ADEMU WORKING PAPER SERIES

Self-Fulfilling Runs and Endogeneous Liquidity Creation

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March 2018

WP 2017/090

Abstract

This paper incorporates endogenous money creation into the liquidity mismatch problem of Diamond and Dybvig (1983). We characterize a nominal economy where demandable deposits are created through lending. Depositors use sight deposits to buy consumption goods and the banks manage reserves to clear payments and to offset liquidity risk. We show that deposit contracts are suboptimal in terms of liquidity risk-sharing. We also observe that the self-fulfilling run depends on the refinancing rate of the central bank. Our analysis emphasizes the importance of effective lender of last resort policies to prevent expectational banking panics.

JEL Codes: G21, E42
Keywords: Bank runs, inside money, risk-sharing

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Acknowledgments

We would like to thank Henrique Basso, Xavier Freixas and Joachim Jungherr for very valuable comments on previous versions of this paper. Rodríguez Mendizábal would like to acknowledge the financial support from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563) and through grant ECO2016-76734-P (AEI/FEDER, UE) as well as that of the Generalitat de Catalunya through Grant 2017 SGR 1571. He is also grateful for financial support from the ADEMU project, "A Dynamic Economic and Monetary Union", funded by the European Union's Horizon 2020 Program under grant agreement No 649396. All errors are our responsibility.

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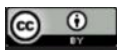
1   Introduction

The prevailing view in banking theory is, as Diamond and Dybvig [12] pointed out, that "illiquidity of assets provides the rationale both for the existence of banks and for their vulnerability to runs".[1] According to this theory, banks channel pre-existing real assets from savers to finance illiquid entrepreneurial projects. In this intermediation process, banks create liquidity, that is, they offer liabilities (deposits) that are immediately available while the assets they hold (loans) are not. In other words, banks promise full recovery of deposits at anytime although disposing of their assets before maturity may only come at a cost. This liquidity mismatch between redeemable deposits and illiquid assets explains the fragility of banks.

In a modern economy, depository institutions create the commitment to implement a payment system by which transactions are cleared. These banks endogenously extend credit whose liability counterpart is the production of generally acceptable means of payment in the shape of different types of nominal deposits. Within this process, payments and withdrawals usually take place as electronic transfers and the settlement of these flows needs of outside money in the form of cash or central bank reserves.[2]

In this paper we reformulate the model of Diamond and Dybvig [12] (DD for short) to incorporate a set of elements aimed at reproducing these basic features of a modern monetary system. First, instead of sticking to the traditional description of banks as financial vehicles that take real assets from savers to lend them to ultimate borrowers, effectively intermediating pre-existing deposits, we consider that when banks originate a new loan they are creating nominal inside money and purchasing power. Thus, the intermediation process performed by banks starts on the asset side of their balance sheets. Second, the maturity and liquidity mismatch between bank assets and liabilities arises automatically when a loan is originated. This is because the counterpart to the provision of a long term loan is the creation of an overnight liability in the form of a disposable deposit. In this sense, the liquidity risk faced by the banking institution is due to the transfer of funds between banks, which is solved by managing a demand for outside money produced by the central bank. Finally, the vast majority of bank loans cannot be recalled nor banks have any say about the liquidation decision of the investment projects pursued by borrowers.

We show how including these elements have important implications on the equilibrium of the model and its predictions on financial fragility as compared with traditional banking theories based on the seminal work of DD. On this respect, DD showed that, with no aggregate uncertainty, (i) banks can reproduce the optimal allocation among depositors with random liquidity needs, and (ii)

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[1] See also Bryant [6], Holmstrom and Tirole [23], and Diamond and Rajan [13],[14].

[2] In practice, central banks support a payment system through which transactions can be easily settled. According to the BIS [5], the pivotal role of central bank money in payment systems "*reflects the layered architecture of financial systems, whereby private individuals and non-financial businesses hold (part of) their liquidity in banks, and banks in turn hold (part of) their liquidity in the central bank*".

there also exists a self-fulfilling equilibrium wherein uninsured depositors rush to withdraw their savings from the banking system. When this happens, banks are unable to honor their repayment obligations and become insolvent. In contrast, we show that the uniqueness of equilibrium depends on the refinancing policy of the central bank. In particular, the possibility of a run only appears at relatively high refinancing rates. Furthermore, we also show that equilibrium is always inefficient from a social point of view.

A few papers have explored the implications of introducing inside money over the withdrawal incentives of depositors and the optimality of deposit contracts in terms of liquidity risk-sharing.[3] As we stated above, financial contracts are usually denominated in nominal terms, and withdrawals do not imply that money is per se converted into cash and drained out of the banking system. Skeie [32] considers these characteristics and shows the existence of a unique and efficient equilibrium when nominal deposits are repayable in inside money. The non-bank run equilibrium is explained by price adjustments in the goods' market only when banks choose the optimal amount of liquidity that is stored in the economy. In the same line, Allen et al. [3] incorporate fiat money issued by the central bank into Allen et al. [4] and find that the efficiency and uniqueness of the equilibrium also holds with aggregate return uncertainty, aggregate liquidity shocks, and bank specific liquidity shocks. Unlike these important contributions on this subject, we find that the uniqueness of equilibrium does not necessarily depend on nominal prices adjusting in response to a run. We connect the self-fulfilling run with the central bank response to the panic. Indeed, we state that expectational runs of the Diamond and Dybvig type can be prevented by effective lender of last resort policies that offset the incentives of depositors to coordinate in a run. In addition, we cast doubt about the capacity of nominal deposit contracts to implement the optimal amount of real liquidity. A key feature of the present work is the lack of commitment of depository institutions to ensure future consumption needs of depositors. With nominal deposit contracts repayable in cash, banks cannot set in the present credible promises about the real value of future payoffs because of their inability to set prices. Furthermore, the implementation of monetary policy by the central bank in our model reproduces actual institutions we find in our economies.

These nominal banking models, including ours, do not support the idea that coordination failures leading to a bank run can be explained exclusively by illiquidity itself. The view of purely self-fulfilling runs was endorsed by Friedman and Schwartz [17]'s explanation of the bank panics that occurred in the United States up to the 1930s but was disputed by Gorton [21] and Calomiris and Mason [8], [9]. More recently, the collapses of Bearn Stearns and Lehman Brothers (Lucas and Stokey [24]), and the run of the UK bank Northern Rock on

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[3]Other papers have developed nominal frameworks to explore other issues for financial stability. Allen and Gale [1] introduce fiat money in a model of banking and show that variations in the price level allow nominal debt to become effectively state contingent so that risk-sharing is improved. Diamond and Rajan [15] find that nominal contracts cannot prevent bank runs when there is idiosyncratic risk on the bank's asset side caused by delays in asset returns.

2007 (Shin [31]) do not seem to be related to a coordination failure, so additional elements on the bank's fundamentals are required to explain bank instability.[4] In our case, this element is the refinancing policy of the central bank.

An interesting feature of the model described here is that it provides an explicit bridge between banking and monetary theories. Typically, banking models are set in real terms and abstract from the ability of depository institutions to endogenously expand or contract the size of their balance sheet.[5] Arguably, one of the drives of these expansions and contractions, and of their potential effects on economic activity and price determination, is the monetary policy stance of the central bank. Because these banking models are built in real terms and do not include a monetary authority, they are not designed to analyze these interactions. On the other hand, monetary models typically abstract from an active banking sector.[6] These models therefore are not able to incorporate the endogenous management of the liabilities of depository institutions into the analysis of the money creation process.

The paper is structured as follows. Section 2 describes the real economy and connects with the DD model. Section 3 reviews the main ingredients of a modern monetary system which will be included in the nominal model below. Section 4 introduces money in the model and shows the main results. Finally, section 5 concludes.

2   The real setup

The real model reproduces the maturity transformation problem described in DD but introducing labor in the production technology. As it will be clear in Section 4, this modification does not alter the production possibility frontier in any respect but allows for the existence of inside money.

2.1   The environment

Consider an economy characterized by a circle with measure 1 and three dates, indexed by $t = 0, 1, 2$. Locations are continuously distributed over the circle. On each location there is a continuum of identical risk averse households with measure 1. Households are composed of a worker and an entrepreneur. Each worker is endowed in period 0 with a unit of time, whereas entrepreneurs have access to a risk-free productive technology.

Households face uncertainty about future liquidity needs in period 0. With probability $\lambda \in (0,1)$ the household becomes impatient ($h = 1$) and prefers to consume in period 1, while with probability $(1 - \lambda)$ the household is patient ($h = 2$) and consumes at $t = 2$. Once households observe types at the beginning

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[4]Goldstein and Pauzner [19], and Rochet and Vives [29] provide the theoretical foundations to fill the gap between both literatures.

[5]See, for example, the models described in Allen and Gale [2] or Freixas and Rochet [16].

[6]In fact, the workhorse model for monetary analysis, the neokeynesian model, dispose not only of banks but also of money all together. See, for example, the models discussed in Gali [18] or Woodford [34].

of period 1 they make choices and obtain a utility $u(c^h)$, where c^h denotes the consumption of a household of type $h \in \{1, 2\}$ at period $t = h$.[7] The function $u(c)$ has the following properties

$$u'(c) > 0, u''(c) < 0, \lim_{c \to 0} u'(c) = \infty, \text{ and } \lim_{c \to \infty} u'(c) = 0.$$

We assume further that the coefficient of relative risk aversion satisfies

$$-c\frac{u''(c)}{u'(c)} > 1 \tag{1}$$

everywhere.

The productive technology transforms each unit of labor employed at $t = 0$ into $\rho > 1$ units of the good at $t = 2$. If a fraction $y \in [0, 1]$ of the production is interrupted at $t = 1$ it will produce a scrap value equal to y. The remaining fraction left until maturity of the production process will yield $\rho(1-y)$ in period 2. In addition to the productive technology, households have also access to storage, $x \in [0, 1]$, without any cost. Obviously, nobody will store anything from $t = 0$ to $t = 1$ since storing is dominated by the production technology in period 0. It may be the case, however, that storage could be used between periods 1 and 2.

A household of type $h \in \{1, 2\}$ faces the problem of choosing in period 1 (i) the fraction y^h of the productive technology to be liquidated, (ii) the amount x^h to store between $t = 1$ and $t = 2$, and (iii) consumption, c^h. Clearly, if households lived in autarky, they would choose to liquidate the whole project in the event of becoming impatient, $y^1 = 1$, and consume $c^1 = 1$ of the good at $t = 1$, storing nothing, $x^1 = 0$. On the other hand, patient households will liquidate none of the project, $y^2 = 0$, and store nothing, $x^2 = 0$, at $t = 1$, consuming $c^2 = \rho > 1$ in period 2.

2.2 Risk sharing

If types were publicly observable at $t = 0$, it is easy to see that a planner who verifies types would choose not to store, while determining c^1 and c^2, together with the aggregate fraction of the productive technology to be liquidated prematurely, y, to maximize

$$\lambda u(c^1) + (1 - \lambda)u(c^2) \tag{2}$$

subject to the feasibility constraints

$$\lambda c^1 \leq y \tag{3}$$

and

$$(1 - \lambda)c^2 = (1 - y)\rho. \tag{4}$$

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[7]Throughout the paper, subscripts will refer to periods ($t = 1, 2$) and superscripts to types ($h = 1, 2$).

The first order conditions of this problem to determine the optimal choice $\{c^{1*}, c^{2*}, y\}$ are

$$u'(c^{1*}) = \rho u'\left(c^{2*}\right),\qquad(5)$$

together with the two resource constraints (3) and (4). These expressions characterize the efficient risk-sharing for this economy and are equivalent to the ones in DD. Since $\rho > 1$ and because of the degree of risk aversion considered in (1), it turns out that $1 < c^{1*} < c^{2*} < \rho$, which means households would prefer to share *ex ante* the risk associated with the timing of consumption.

2.3   A time bank

Insurance against consumption uncertainty could be provided by introducing a contingent time bank. Workers can deposit their time at $t = 0$ in a time depository institution. The time bank then designs a contingent deposit contract at $t = 0$ providing c^{1b} units of consumption at $t = 1$ to those withdrawing their deposits in that period, or c^{2b} units at $t = 2$ for those who wait to withdraw at that period. The time bank then puts to work all the depositors in the productive technology in period 0 and chooses the aggregate liquidation of the productive investment, y^b, to maximize the expected utility of depositors

$$\lambda u(c^{1b}) + (1 - \lambda)u(c^{2b})\qquad(6)$$

subject to the feasibility constraint

$$\lambda c^{1b} \leq y^b,$$

and

$$(1 - \lambda)c^{2b} \leq (1 - y^b)\rho.$$

Obviously, this problem yields the optimal allocation found in the planner's problem above.

This time deposit contract $\{c^{1b}, c^{2b}\}$ provides efficient risk-sharing because it determines implicit contingent wages to be paid to households at $t = 1$ depending on their realized types. The $t = 1$ equivalent contingent wages depositors are receiving for providing time at $t = 0$ are

$$w_1^1 = \frac{y^b}{\lambda} = c^{1b} \ \text{ and } w_1^2 = \frac{1 - y^b}{1 - \lambda} = \frac{c^{2b}}{\rho},$$

for workers belonging to impatient and patient households, respectively. Because of the assumed degree of risk aversion (1), it must be the case that

$$w_1^1 = \frac{y^b}{\lambda} > 1 > \frac{1 - y^b}{1 - \lambda} = w_1^2,$$

which allows to obtain the first-best allocation.

The contingent time deposit contract supports, however, a suboptimal equilibria. The bank designs such contract inferring that there will be λ withdrawals

in period 1. As in the seminal work of DD, patient households have incentives to withdraw before time if they anticipate that the bank will be forced to liquidate a significant amount of its long term investment to service the increasing demand of early withdrawals. Let $\bar{\lambda}$ be the minimum amount of early withdrawals in period 1 to have a self-fulfilling run. This threshold satisfies

$$\bar{\lambda} = \frac{y^b}{\rho(1 - y^b) + y^b}.$$

If late consumers expect the fraction of early withdrawals to be larger than $\bar{\lambda}$, it will be optimal for them to withdraw at date 1 and store the proceeds until $t = 2$. In this second equilibrium the bank suffers a run since anyone who waits until the last period will get nothing.

3   Main features of a modern monetary system

This section reviews some of the features of modern depository institutions not included in traditional models of banking that we believe are crucial to understand banks' contribution in our economies.

3.1   The production of loans and deposits

The first challenge of traditional banking models rests on the way the production of loans and deposits is described. In those models, this process starts on the liability side of banks' balance sheets when a saver deposits some pre-existing real assets. The bank then transfers those resources to a borrower who puts them into some productive use.

This description is at odds with current procedures in depository institutions. As a matter of practice, commercial *banks create money*, in the form of bank deposits, when making new loans. This is how the bulk of deposits we use to make payments is originated.[8] If you could trace back the life of a deposit someone has recently transferred to you, invariably it was born with a loan to someone somewhere in the past. This view in which money is created through credit is shared both by academicians (see Goodhart [20]), as well as central bankers (see, among others McLeay et al. [25], from the Bank of England, Holmes [22], from the Federal Reserve Bank of New York, or Constancio [11] from the ECB), regulators (see Turner [33]), and market practitioners (see Sheard [30]).

This deposit creation power is the distinguishing characteristic of depository institutions.[9] Of course, this ability to create its own liabilities on the spot does not provide banks with an unlimited capacity to expand their balance sheets. This is because, among other constraints, the process of loan and money creation exposes banks to a number of risks. Among the exposures faced by

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[8] See McLeay et al. [25].

[9] The accounting conventions that allow banks to create money out of nothing are described in Werner [35] and [36].

banks we consider liquidity risk. This risk is associated with having enough generally acceptable assets to cover the net flow of payments ordered by the bank's clients. Notice that the process of endogenous money creation through loan provision endorsed in this paper automatically exposes banks to both a maturity as well as a liquidity mismatch between assets and liabilities. This is because the counterpart of a long term loan is the creation of demandable short term deposits. In this sense, agents do not ask for a loan to sit on the funds created but to make a payment they lacked the funds for. Thus, deposits will, and are designed to, exchange hands at the very moment they are created. Loans, on the contrary, are not an asset banks can dispose of that easily. Banks thus need an asset with the same degree of immediacy as the (net flow of) deposits they hold in the liability side of their balance sheets. Reserves, in the form of current accounts at the central bank, are these assets. Because reserves need to be borrowed, they are onerous to obtain. As producing loans necessarily means obtaining reserves to service the deposit these loans create, the borrowing cost of those reserves also constrain how much banks want to expand their balance sheets.[10]

3.2 The endogenous nature of financial intermediation

The second challenge of traditional models of banking has to do with the prede-termined nature of the *volume of intermediation*. In traditional models, savers hold a pre-existing volume of savings, in the form of a stock of real assets, that needs to be transferred to borrowers. This means that the amount of interme-diation that takes place in these models, that is, the size of the balance sheets of commercial banks, is bounded by the amount of already existing savings to be transferred.

Unlike this description of traditional models of banking, our endogenous view of the intermediation process performed by banks is not constrained by existing savings or deposits. Apart from the constraints mentioned in the previous sec-tion, banks could produce more financial assets, to be held by the nonfinancial sector, as long as they expect the corresponding credit that originated those assets to be paid in the future. The purchasing power created with these new loans could be used to pay for existing real assets as well as for new consumption or investment goods or even for the purchase of other financial assets.

This endogenous view of bank intermediation has important implications for the way we should analyze bank's balance sheet dynamics and its connections to money creation. If we look at depository institutions as a whole, when a bank decides to expand its balance sheet by granting a loan, effectively it is increasing the asset side of depository institutions until the time the loan matures, provided the bank keeps the loan in its books for the whole time. This means the banking sector needs to maintain a matching liability also throughout that period. As mentioned before, at the very moment the loan is granted, the matching liability

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[10]A second exposure for banks is solvency risk given that there is the possibility that the loans banks provide are not repaid and the value of assets could drop below the value of their liabilities. To stay as close as possible to DD we do not consider this risk here.

is a sight deposit which quickly is transferred to another party as a payment for an exchange. The party receiving these funds now has to decide what to do with them. This portfolio choice will split the funds that originally started as sight deposits between liquid or illiquid bank liabilities. The former are accepted as a general means of payment and included in a broad monetary aggregate while the latter are not. Throughout the life of the loan that originated these bank liabilities, the different owners of those funds will be transferring them and splitting them in different ways, changing the amount and composition of broad monetary aggregates.

Notice this split of the particular liabilities of the bank associated with a particular loan will depend both on the demand for those assets by customers as well as on the supply of those liabilities by the bank themselves. This way, the funding problem of expanding the balance sheet of a bank is summarized by the ability of the bank to convince someone to hold a matching liability until the initial loan matures and to manage the liquidity risks associated with each type of liability. The only way out of this service obligation is to take the original loan out of the balance sheet either by selling it out directly or through securitization.

3.3   The role of reserves and the implications of bank runs

The third challenge of the traditional view of banking is the *role reserves play* in the process of money creation. In the traditional view, reserves are just deposited assets that are left idle or invested in an inferior technology that allows full recovery at anytime. In reality, reserves, in the form of current accounts at the central bank, are a completely different object than customer's deposits at commercial banks or the loans these banks provide to their borrowers. Reserves are produced by the central bank, while loans and deposits are produced by commercial banks. Banks maintain reserves for two reasons. The first reason is to satisfy depositor's payments demand. Whenever a client wishes to make a payment to be transferred to another bank, this payment is usually done with reserves. The second reason is to satisfy reserve requirements wherever these requirements are in place. Thus, reserve demand is driven both by regulation as well as by the netting of payments derived from the loan and deposit creation to finance economic activity. Reserve supply, on the other hand, is characterized by the monetary policy stance of the central bank. This monetary policy stance is typically defined as a target on very short rates (i.e. overnight) in money markets. In implementing its monetary policy, monetary authorities are usually ready to supply, at the target rate, as much reserves as depository institutions demand. When the monetary policy stance changes, the central bank modifies its interest rate target but still "reads" the amount of reserves needed to support that new target from demand by commercial banks.

An important conclusion can be drawn from the description in the previous paragraph. In modern monetary systems the amount of reserve holdings should not constrain loan and deposit production by commercial banks. As long as central banks are willing to supply reserves at the specified refinancing

rate, reserve demand is determined by reserve requirements and the netting of payments both of which depend on loan and deposit creation.[11]

At this point it is important to connect the notion of reserves with the intermediation performed by commercial banks and the risk of facing a run. As mentioned above, because in traditional models banks intermediate real assets that are put directly into an illiquid production process, reserves are just deposited assets that are left idle or invested in an inferior technology that allows full recovery at anytime. In the event that depositors demand funds above the reserves previously accumulated by the bank, this financial institution will be required to force borrowers to repay back the loan prematurely by liquidating their production projects with the corresponding efficiency costs.

In our view of the intermediation process, when depositors do not trust a bank they will demand their liquid deposits to be converted to a financial asset produced by a different financial institution. This could be cash or a deposit in a different bank. To honor this convertibility promise, banks need to borrow reserves either from the central bank or from other depository institutions or else, sell existing assets in exchange for these reserves. However, liquidating an asset, a loan for example, is not the same as liquidating the productive investment this loan has financed, nor it means recalling the loan. This is for several reasons. First, the vast majority of loans are noncallable. This is the case of basically all mortgages and, according to the Board of Governors of the Federal Reserve System, of 87.5 percent of all C&I loans.[12] Second, liquidating a loan means selling in the market the right to the future cash flows the loan generates. The borrower will continue with his/her investment with the loan payments now accruing to a different creditor. The anticipation that banks will not be able to honor these promises, either because they will not raise enough liquid funds in the market or from the central bank or because they will sell assets at a significant discount, is the reason why holders of its short term debt run the bank. But this fact, by itself, does not mean that real investments are affected by the run as the traditional view contends. Any effect on real investment decisions should indirectly come from general equilibrium effects through prices.

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[11]An important element to the description in the main text is the possible connections between the amount of reserves and the amount of deposits. These connections are exemplified by the Treasury accounts at the central bank and cash holdings by the nonfinancial sector. For example, as we withdraw cash from ATMs, banks use their reserves to get the banknotes needed to replenish their cash machines. In normal times, however, these movements are not significant and the central bank usually accommodates them to restore the levels of reserves held previously by banks.

[12]See Board of Governors of the Federal Reserve System, Statistical Release E2. This figure is the average fraction of noncallable loans, weighted by volume, between the second quarter of 1997 and the first quarter of 2003. In 2003 the Board stopped including the amount of C&I loans that are callable because, representing a small fraction of total loans, their behavior did not significantly differed from loans which are not callable. See Board of Governors of the Federal Reserve System [7].

4 The nominal economy

4.1 The setup

In this section we add nominal deposit contracts and a flow of nominal funds into the real economy of Section 2. To do so, we incorporate the ideas described in Section 3. In particular, we include banks that (i) create endogenous money in the form of deposits when providing loans, and (ii) manage central bank reserves to honor the convertibility promise associated with deposits. We also separate the liquidation decision of financial positions from that of real investments. We then explore the extent to which these nominal contracts achieve optimal risk-sharing and study whether self-fulfilling panics do occur.

Production technologies and preferences remain equal as described in Section 2. That is, entrepreneurs use labor hired at $t = 0$ in the productive technologies whose proceeds will be collected at either $t = 1$ or $t = 2$. To introduce a role for banks, assume entrepreneurs hire workers in a competitive labor market. Furthermore, when workers are hired, at $t = 0$, these entrepreneurs lack the credibility to convince those workers they will get paid in the future, when production takes place either at $t = 1$ or $t = 2$.

The function of banks under this setting is to intermediate between households in this payment process. Assume each location is served by a continuum of banks with measure 1. The timing of events is as follows. At $t = 0$, entrepreneurs borrow inside money, W, from one of the banks located in the same location they live in. This loan produces a double entry in the bank's balance sheet. On the asset side, the bank annotates the right associated with the loan taken by the entrepreneur. On the other hand, means of payments are created, and the liability side reflects the right of the entrepreneur to dispose of those funds to make payments. The interest rate of these loans is i^l to be paid at the end of period $t = 2$

Still at time $t = 0$, the loan is used by entrepreneurs to pay workers in advance for their labor services. Notice the introduction of banks solves the commitment problem of the entrepreneurs as wages are paid in advance. Furthermore, it also solves any commitment problem on the part of workers as the receipt from these transfers is proof of the wage payments and, therefore, can be used by entrepreneurs to claim the workers' labor services. Additionally, deposits are homogeneous units of account that can be used by households to buy goods from entrepreneurs at either $t = 1$ or $t = 2$. When households pay for consumption goods, they transfer these deposits to an entrepreneur. A nominal price for consumption goods will be formed as deposits are exchanged for goods. Entrepreneurs then use these revenues from selling the goods they produce to pay back the loan they asked for at period 0. That is the reason these deposits are accepted back by entrepreneurs in exchange for consumption goods. Thus, with the introduction of depository institutions, loan and deposit creation by banks are used to bridge the intertemporal gap between the wage and goods payments in this economy.

Once households receive income W from the payment of wages, they make

a portfolio choice by which they split those funds between liquid and illiquid assets. Liquid assets, S_1, have the form of a sight deposits disposable at any time. Illiquid assets, D, have the form of a time deposits that pays off at $t = 2$. At $t = 0$ households face the constraint

$$S_1 + D \leq W. \tag{7}$$

At the beginning of the interim period, $t = 1$, households receive the remuneration from their sight deposits at the interest rate i_1^s. Then, the liquidity shock realizes and households learn whether they are of the patient or impatient type. At this point, households have the opportunity to buy goods. This means these households will transfer part of their liquid funds to entrepreneurs in exchange for goods produced at $t = 1$. These liquid funds consists of the gross sight deposits, $(1 + i_1^s)S_1$. Banks also allow households to liquidate part of their time deposits, Δ, at a cost. The bank will charge an early liquidation fee $0 \leq \phi \leq 1$ per unit of liquidated time deposit. The parameter ϕ is a measure of how illiquid these other bank liabilities are as compared with sight deposits. Thus, the total amount of liquid funds to be used for goods purchases is $(1+i_1^s)S_1+(1-\phi)\Delta$. Let P_t be the nominal price of goods in period t. Households then could buy, at this price, goods for consumption, c, or for storage, x.

Because at the time households make decisions on period $t = 1$ they already know their type, their choices will depend on of whether the household is impatient, $h = 1$, or patient, $h = 2$. Goods purchases at $t = 1$ are then subject to the following cash in advance constraints

$$P_1 c^1 + P_1 x^1 \leq (1 + i_1^s)S_1 + (1 - \phi)\Delta^1 \tag{8}$$

and

$$P_1 x^2 \leq (1 + i_1^s)S_1 + (1 - \phi)\Delta^2, \tag{9}$$

for impatient and patient households, respectively. Notice patient households do not buy goods for consumption but could decide to buy them for storage.

Once goods purchases take place, the household makes another portfolio choice allocating $t = 1$ resources into either cash, M^h, or sight deposits, S_2^h.[13] The available resources are whatever funds are left from the purchases of goods plus the revenues from selling goods obtained from liquidating part of the productive technology, y^h. Thus, in making these choices, households face the portfolio constraint

$$M^1 + S_2^1 \leq (1 + i_1^s)S_1 + (1 - \phi)\Delta^1 - P_1\left(c^1 + x^1\right) + P_1 y^1 \tag{10}$$

and

$$M^2 + S_2^2 \leq (1 + i_1^s)S_1 + (1 - \phi)\Delta^2 - P_1 x^2 + P_1 y^2. \tag{11}$$

At the beginning of period $t = 2$, only patient households are buying goods. Therefore, they face the cash-in-advance constraint

$$P_2 c^2 \leq M^2 + (1 + i_2^s)S_2^2 + (1 + i^d)\left(D - \Delta^2\right). \tag{12}$$

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[13]Because the economy stops at $t = 2$, banks only supply sight deposits at $t = 1$.

In this case, liquid funds are the cash hoarded from period $t = 1$, M^2, plus the return on sight deposits, $(1 + i_2^s)S_2^2$, and on the remaining time deposits which are due precisely at period $t = 2$, $(1 + i^d)(D - \Delta^2)$. Then, after these goods are bought, the household has to pay back the original loan taken by the entrepreneur, so, it must be the case that

$$(1+i^l)W \leq M^2 + (1+i_2^s)S_2^2 + (1+i^d)(D - \Delta^2) - P_2c^2 + P_2x^2 + P_2(1-y^2)\rho. \quad (13)$$

That is, the loan needs to be repaid with whatever resources are left from buying goods, plus revenues from selling goods obtained either from the storage performed at $t = 1$ or from the productive technology not liquidated at $t = 1$. On the other hand, impatient agents do not buy goods at $t = 2$ and they only care about repaying back the loan, that is,

$$(1+i^l)W \leq M^1 + (1+i_2^s)S_2^1 + (1+i^d)(D - \Delta^1) - P_2c^1 + P_2x^1 + P_2(1-y^1)\rho. \quad (14)$$

This setup includes several features worth mentioning. First, the intermediation role played by banks starts when a borrower asks for a loan at time $t = 0$. The loan is produced because the borrower (entrepreneur) lacks the means of payment to make a purchase (wage payment). Then, once these means of payment, in the form of deposits, are created, households split those assets into liquid (sight deposits) and illiquid (time deposit) funds. In this sense, the liquidity problem of the household is financial rather than technological. That is, to make a purchase, households need financial claims which are generally acceptable. Because at time $t = 0$ they face the risk of needing to buy goods at $t = 1$, they have to maintain liquid financial funds for precautionary reasons at the corresponding opportunity cost. These liquid funds then circulate in the economy as broad money as long as the loan does not mature. Notice this sequence of events is opposite to the one in DD and the subsequent literature where the intermediation process starts when a saver deposit assets in the bank to be loaned out to a borrower.

Second, this intermediation service implies two obligations to depository institutions. From a liability side perspective, the production of deposits means the bank has to service the payment orders of depositors. Obviously, if the owner of the deposits make payments to other clients of the same bank, this service obligation is very easy to fulfill. The bank just renames the owner of the deposits. However, if the destination of the payment is a client of another bank, then the transfer of deposits must be met by a transfer of liquid assets. This will also happen if a household wants to convert the deposit into outside money (cash). Unlike DD, these liquid assets are not deposits that are left idle or invested in an inferior short term technology (remember the intermediation process starts on the asset side of the bank's balance sheet). These liquid assets are borrowed from the central bank either in the form of cash in banks' vaults or in the form of reserves (current accounts in the central bank). The refinancing rate of the central bank will be denoted i^o.

Thus, the role outside money plays in this model differs from that on the existing literature at least on two instances. *Ex ante*, central bank liquidity is

not needed for the creation of loans and deposits by banks. These financial institutions are autonomous in that respect. On the other hand, *ex post*, once payments are made, the liquidity risk banks face, and therefore, the need to borrow outside money from the central bank, is not so much related to depositors disposing of their deposits as to the net transfer of funds between banks and into outside money the use of these deposits imply. For example, in the model here, under the assumption that all payments are distributed evenly across all banks, depositors will be disposing of their deposits but there would not be any need for banks to hold liquid assets as all net flows between them will be zero. It is precisely the fact that payments are unevenly distributed across banks the reason why outside money is needed to settle accounts among them.

From an asset side perspective, the other obligation banks face has to do with the possibility that the value of their assets falls below that of their liabilities. This may well happen if a fraction of loans are not repaid in full. Notice the value of deposits, and the corresponding value of the obligations they generate, should not be affected by that event. This obligation is met by a new category of liabilities, capital, which should absorb fluctuations in the value of assets. In the model here, because loans are to be repaid, there are no solvency issues associated with the riskiness of assets and capital is not needed.

The third point to stress is the maturity mismatch between bank assets and liabilities. In the model, loans take two periods to mature while deposits are available to depositors anytime. Notice this maturity mismatch is an inevitable consequence of loan provision. Of course, the bank can manage its balance sheet to reduce or eliminate that maturity mismatch. In particular, the bank will manage the supply of liquid (sight deposits) and illiquid (time deposits) to take this mismatch into account. However, because loans are provided essentially to produce the means of payments a borrower lacks and, therefore, to be disposable immediately, automatically the asset the loan creates will have a longer maturity than the liability associated with it.

To understand the problems associated with the maturity mismatch of assets and liabilities of the bank and the solvency problems this could cause, we can look at the net worth of the bank at $t = 2$. This net worth is equal to

$$
\begin{aligned}
NW_2 &= (1+i^l)W - \\
&\quad -\lambda \left[(1+i^o)M^1 + (1+i_2^s)S_2^1 + (1+i^d)\left(D - \Delta^1\right)\right] \\
&\quad -(1-\lambda)\left[(1+i^o)M^2 + (1+i_2^s)S_2^2 + (1+i^d)\left(D - \Delta^2\right)\right]. \quad (15)
\end{aligned}
$$

That is, the net worth will be equal to the repayment of the loan, $(1 + i^l)W$, which is assumed to be paid in full, minus the obligations to depositors, both from patient and impatient households. These obligations are the gross payment of sight, $(1 + i_2^s)S_2^{h1}$, and time, $(1 + i^d)\left(D - \Delta^h\right)$, deposits plus the cost of providing cash, $(1+i^o)M^h$. This cash has to be borrowed from the central bank at the rate i^o. Having NW_2 positive is the condition households evaluate at $t = 1$ to predict whether the bank will be solvent at $t = 2$. Anticipation that the net worth of the bank could be negative at $t = 2$ could trigger a run at $t = 1$. Clearly, given that rates i^l and i^d are set on period $t = 0$, high policy

rates i^o together with a strong enough desire to convert sight deposits into cash, could render the bank insolvent.

Importantly, contrary to the literature based on DD and the intermediation of real assets, this model separates the liquidation of financial positions from the liquidation of investment projects. The loan the bank holds provides a right to a future flow of funds for the bank. This flow of funds originates from the production and selling activities of the borrowers (the entrepreneurs in the model). In the event of insolvency, forcing the bank to liquidate that asset to respond to a deposit outflow, does not necessarily imply the production activity financed with that loan has to be liquidated too. As mentioned on Section 3, loans are usually noncallable so, in general, banks cannot force borrowers to pay earlier than what is established in the loan contract. What the bank can do is to try to sell the asset in the market, possibly at a discount. This means the bank is only obtaining a fraction of the present discounted value of cash flows produced by the loan. But selling the loan this way, *per se*, has nothing to do with the ability of the borrower to pay back the loan. Thus, the liquidation of bank assets is just a redistribution of future flows between market participants and does not need to imply a real cost for society as a whole.

Of course, the fact that financial liquidation is separated from real investment liquidation does not mean these two decisions are not linked. But any connection between them must go through general equilibrium effects as changes in prices and interest rates in response to a generalized failure of the banking system may induce households to take them simultaneously. We look at this possibility in the solution of the model below.

4.2 Solution

4.2.1 Individual problems

Each household $h \in \{1, 2\}$ faces the problem of choosing consumption, c^h, storage, x^h, liquidation of the productive technology, y^h, the portfolio allocation at $t = 0$ between sight deposits, S_1 and time deposits, D, the liquidation of time deposits, Δ^h, as well as the portfolio allocation at $t = 1$, between cash, M^h, and sight deposits, S_2^h, to maximize utility $u(c^h)$ subject to constraints (8) through (14) depending on whether the household is impatient, $h = 1$, or patient, $h = 2$. In making these choices, households take as given prices, namely, nominal good prices in each period, P_1 and P_2, and interest rates on loans, i^l, time deposits, i^d, and sight deposits, i_1^s and i_2^s.

To describe the problem of households, notice that all nominal variables can be normalized by the initial level of the loan, W. Denote normalized nominal variables by the corresponding lower case letter. Take now a household of type h, entering period $t = 1$ with (normalized) sight, s_1, and time, d, deposits. Let $v^h(s_1, d)$ be the maximum level of utility this household is going to obtain as a function of its type and its portfolio choice. The problem the household solves is then

$$v^1(s_1, d) = \max u(c^1) \tag{16}$$

subject to

$$p_1 x^1 + p_1 c^1 \leq (1 + i_1^s)s_1 + \delta^1(1 - \phi), \tag{17}$$

$$m_2^1 + s_2^1 \leq (1 + i_1^s)s_1 + \delta^1(1 - \phi) - p_1(x^1 + c^1) + p_1 y^1, \tag{18}$$

and

$$1 + i^l \leq p_2 x^1 + p_2(1 - y^1)\rho + m_2^1 + (1 + i_2^s)s_2^1 + (1 + i^d)(d - \delta^1) \tag{19}$$

if the household is impatient ($h = 1$), or

$$v^2(s_1, d) = \max u(c^2) \tag{20}$$

subject to

$$p_1 x^2 \leq (1 + i_1^s)s_1 + \delta^2(1 - \phi), \tag{21}$$

$$m_2^2 + s_2^2 \leq (1 + i_1^s)s_1 + \delta^2(1 - \phi) - p_1 x^2 + p_1 y^2, \tag{22}$$

$$p_2 c_2^2 \leq m_2^2 + (1 + i_2^s)s_2^1 + (1 + i^d)(d - \delta^2), \tag{23}$$

and

$$1 + i^l \leq p_2 x^2 + p_2(1 - y^2)\rho + m_2^2 + (1 + i_2^s)s_2^2 + (1 + i^d)(d - \delta^2) - p_2 c_2^2 \tag{24}$$

if the household is patient ($h = 2$).

On period $t = 0$ the household chooses its portfolio to maximize expected utility

$$v = \max \lambda v^1(s_1, d) + (1 - \lambda)v^2(s_1, d) \tag{25}$$

subject to the budget constraint

$$s_1 + d = 1. \tag{26}$$

On the other hand, banks make choices to maximize their net worth at $t = 2$, NW_2, specified in (15). This expression can also be normalized by W, so banks decide on their supply of (normalized) sight and time deposits to maximize

$$\begin{aligned} nw_2 &= 1 + i^l - \lambda \left[(1 + i^o)m^1 + (1 + i_2^s)s_2^1 + (1 + i^d)\left(d - \delta^1\right)\right] \\ &\quad - (1 - \lambda)\left[(1 + i^o)m^2 + (1 + i_2^s)s_2^2 + (1 + i^d)\left(d - \delta^2\right)\right]. \end{aligned} \tag{27}$$

4.2.2   Equilibrium with valued deposits

In this economy, an equilibrium is defined as usual.

Definition 1 *An equilibrium is a collection of allocations $\{c^h,\ x^h,\ y^h,\ s_1,\ d,\ \delta^h,\ m^h,\ and\ s_2^h\}$ for $h \in \{1, 2\}$ and prices $\{p_1,\ p_2,\ i^l,\ i^d,\ i_1^s,\ and\ i_2^s\}$ such that:*

1. *given prices, allocations solve individual problems both of households and banks, and*

2. *prices are such that goods markets clear, for $t = 1$*

$$\lambda(c^1 + x^1) + (1 - \lambda)x^2 = \lambda y^1 + (1 - \lambda)y^2,$$

and $t = 2$

$$(1 - \lambda)c^2 = \lambda \left[(1 - y^1)\rho + x^1\right] + (1 - \lambda)\left[(1 - y^2)\rho + x^2\right].$$

At $t = 1$, impatient households, representing a fraction λ of the population, demand goods to cover for consumption, c^1, and storage, x^1, while supplying the part of the productive technology they have liquidated, y^1. On the other hand, patient households, representing a fraction $1 - \lambda$ of the population, demand goods only storage, x^2, and supply the part of the productive technology they have liquidated, y^2. At $t = 2$, only impatient agents demand goods, this time for consumption, c^2, while both impatient and patient households supply goods from the return of the productive investment not liquidated at $t = 1$ together with the storing carried over from the previous period. Notice the definition imposes that labor markets clear as workers supply labor inelastically. Also, financial markets clear because the objective function of banks is linear in choice variables, so that the supply of financial services is perfectly elastic at market rates and effectively is demand determined.

We are interested in figuring out whether an equilibrium with valued deposits exists and whether it is unique or not. In such an equilibrium households are willing to hold bank liabilities at $t = 1$. Because at $t = 1$ competition is between cash and sight deposits, we define an equilibrium with valued deposits as follows:

Definition 2 *An equilibrium with valued deposits is an equilibrium in which either $s_2^1 > 0$, $s_2^2 > 0$, or both.*

Equilibrium in this economy seems a complicated object as it involves a total of 20 variables, 14 of which are allocations and 6 are prices. Notice, however, this complexity gets significantly reduced once we apply the following lemma.

Lemma 1 *In an equilibrium with valued deposits and liquidation of the productive technology, so that $y^h > 0$ for some $h \in \{1, 2\}$, $x^h = m^h = \delta^h = 0$ for all $h \in \{1, 2\}$.*

Proof. See the Appendix.

Although the proof of the lemma is in the Appendix, its intuition is clear once we compare the returns associated with households' choices. For that, notice households have five margins with which to transfer resources between $t = 1$ and $t = 2$. First, they could liquidate the production technology, y^h. Reducing the liquidation of the project by one monetary unit at $t = 1$ raises $\rho p_2 / p_1$ monetary units at $t = 2$. Second, households could store goods, x^h. Buying one monetary unit worth of storage at $t = 1$ will produce a revenue of p_2 / p_1 monetary units at $t = 2$. Third, they could hoard cash, m^h. The nominal return of this investment is just 1. Fourth, households could accumulate sight deposits, s_2^h, with a gross

nominal return of $1 + i_2^s$. Finally, there is the liquidation of the time deposits, δ^h. Reducing the liquidation of the time deposit by one monetary unit increases resources at $t = 2$ by $(1 + i^d)/(1 - \phi)$. This is because of the liquidation fee ϕ.

Clearly, since $\rho > 1$, it is in the interest of the households to reduce the liquidation of the productive technology by reducing storing. Thus, if in equilibrium it is optimal to liquidate part of the production technology, $y^h > 0$, it should imply that storing is zero. At the same time, if sight deposits are valued it must be the case that $i_2^s > 0$ and dominate cash in rate of return. To see this, assume that $i_2^s \leq 0$. In such a case, households would demand cash instead of sight deposits at $t = 1$. But, to obtain the cash demanded by their customers, banks would need to borrow it from the central bank at the rate $i^o > 0$. Thus, banks have incentives to increase the remuneration of sight deposits above 0. But in that case, households would cease to demand cash and accumulate sight deposits instead. Finally, below we will see that in equilibrium it must be the case that $i^d \geq i_2^s$. Thus, the cost of liquidating the time deposit at $t = 1$ exceeds the return on investing in either cash or sight deposits and it is in the interest of households to set $\delta^h = 0$.

With this lemma in hand, we can now show the following proposition.

Proposition 1 *There exists an equilibrium with valued deposits in which consumption levels are*

$$c^1 = 1, \ c^2 = \rho, \tag{28}$$

and the liquidation of the productive technology satisfies

$$\lambda y^1 + (1 - \lambda)y^2 = \lambda. \tag{29}$$

Furthermore, interest rates obey

$$1 + i_2^s = \rho \frac{p_2}{p_1}, \tag{30}$$

$$1 + i^l = 1 + i^d = (1 + i_1^s)(1 + i_2^s) > 1, \tag{31}$$

while the inflation rate is bounded by

$$\frac{1}{\rho} < \frac{p_2}{p_1} < \frac{1 + i^d}{\rho(1 - \phi)}. \tag{32}$$

Proof. See the Appendix.

This is as much of the equilibrium as it can be characterized. However, even without solving for specific values for the endogenous variables, some conclusions can be drawn. First, as shown in (28), the equilibrium in the nominal economy does not provide households with any degree of insurance. The intuition of this result is as follows. Banks in this economy cannot make any promise about the real value of deposits, nor they can condition the rates on sight deposits on household types. Thus, effectively they cannot write deposit contracts with real contingent payouts.

One could think that a way out of this inefficient outcome could be to allow banks to condition the rate on sight deposits at $t = 2$, i_2^s, on household types. This could be done simply by indexing that interest rate to the withdrawals the household makes at $t = 1$ since impatient households have a higher propensity to spend at that period. We are, however, reluctant to explore that possibility for several reasons. On the one hand, in reality sight deposits do not work that way since, by definition, they are spot contracts. Each period a rate is determined for everyone that is not history dependent. This type of contingencies is precisely what time deposits try to accomplish, not sight deposits. On the other hand, it is not clear how this type of remuneration would affect the equilibrium. Because of perfect competition in the banking industry, a household facing a reduced rate in a bank because of its withdrawal history may decide to move its funds to a different bank. It is not clear the recipient bank has incentives to apply the same reduced rate to these new funds.

Looking at the CIA constraint (17) together with the equilibrium values for consumption and storage, it must be the case that

$$p_1 \leq (1 + i_1^s)s_1.$$

Notice the corresponding constraint for patient households (23) is not binding for sure. Thus, s_1 has the interpretation of a precautionary demand for liquidity which has to be strictly positive.

As a second result, notice in equilibrium some liquidation of the productive technology has to be done if impatient households are to consume. This means that $y^h > 0$ for some $h \in \{1, 2\}$. Thus, storage is not used as it was explained in Lemma 1. In fact, the only other margin used to transfer resources from $t = 1$ to $t = 2$ is sight deposits which implies that their real return should equal that of the productive technology, ρ, as (30) states. In other words, because households have accumulated financial claims at $t = 1$, in an equilibrium with valued deposits the rate of such claims should be such that they are maintained until the corresponding financial asset, loans in this case, mature. In such scenario, individual households are indifferent between liquidating the productive technology at $t = 1$, deposit the revenues from selling those goods and obtaining the corresponding proceeds at $t = 2$, or else, maintaining the initial real investment until maturity. However, in equilibrium a significant fraction of households should liquidate the productive project so that impatient households can consume. That is why expression (29) does not pin down individual productive liquidation rates but the aggregate one.

Third, from the point of view of the banks, lending out an additional monetary unit produces a revenue of $1 + i^l$ at $t = 2$. However, given that households do not demand cash, the cost for a bank of producing that loan is remunerating the corresponding liability, either in the form of a time or a sight deposit. In equilibrium, because time deposits are not liquidated at $t = 1$, the marginal cost of maintaining these liabilities between $t = 0$ and $t = 2$ should be the same and equal to the marginal revenue of providing the loan, as specified in (31). Regarding nominal lending, the equilibrium does not determine the size of the

banking sector as represented by the initial loan W. All nominal variables are proportional to it.

Finally, expression (32) claims that, for deposits to be valued, the inflation rate between $t = 1$ and $t = 2$ should be large enough to discourage the use of cash. Also, that inflation rate should be low enough so that households do not liquidate the time deposit and accumulate sight deposits instead. Notice the equilibrium implies price indeterminacy. As long as expressions (30), (31) and (32) are satisfied, the real allocation is independent of the particular values assigned to prices in the economy.

4.2.3 The possibility of a run on a single bank

In a bank run, households decide in $t = 1$ they do not trust their bank to be solvent and withdraw their funds. Notice both impatient and patient households need funds at $t = 2$ to pay back the loan they asked for at $t = 0$. Thus, when households are concerned about solvency, all of them, independent of their type, may have incentives to transfer funds between $t = 1$ and $t = 2$ by means of a different asset than the deposits at their bank.

To make the run comparable with the one in DD, we assume it takes place at the beginning of $t = 1$, before households start purchasing goods but after they know their types. When the run affects only a single bank, depositors of that depository institution decide to withdraw their deposits, totally liquidating their time deposits, $\delta^{hr} = d$. Here, the superscript r denotes that the run is in a single bank. Because there are no solvency concerns with respect to other banks, and because deposits still dominate cash in rate of return, these funds are then transferred to a different financial institution as sight deposits. As the bank that is run is an atomistic agent in the economy, this means that prices remain at their equilibrium values

$$1 + i^l = 1 + i^d = (1 + i_1^s)(1 + i_2^s),$$

and

$$\rho \frac{p_2}{p_1} = 1 + i_2^s > 1.$$

Also, assuming all customers of the running bank distribute themselves among the rest of banks in the economy, the balance sheets of the recipient banks are not altered.

Notice that, as the run is assumed to happen at $t = 1$, all households have already chosen the split between s_1 and d. These choices stay at their equilibrium values since they were determined at $t = 0$. Let m_1^r be the normalized amount withdrawed from the bank at the beginning of $t = 1$ and transferred to a different financial institution, aggregated across all depositors of the bank. This is the aggregate amount the bank needs to borrow from the central bank. At $t = 2$, for the bank to be solvent it needs to be the case that

$$nw^r = 1 + i^l - (1 + i^o)m_1^r \geq 0.$$

Thus, the withdrawal customers can make should be

$$m_1^r = \min\left\{(1+i_1^s)s_1 + d(1-\phi), \frac{1+i^l}{1+i^o}\right\}$$

given that $(1+i_1^s)s_1 + d(1-\phi)$ is the value of deposits at $t = 1$. That is, in the event of a run at a bank, m_1^r is the maximum amount of reserves the central bank will be willing to lend to that bank. Assuming customers are served as they place the order the transfers, the fraction

$$\lambda^r = \min\left\{\frac{1+i^l}{(1+i^o)\left[(1+i_1^s)s_1 + d(1-\phi)\right]}, 1\right\}$$

of first depositors withdrawing, will get $(1+i_1^s)s_1 + d(1-\phi)$ while the remaining fraction $1 - \lambda^r$ will get 0.

We then have the following result.

Proposition 2 *Households do not have incentives to coordinate in a run in their bank, given that the remaining banks are solvent, as long as the refinancing rate is low enough, in particular, as long as*

$$1 + i^o \leq \frac{1+i^l}{1+i_1^s - (i_1^s + \phi)d}. \tag{33}$$

If this condition is not satisfied, a self-fulfilling run on any of the banks is supported in equilibrium. In such a case,

$$c^{1r} < c^1 = 1 \text{ and } c^{2r} < c^2 = \rho$$

so that both patient and impatient households are worse off as compared with the equilibrium without the run.

Proof: See the Appendix.

When households decide on the possibility of joining a run in their bank, they evaluate the extent to which the bank will be solvent at $t = 2$. Solvency will now depend on the relative costs of the reserves needed to satisfy the transfers demanded by the bank's customers. According to this Proposition, multiplicity of equilibria, and the possibility of a self-fulfilling run on a particular bank, depends on the level of the official rate of the central bank. For relatively low rates, there is only one equilibrium with solvent banks, while for relatively high rates, there is also an inferior equilibrium in which depositors coordinate in a run. Condition (33) specifies the threshold for the refinancing rate, above which the bank becomes insolvent at $t = 2$ and the run takes place. In such a run, both households are worse off since they liquidate the time deposit at a cost.

4.2.4 The possibility of an aggregate bank run

Unlike the run on a single bank, in a system-wide run there is no other bank to turn into and the withdrawals are done either in cash or in goods. Thus, with a run, $s_2^{hR} = 0$ and $\delta^{hR} = d$ for both $h = \{1, 2\}$. Here the superscript R denotes the fact that there is a run in the whole banking system of the economy. Notice that, again, as the run is assumed to happen at $t = 1$, all households have already chosen the split between s_1 and d. These choices, together with the interest for loans, i^l, and $t = 0$ sight deposits, i_1^s, stay at their equilibrium values as they were determined at $t = 0$. Furthermore, because all banks are affected by a run, aggregate prices, p_1 and p_2 could be affected.

As before, let m_1^R the normalized amount withdrawed from any bank at the beginning of $t = 1$ and converted into cash. This is the aggregate amount each bank needs to borrow from the central bank. At $t = 2$, for the bank to be solvent it needs to be the case that

$$nw_2^R = 1 + i^l - (1 + i^o)m_1^R \geq 0.$$

Thus, the withdrawal customers can make should be

$$m_1^R = \min\left\{ (1 + i_1^s)s_1 + d(1 - \phi), \frac{1 + i^l}{1 + i^o} \right\}.$$

Again, assuming customers are served as they order the transfers, the fraction

$$\lambda^R = \min\left\{ \frac{1 + i^l}{(1 + i^o)\left[(1 + i_1^s)s_1 + d(1 - \phi)\right]}, 1 \right\}$$

of first depositors withdrawing, will get $(1 + i_1^s)s_1 + d(1 - \phi)$ while the remaining fraction $1 - \lambda^R$ will get 0.

With this, the individual problems become choosing c^{hR}, x^{hR}, y^{hR}, and m^{hR} to maximize

$$u(c^{1R}) \tag{34}$$

subject to

$$p_1^R x^{1R} + p_1^R c^{1R} \leq m_1^R \tag{35}$$

$$m_2^{1R} \leq m_1^R - p_1^R(x^{1R} + c^{1R}) + p_1^R y^{1R} \tag{36}$$

and

$$1 + i^l \leq p_2^R x^{1R} + p_2(1 - y^{1R})\rho + m_2^{1R} \tag{37}$$

if the household turns out to be impatient, or

$$u(c^{2R}) \tag{38}$$

subject to

$$p_1^R x^{2R} \leq m_1^R, \tag{39}$$

$$m_2^{2R} \leq m_1^R - p_1^R x^{2R} + p_1^R y^{2R}, \tag{40}$$

$$p_2^R c^{2R} \leq m_2^{2R}, \tag{41}$$

and

$$1 + i^l \leq p_2^R x^{2R} + p_2^R (1 - y^{2R})\rho + m_2^{2R} - p_2^R c^{2R}, \tag{42}$$

if the household turns out to be patient. Here, potentially all endogenous variables are affected with the exception of s_1, d, i^l, and i_1^s which are determined at $t = 0$.

Because all banks need to borrow funds equal to $(1 + i_1^s)s_1 + d(1 - \phi)$ from the central bank, their normalized net worth at $t = 2$ would be

$$nw_2^R = 1 + i^l - (1 + i^o)\left[(1 + i_1^s)s_1 + d(1 - \phi)\right].$$

We have the following result.

Proposition 3 *Households do not have incentives to coordinate in a run in their bank, given that runs are occurring at the remaining banks, as long as the refinancing rate is low enough, in particular, as long as*

$$1 + i^o \leq \frac{1 + i^l}{1 + i_1^s - (i_1^s + \phi)d}. \tag{43}$$

If this condition is not satisfied, money is not valued in equilibrium, that is, both

$$p_1^R \to \infty \ \text{and} \ p_2^R \to \infty.$$

Proof: See the Appendix.

The intuition of the first part of the proposition is as before. If the bank can afford the reserves needed to satisfy liquidation of deposits, then it will remain solvent and customers will not join the run. The second part is a little bit trickier. For a successful run to exist, money should be valued, otherwise customers will have no asset in which to transfer purchasing power from $t = 1$ to $t = 2$. However, for an equilibrium with valued money to exist, two conditions must be fulfilled. First, markets for goods must clear. For goods markets at $t = 1$ this means some liquidation of the productive technology must happen. As Lemma 1 stated, in such a case, storage will not be used. So, for goods markets at $t = 2$ to clear, liquidation at $t = 1$ cannot be total. The second condition involves money to be valued. For that, money should have the same real rate of return as the productive technology, namely, there should be a deflation equal to $1/\rho$. In other words, in an aggregate run, the marginal rate of transformation between $t = 1$ and $t = 2$ still should be ρ as in the equilibrium without the run.

As shown in the Appendix, substituting (35) and (36) into (37) as well as (39) and (40) into (42) produces

$$c^{2R} = \rho c^{1R} = \rho \left[1 - \frac{1}{p_1^R}\left(1 + i^l - m_1^R\right)\right] < \rho$$

so that $c^{2R} < \rho = c^2$ and $c^{1R} < 1 = c^1$. However, from the market clearing conditions, the only combination of consumption consistent with the marginal rate of transformation ρ is $(1, \rho)$. Thus, unless nominal prices satisfy $p_1^R \to \infty$ and $p_2^R \to \infty$ and money is not valued, there is no equilibrium. Basically, with the run, the economy reverts to autarky.

5 Conclusions and Implications

In this paper we have provided a model for analyzing the maturity transformation and liquidity insurance dispensed by banks in a nominal economy with endogenous money creation. In the model banks commit to implement a payment system. In this setup, we make an explicit separation between the liquidity created through lending and the reserves held by depository institutions at the central bank. Sight deposits are used by depositors to acquire consumption goods while reserves are employed by the banks to service payment orders and offset liquidity risk. Moreover, we also differentiate between the liquidation of financial (nominal) assets and real investments. Withdrawals generally imply the convertibility to sight deposits produced by other banks. The liquidation of bank assets to honor this commitment does not imply, however, the extinction of the real investment loans finance. The reason is that most of the assets originated are noncallable, and the liquidation of loans results in selling the right to the future cash flows they generate.

Our paper can be viewed as corroborating the hypothesis, already included in Diamond and Dybvig [12], about the equivalence between deposit insurance and the lender of last resort function of central banks when technology is riskless. There is one caveat, though. We observe that, in equilibrium, nominal deposit contracts do not reproduce the efficient allocation traditionally found in the previous literature. The interpretation of this result is that, in a nominal setup, depository institutions cannot commit to support a particular consumption bundle according to the future liquidity needs of their depositors. Since inside money creation in our model is linked to loan origination, the counterpart to loan provision is the creation of debt contracts redeemable on demand. The debt holder chooses the demandability of these liabilities, namely sight and time deposits, but it does not achieve the optimality in terms of liquidity risk-sharing.

On the policy front, we show that the existence of a self-fulfilling equilibria depends on the refinancing rate of the central bank. In the model depositors only have incentives to withdraw early if they anticipate the insolvency of the depository institution. We study the possibility of an individual bank run as in Diamond and Dybvig [12], as well as the existence of a system-wide run in which depositors of the entire banking system coordinate in a run. In both cases, we state that whenever the central bank provides outside money at proper rates, depositors will anticipate the solvency of the banking system and the eventual illiquidity of the bank will not imply its bankruptcy.[14]

The failing response of the Fed to offset the banking panics during the Depression can be consider as a case study to support the view that an effective discount-window lending policy can prevent bank runs. As noted by Meltzer [26], [27], the Fed was at the center of the forces creating the banking panics of 1930-1933. The misreading of monetary conditions was explained by the implementation of an ineffective discount window mechanism, manifested by the

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[14]This eventuality was also anticipated by Diamond and Dybvig [12] when they stated that the central bank would buy bank assets "... for prices greater than their liquidating value."

massive decline in the money supply, a collapse of the public's deposit currency ratio, and the exclusive access to this facility to member banks.

Further evidence of the role of central banks to prevent a wave of panics can be found in the response of the Federal Reserve of Atlanta in the period 1929-1933. The extraordinary and aggressive measures adopted by the Atlanta Fed to inject liquidity in the region helped to prevent the banking panic in Florida in 1929 (Carlson et al. [10]), and to contain the failures of Mississippi's banks during the initial banking panic of the 1930s (Richardson and Troost [28]). This evidence highlights the importance of the Bagehot's doctrine in the event of expectational panics of the Diamond and Dybvig type: a lender of last resort policy that commits to provide liquidity to banks can be effective to prevent a system-wide run and restore the confidence of depositors in the banking system.

Finally, another interesting feature of the model described here is that it provides an explicit bridge between banking and monetary theories. At least from a quantitative point of view, bridging this gap seems important since liabilities of depository institutions make the bulk of broad monetary aggregates. For the US, travelers checks and checkable deposits have represented about 67 percent of M1 over the period 1959-2017, while these items together with small time deposits and savings deposits have represented an average 85 percent of M2 over the same period. For the euro area, the liabilities of depository institutions have represented an average of 84, 91 and 83 percent of M1, M2 and M3, respectively between 1997 and 2017. In the model, both the size of banks' balance sheet and the split between monetary and nonmonetary liabilities are endogenous. Furthermore, the model introduces reserves as a liability of the central bank which is different from the monetary units depository institutions create and connects it with the payment flows between banks. These connections make the model a promising tool to open the box as of how central banks are able to manage economic activity by controlling the cost of expanding bank's balance sheet through changes in the level of the refinancing rate.

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6 Appendix: Proofs

6.1 Proof of Lemma 1

This section presents a proof of Lemma 1. Here we prove that an equilibrium with valued deposits implies that depositors do not transfer purchasing power from period 1 to period 2 in form of storage, time deposits, or hoarding money.

Differentiating the household's problem (25) with respect to s_1 and d we obtain

$$\left[\lambda \frac{\partial v^1(s_1, d)}{\partial s_1} + (1 - \lambda) \frac{\partial v^2(s_1, d)}{\partial s_1} - \eta_0 \right] s_1 = 0 \tag{44}$$

and

$$\left[\lambda \frac{\partial v^1(s_1, d)}{\partial d} + (1 - \lambda) \frac{\partial v^2(s_1, d)}{\partial d} - \eta_0 \right] d = 0, \tag{45}$$

where η_0 is the Lagrange multiplier associated with (26).

The FOCs associated with problems (16) and (20) are, with respect to consumption, c^h,

$$\frac{u'(c^1)}{p_1} = \theta_1^1 + \eta_1^1, \tag{46}$$

and

$$\frac{u'(c^2)}{p_2} = \theta_2^2 + \eta_2^2, \tag{47}$$

where θ_1^1 is the Lagrange multiplier associated with (17), η_1^1 is the Lagrange multiplier associated with (18), θ_2^2 is the Lagrange multiplier associated with (23) and η_2^2 is the Lagrange multiplier associated with (24).

Moreover, differentiating with respect to storage, x^h, we obtain

$$\left[p_2 \eta_2^1 - p_1 \left(\theta_1^1 + \eta_1^1 \right) \right] x^1 = 0, \tag{48}$$

and

$$\left[p_2 \eta_2^2 - p_1 \left(\theta_1^2 + \eta_1^2 \right) \right] x^2 = 0, \tag{49}$$

where η_2^1 is the Lagrange multiplier associated with (19), θ_1^2 is the Lagrange multiplier associated with (21) and η_1^2 is the Lagrange multiplier associated with (22).

At the same time, we can differentiate against the liquidation of the productive technology, y^h,

$$\left[p_1 \eta_1^1 - p_2 \rho \eta_2^1 \right] y^1 = 0, \tag{50}$$

and

$$\left[p_1 \eta_1^2 - p_2 \rho \eta_2^2 \right] y^2 = 0; \tag{51}$$

and with respect to liquidation of the time deposit, δ^h,

$$\left[(1 - \phi) \left(\theta_1^1 + \eta_1^1 \right) - (1 + i^d) \eta_2^1 \right] \delta^1 = 0, \tag{52}$$

$$\left[(1 - \phi) \left(\theta_1^2 + \eta_1^2 \right) - (1 + i^d)(\theta_2^2 + \eta_2^2) \right] \delta^h = 0; \tag{53}$$

and, finally, with respect to cash holdings, m_2^h,

$$\left(\eta_2^1 - \eta_1^1 \right) m_2^1 = 0, \tag{54}$$

and

$$\left(\theta_2^2 + \eta_2^2 - \eta_1^2 \right) m_2^2 = 0, \tag{55}$$

and with respect to sight deposits, s_2^h,

$$\left[(1 + i_2^s)\eta_2^1 - \eta_1^1 \right] s_2^1 = 0, \tag{56}$$

and

$$\left[(1 + i_2^s)(\theta_2^2 + \eta_2^2) - \eta_1^2 \right] s_2^2 = 0. \tag{57}$$

There is also the envelope conditions with respect to initial sight deposits, s_1,

$$\frac{\partial v^1(s_1, d)}{\partial s_1} = (1 + i_1^s) \left(\theta_1^1 + \eta_1^1 \right), \tag{58}$$

and

$$\frac{\partial v^2(s_1, d)}{\partial s_1} = (1 + i_1^s) \left(\theta_1^2 + \eta_1^2 \right), \tag{59}$$

and initial time deposit, d,

$$\frac{\partial v^1(s_1, d)}{\partial d} = (1 + i^d)\eta_2^1, \tag{60}$$

and

$$\frac{\partial v^2(s_1, d)}{\partial d} = (1 + i^d) \left(\theta_2^2 + \eta_2^2 \right). \tag{61}$$

Given the set of FOCs specified above, we have to prove that, in equilibrium with valued deposits, so that $s_2^h > 0$, if $y^h > 0$, then $x^h = m^h = \delta^h = 0$, for all $h = \{1, 2\}$. Given that impatient agents want to consume at $t = 1$, start assuming $y^1 > 0$. Then, from (50),

$$\frac{\eta_1^1}{\eta_2^1} = \frac{p_2 \rho}{p_1}. \tag{62}$$

From (48) it must be the case that

$$p_2 \eta_2^1 - p_1 \left(\theta_1^1 + \eta_1^1 \right) < 0$$

so that

$$x^1 = 0. \tag{63}$$

As we are searching for an equilibrium with valued deposits, assume that sight deposits have positive remuneration in period 2, i.e. $i_2^s > 0$ so, from (54), (55), (56), and (57), $s_2^1 > 0$, $s_2^2 > 0$, and

$$m_2^1 = m_2^1 = 0, \tag{64}$$

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together with

$$\frac{\eta_1^1}{\eta_2^1} = \frac{\eta_1^2}{\theta_2^2 + \eta_2^2} = 1 + i_2^s = \frac{p_2 \rho}{p_1}. \qquad (65)$$

This means that

$$p_1 \eta_1^2 - p_2 \rho \eta_2^2 = p_2 \rho \theta_2^2 \geq 0.$$

Thus, either $\theta_2^2 = 0$ which, from (51) and (65) would imply an interior solution for y^2 and from (23)

$$p_2 c^2 < m_2^2 + (1 + i_2^s)s_2^1 + (1 + i^d)(d - \delta^2),$$

or else, $\theta_2^2 > 0$ which, from (51) and (65) would imply $y^2 = 1$ and from (23)

$$p_2 c^2 = m_2^2 + (1 + i_2^s)s_2^1 + (1 + i^d)(d - \delta^2).$$

In either case, from (49)

$$-p_1 \left(\theta_1^2 + \eta_1^2 \right) + p_2 \eta_2^2 < 0$$

which implies

$$x^2 = 0 \qquad (66)$$

and $\theta_1^2 = 0$. Then, if $\theta_2^2 > 0$, from (24), having condition (23) satisfied with equality and $y^2 = 1$, would imply $1 + i^l = 0$ which is a contradiction. Then, it must be the case that $\theta_2^2 = 0$ and constraint (23) is not binding.

Summarizing

$$\frac{\eta_1^1}{\eta_2^1} = \frac{\eta_1^2}{\eta_2^2} = 1 + i_2^s = \frac{p_2 \rho}{p_1}. \qquad (67)$$

Next, assume agents contract some time deposit so that $d > 0$. From (45)

$$(1 + i^d) \left[\lambda \eta_2^1 + (1 - \lambda)\eta_2^2 \right] = \eta_0.$$

Substituting for η_0 in (44) yields

$$\lambda \left[(1 + i_1^s) \left(\theta_1^1 + \eta_1^1 \right) - (1 + i^d) \right] \eta_2^1 + (1 - \lambda) \left[(1 + i_1^s) \left(\theta_1^2 + \eta_1^2 \right) - (1 + i^d) \right] \eta_2^2 \leq 0, \qquad (68)$$

otherwise $s_1 = 1$ and $d = 0$, contradicting the assumption that $d > 0$. Looking at this expression together with (52) and (53) it cannot be the case that both $\delta^1 > 0$ and $\delta^2 > 0$ simultaneously. To solve for the δ^h assume first that $\delta^2 > 0$ so that $\delta^1 = 0$. Then, from (53) and the results above,

$$\frac{1 + i^d}{1 - \phi} = \frac{\eta_1^2}{\eta_2^2} = \frac{\eta_1^1}{\eta_2^1} = 1 + i_2^s = \frac{p_2 \rho}{p_1}. \qquad (69)$$

But then, from (52)

$$(1 - \phi)\theta_1^2 + (1 - \phi)\eta_1^2 - (1 + i^d)\eta_2^2 > 0$$

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which would make the household choose $\delta^1 = d > 0$ which contradicts the assumption that $\delta^1 = 0$. Now assume that $\delta^1 > 0$ so that $\delta^2 = 0$. Then, from (52) and the results above,

$$\frac{\theta_1^1 + \eta_1^1}{\eta_2^1} = \frac{1 + i^d}{1 - \phi} > \frac{\eta_1^1}{\eta_2^1} = \frac{\eta_1^2}{\eta_2^2} = 1 + i_2^s = \frac{p_2 \rho}{p_1}.$$

But if $(1 - \phi)\left(\theta_1^1 + \eta_1^1\right) = (1 + i^d)\eta_2^1$, then it must be that $(1 + i_1^s)\left(\theta_1^1 + \eta_1^1\right) > (1 + i^d)\eta_2^2$ so that $(1 + i_1^s)\eta_1^2 > (1 + i^d)\eta_2^2$ or

$$1 + i^d < (1 + i_1^s)\frac{\eta_1^2}{\eta_2^2} = (1 + i_1^s)(1 + i_2^s).$$

But if this is the case, rolling over the sight deposit is more profitable than contracting the time deposit and $d = 0$, which contradicts the idea that $\delta^1 > 0$. Thus, in equilibrium it must be the case that

$$\delta^1 = \delta^2 = 0.$$

To sum up, we have that, when $s_2^h > 0$, if $y^h > 0$, then $x^h = m^h = \delta^h = 0$ for all $h = \{1, 2\}$. $\square$

6.2 Proof of Proposition 1

By Lemma 1, replacing equilibrium values in (19) and (24) yields

$$1 + i^l = p_2 \rho - p_2 \rho c^1 + (1 + i_1^s)(1 + i_2^s)s_1 + (1 + i^d)d \qquad (70)$$

and

$$1 + i^l + p_2 c^2 = p_2 \rho + (1 + i_1^s)(1 + i_2^s)s_1 + (1 + i^d)d. \qquad (71)$$

From these two expressions we get $c_2^2 = \rho c^1$. This expression together with the market clearing conditions in the good markets

$$\lambda c^1 = \lambda y^1 + (1 - \lambda)y^2$$

and

$$(1 - \lambda)c^2 = \lambda(1 - y^1)\rho + (1 - \lambda)(1 - y^2)\rho,$$

yields

$$c^1 = 1 \quad \text{and} \quad c^2 = \rho$$

so that

$$\lambda = \lambda y^1 + (1 - \lambda)y^2.$$

To get the equilibrium conditions for the interest rates, we know that the bank's net worth at $t = 2$ is determined as follows

$$
\begin{aligned}
nw_2 \quad \equiv \quad \frac{NW_2}{W} &= 1 + i^l - \lambda\left[(1 + i_2^s)s_2^1 + (1 + i^d)d\right] \\
&\quad -(1 - \lambda)\left[(1 + i_2^s)s_2^2 + (1 + i^d)d\right] \\
&= 1 + i^l - (1 + i^d)d - (1 + i_1^s)(1 + i_2^s)s_1 \\
&= \left[1 + i^l - (1 + i^d)\right]d + \left[1 + i^l - (1 + i_1^s)(1 + i_2^s)\right]s_1,
\end{aligned}
$$

so an equilibrium with both $s_1 > 0$ and $d > 0$, implies

$$1 + i^l = 1 + i^d = (1 + i_1^s)(1 + i_2^s). \tag{72}$$

Finally, using expression (69), for $i_2^s > 0$, so that $s_2^h > 0$ and deposits are valued, it must be the case that

$$\frac{1}{\rho} < \frac{p_2}{p_1} < \frac{1 + i^d}{\rho(1 - \phi)}.$$

□

6.3 Proof of Proposition 2

We have to find the conditions that support a self-fulfilling equilibrium when there is a run on a single depository institution. With this, the individual problems become choosing c^{hr}, x^{hr}, y^{hr}, m_1^r, s_2^{hr} and m_2^{hr} to maximize

$$v^1(s_1, d) = \max u(c^{1r}) \tag{73}$$

subject to

$$p_1 x^{1r} + p_1 c^{1r} \leq m_1^r, \tag{74}$$

$$m_2^{1r} + s_2^{1r} \leq m_1^r - p_1(x^{1r} + c^{1r}) + p_1 y^{1r}, \tag{75}$$

and

$$1 + i^l \leq p_2 x^{1r} + p_2(1 - y^{1r})\rho + m_2^{1r} + (1 + i_2^s)s_2^{1r} \tag{76}$$

if the household turns out to be impatient, or

$$v^2(s_1, d) = \max u(c^{2r}) \tag{77}$$

subject to

$$p_1 x^{2r} \leq m_1^r, \tag{78}$$

$$m_2^{2r} + s_2^{2r} \leq m_1^r - p_1 x^{2r} + p_1 y^{2r}, \tag{79}$$

$$p_2 c^{2r} \leq m_2^{2r} + (1 + i_2^s)s_2^{2r}, \tag{80}$$

and

$$1 + i^l \leq p_2 x^{2r} + p_2(1 - y^{2r})\rho + m_2^{2r} + (1 + i_2^s)s_2^{2r} - p_2 c^{2r} \tag{81}$$

if the household turns out to be patient. Notice the household still uses sight deposits at $t = 1$, s_2^{hr}, but these funds are deposited at a different institution.

The FOCs associated with problems (73) and (77) are now, with respect to consumption, c^{hr},

$$\frac{u'(c^{1r})}{p_1} = \theta_1^{1r} + \eta_1^{1r}, \tag{82}$$

and

$$\frac{u'(c^{2r})}{p_2} = \theta_2^{2r} + \eta_2^{2r}, \tag{83}$$

where θ_1^{1r} is the Lagrange multiplier associated with (74), η_1^{1r} is the Lagrange multiplier associated with (75), θ_2^{2r} is the Lagrange multiplier associated with (80) and η_2^{2r} is the Lagrange multiplier associated with (81); with respect to storage, x^{hr},

$$\left[p_2\eta_2^{1r} - p_1\left(\theta_1^{1r} + \eta_1^{1r}\right)\right]x^{1r} = 0, \tag{84}$$

and

$$\left[p_2\eta_2^{2r} - p_1\left(\theta_1^{2r} + \eta_1^{2r}\right)\right]x^{2r} = 0, \tag{85}$$

where η_2^{1r} is the Lagrange multiplier associated with (76), θ_1^{2r} is the Lagrange multiplier associated with (78) and η_1^{2r} is the Lagrange multiplier associated with (79); with respect to liquidation of the productive technology, y^{hr},

$$\left[p_1\eta_1^{1r} - p_2\rho\eta_2^{1r}\right]y^{1r} = 0, \tag{86}$$

and

$$\left[p_1\eta_1^{2r} - p_2\rho\eta_2^{2r}\right]y^{2r} = 0; \tag{87}$$

and with respect to cash holdings, m_2^{hr},

$$\left(\eta_2^{1r} - \eta_1^{1r}\right)m_2^{1r} = 0, \tag{88}$$

and

$$\left(\theta_2^{2r} + \eta_2^{2r} - \eta_1^{2r}\right)m_2^{2r} = 0; \tag{89}$$

and with respect to sight deposits, s_2^{hr},

$$\left[-\eta_1^{1r} + (1 + i_2^s)\eta_2^{1r}\right]y^{1r} = 0, \tag{90}$$

and

$$\left[-\eta_1^{2r} + (1 + i_2^s)\left(\theta_2^{2r} + \eta_2^{2r}\right)\right]y^{2r} = 0. \tag{91}$$

Because in the equilibrium with valued deposits we need that $i_2^s > 0$ so, from (88), (89), (90), and (91), we have that $s_2^1 > 0$, $s_2^2 > 0$, and

$$m_2^{1r} = m_2^{2r} = 0, \tag{92}$$

together with

$$\frac{\eta_1^{1r}}{\eta_2^{1r}} = \frac{\eta_1^{2r}}{\theta_2^{2r} + \eta_2^{2r}} = 1 + i_2^s = \frac{p_2\rho}{p_1}. \tag{93}$$

This means that

$$p_1\eta_1^{1r} = p_2\rho\eta_2^{1r},$$

so that from (86) $y^{1r} > 0$ while from (84) we obtain $x^{1r} = 0$. Furthermore, from (92)

$$p_1\eta_1^{2r} - p_2\rho\eta_2^{2r} = p_2\rho\theta_2^{2r} \geq 0.$$

Thus, we can distinguish between two possibilities. Either $\theta_2^{2r} > 0$ which, from (80) leads to $p_2c_2^{2r} = (1 + i_2^s)s_2^{2r}$, and from (87) leads to $y^{2r} = 1$. This means, from (81), that $1 + i^l = 0$, which clearly is a contradiction. The second possibility

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implies $\theta_2^{2r} = 0$ which, from (80) leads to $p_2 c_2^{2r} < (1 + i_2^s) s_2^{2r}$, and from (87) implies that y^{2r} has an interior solution. Then, (93) implies

$$\frac{\eta_1^{1r}}{\eta_2^{1r}} = \frac{\eta_1^{2r}}{\eta_2^{2r}} = 1 + i_2^s = \frac{p_2 \rho}{p_1}. \tag{94}$$

Then, from (85), we have that $x^{2r} = 0$ and from (78) we obtain that $\theta_1^{2r} = 0$.

Substituting equilibrium values in (76) and (81) yields

$$1 + i^l = p_2 \rho - p_2 \rho c^{1r} + (1 + i_2^s) m_1^r \tag{95}$$

and

$$1 + i^l + p_2 c^{2r} = p_2 \rho + (1 + i_2^s) m_1^r. \tag{96}$$

Combining both expressions we get $c^{2r} = \rho c^{1r}$.

Now, two things may happen according to the refinancing rate, i^o. In the first case, we can assume that the refinancing rate of the central bank is relatively low

$$1 + i^o \leq \frac{1 + i^l}{(1 + i_1^s) s_1 + (1 - \phi) d}.$$

In that case, households obtain all the cash they demand, $m_1^r = (1 + i_1^s) s_1 + (1 - \phi) d$, and the bank is solvent. Then consumption would be

$$c^{1r} = 1 - \frac{1}{P_1} (i_1^s + \phi) D < 1 = c^1 \tag{97}$$

and

$$c^{2r} = \rho - \frac{\rho}{P_1} (i_1^s + \phi) D < \rho = c^2. \tag{98}$$

Because households loose consumption and the bank ends up being solvent, depositors will not have incentives to coordinate in a run if they anticipate that the central bank will provide liquidity insurance to the depository institution at a lower rate.

In the second case, we can consider that the refinancing rate of the central bank is relatively high, such that

$$1 + i^o > \frac{1 + i^l}{(1 + i_1^s) s_1 + (1 - \phi) d}.$$

In that case, households are restricted in the amount of cash they can withdraw, $m_1^r = (1 + i^l)/(1 + i^o)$, because the bank will end up being insolvent. Then consumption would be

$$c^{1r} = 1 - \frac{1}{p_1} \left[1 + i_1^s - \frac{1 + i^l}{1 + i^o} \right], \tag{99}$$

and

$$c^{2r} = \rho - \frac{\rho}{p_1} \left[1 + i_1^s - \frac{1 + i^l}{1 + i^o} \right]. \tag{100}$$

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Notice, because

$$1 + i^o > \frac{1 + i^l}{(1 + i_1^s)s_1 + (1 - \phi)d} = \frac{1 + i^l}{1 + i_1^s - (i_1^s + \phi)d},$$

we have

$$1 + i_1^s > \frac{1 + i^l}{1 + i^o} + (i_1^s + \phi)d > \frac{1 + i^l}{1 + i^o},$$

so that $c^{1r} < 1$ and $c^{2r} < \rho$. In this case, although all households loose with the run, they will join as they would obtain nothing if they do not. $\square$

6.4 Proof of Proposition 3

This section shows the derivation of the optimal conditions when there is a system-wide run in the economy. We also show the equilibrium condition that supports a self-fulfilling run in such a case.

The FOCs associated with problems (34) and (38) are now, with respect to consumption, c_h^{hR},

$$\frac{u'(c_1^{1R})}{p_1^R} = \theta_1^{1R} + \eta_1^{1R}, \tag{101}$$

and

$$\frac{u'(c_2^{2R})}{p_2^R} = \theta_2^{2R} + \eta_2^{2R}, \tag{102}$$

where θ_1^{1R} is the Lagrange multiplier associated with (35), η_1^{1R} is the Lagrange multiplier associated with (36), θ_2^{2R} is the Lagrange multiplier associated with (41) and η_2^{2R} is the Lagrange multiplier associated with (42); with respect to storage, x^{hR},

$$\left[p_2^R \eta_2^{1R} - p_1^R \left(\theta_1^{1R} + \eta_1^{1R} \right) \right] x^{1R} = 0, \tag{103}$$

and

$$\left[p_2^R \eta_2^{2R} - p_1^R \left(\theta_1^{2R} + \eta_1^{2R} \right) \right] x^{2R} = 0, \tag{104}$$

where η_2^{1R} is the Lagrange multiplier associated with (37), θ_1^{2R} is the Lagrange multiplier associated with (39) and η_1^{2R} is the Lagrange multiplier associated with (40); with respect to liquidation of the productive technology, y^{hR},

$$\left[p_1^R \eta_1^{1R} - p_2^R \rho \eta_2^{1R} \right] y^{1R} = 0, \tag{105}$$

and

$$\left[p_1^R \eta_1^{2R} - p_2^R \rho \eta_2^{2R} \right] y^{2R} = 0; \tag{106}$$

and with respect to cash holdings, m_2^{hR},

$$\left(\eta_2^{1R} - \eta_1^{1R} \right) m_2^{1R} = 0, \tag{107}$$

and

$$\left(\theta_2^{2R} + \eta_2^{2R} - \eta_1^{2R} \right) m_2^{2R} = 0. \tag{108}$$

For an equilibrium to exist it must be the case that either $y^{1R} > 0$, or $y^{2R} > 0$ or both. Given that impatient agents want to consume at $t = 1$, start assuming $y^{1R} > 0$. Then, from (105),

$$\frac{\eta_1^{1R}}{\eta_2^{1R}} = \frac{p_2^R \rho}{p_1^R}. \tag{109}$$

From (103) it must be the case that

$$p_2^R \eta_2^{1R} - p_1^R \left(\theta_1^{1R} + \eta_1^{1R} \right) < 0$$

so that in the run still no productive investment is liquidated by impatient agents

$$x^{1R} = 0. \tag{110}$$

Since $y^{1R} > 0$ and $x^{1R} = 0$, from (36), $m_2^{1R} > 0$, so (107) and (109) imply

$$\frac{\eta_1^{1R}}{\eta_2^{1R}} = 1 = \frac{p_2^R \rho}{p_1^R}, \tag{111}$$

which means

$$\frac{p_2^R}{p_1^R} = \frac{1}{\rho} < 1 \tag{112}$$

and the economy enters a deflation.

On the other hand, from (42), $m_2^{2R} > 0$ too, otherwise $c_2^{2R} = 0$, so (108) implies

$$\frac{\eta_1^{2R}}{\theta_2^{2R} + \eta_2^{2R}} = 1. \tag{113}$$

But then

$$\eta_1^{2R} = \theta_2^{2R} + \eta_2^{2R} \geq \eta_2^{2R}.$$

This, together with the result on prices (109) means

$$-p_1 \theta_1^{2R} - p_1 \eta_1^{2R} + p_2 \eta_2^{2R} < 0$$

so that, from (104)

$$x^{2R} = 0 \tag{114}$$

also. However from (39), with $x^{2R} = 0$, the constraint must be slack and $\theta_1^{2R} = 0$.

Using these equilibrium values in (37) and (42) yields

$$1 + i^l = (1 + i_1^s)s_1 + d(1 - \phi) + p_2^R \rho - p_1^R c_1^{1R}$$

and

$$1 + i^l + p_2^R c_2^{2R} = (1 + i_1^s)s_1 + d(1 - \phi) + p_2 \rho.$$

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From these two expressions we get $c_2^{2R} = \rho c_1^{1R}$. This expression together the market clearing conditions in the good markets

$$\lambda c_1^{1R} = \lambda y^{1R} + (1-\lambda)y^{2R}$$

and

$$(1-\lambda)c_2^{2R} = \lambda(1-y^{1R})\rho + (1-\lambda)(1-y^{2R})\rho,$$

yields

$$c_1^{1R} = 1 \quad \text{and} \quad c_2^{2R} = \rho.$$

From the net worth of any bank at the end of period $t = 2$ normalized by wealth can be written as

$$
\begin{aligned}
nw_2^R \;\equiv\; \frac{NW_2^R}{W} &= 1 + i^l - \lambda(1+i^o)m_2^{1R} - (1-\lambda)(1+i^o)m_2^{2R} \\
&= 1 + i^l - (1+i^o)(1+i_1^s)s_1 - (1+i^o)(1-\phi)d.
\end{aligned}
$$

Thus, for the bank to be solvent, the refinancing rate must satisfy

$$1 + i^o \leq \frac{1+i^l}{1+i_1^s - (\phi + i_1^s)d}. \tag{115}$$