 - a_H) \bar{W}_t}{2a_H (\phi - 1) + 1} \left\{ \begin{array}{l} 2a_H (\phi - 1) + 1 + \frac{(2a_H - 1)}{\beta x_2} \\ \text{sign}(-B_t) \end{array} \right\},
\]

switches sign and becomes negative. This will be the case for values of \( \phi \) below the threshold

\[
\phi \leq \frac{(2a_H - 1) (\beta x_2 - 1)}{2a_H} \leq \frac{(2a_H - 1)}{2a_H} < 1.
\]

where an expansionary monetary policy stance is motivated by an inefficiently low domestic demand. The monetary boosts causes the output gap to become consistently positive. On impact, the optimal response can be written as follows

\[
\tilde{Y}_{H,t} = -\frac{(1 - a_H) \bar{W}_t}{2a_H (\phi - 1) + 1} \left\{ \begin{array}{l} (\beta x_2 - 1) [2a_H (\phi - 1) + 1]^2 + \frac{\beta x_2}{4a_H \phi (\phi - 1)} \\ \text{sign}(\bar{B}_t) \end{array} \right\},
\]

where the term outside the curly brackets has the same sign as capital inflows \( \bar{B}_t \). Therefore, whether the optimal policy turns the output gap positive or negative depends on the sign of the term in curly brackets, in turn a function of \( \phi \). For \( \phi < 1 \), the second term in the curly brackets \( (4a_H^2 \phi (\phi - 1)) \) is always negative, and converges to zero as \( \phi \rightarrow 0 \). The first term is a square and thus always positive, but converges to zero as \( \phi \) converges to the cutoff point \( \frac{2a_H - 1}{2a_H} \) from above; it then becomes increasingly positive for lower values of elasticities. This implies that, under the optimally expansionary monetary policy, there is a range of elasticities around the cutoff point for which the output gap is positive. This range becomes larger, the closer \( \beta x_2 \) is to 1, i.e., the stickier prices are, since the first term in curly brackets goes to zero. Importantly, this is in contrast to the natural rate allocation in which the output gap is negative for any value of the elasticity (see Table 5).

An expansionary stance dictated by concerns with domestic stabilization of course exacerbates the real misalignment. Under the optimal policy, the response of the terms-of-trade gap is

\[
\tilde{T}_t = \frac{2\tilde{Y}_{H,t} - (2a_H - 1) \bar{W}_t}{4a_H (1 - a_H) (\phi - 1) + 1} = \frac{-\bar{W}_t}{2a_H (\phi - 1) + 1} \left\{ \begin{array}{l} \beta x_2 [2a_H (\phi - 1) + 1] \\ \text{sign}(\bar{B}_t) \end{array} \right\},
\]

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where the term in curly brackets is positive if
\[
\phi \geq 1 + \frac{(2a_H - 1)(\beta x_2 - 1)}{\beta x_2} < 1,
\]
and is negative if \( \phi \leq \frac{2a_H - 1}{2a_H} \). It follows that, for \( \phi \leq \frac{2a_H - 1}{2a_H} \), the optimal Home monetary expansion causes the terms of trade to be excessively weak.

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