ADEMU

A Dynamic Economic and Monetary Union

ADEMU WORKING PAPER SERIES

Generalized Compensation Principle

Aleh Tsyvinski[ł]
Nicolas Werquin[ŧ]

July 2018

WP 2018/137

www.ademu-project.eu/publications/working-papers

Abstract

We generalize the classic concept of compensating variation and the welfare compensation principle to a general equilibrium environment with distortionary taxes. We derive in closed-form the solution to the problem of designing a tax reform that compensates the welfare gains and losses induced by an arbitrary economic disruption. In partial equilibrium, average taxes simply increase or decrease to counteract the revenue gains or losses caused by the disruption. In general equilibrium, the compensation features three elements that depart from this benchmark and respectively account for (i) the incidence of the initial exogenous shock, and the fact that the tax reform itself induces indirect welfare effects caused by (ii) the non-constant marginal product of labor and (iii) the skill complementarities in production. This leads to a progressive compensating tax reform, with average tax rates increasing at a rate given by the ratio of the elasticity of labor demand and the elasticity of labor supply net of the rate of progressivity of the pre-existing tax code. We also derive a closed form formula for the fiscal surplus of the wage disruption and the compensation, thus generalizing the traditional Kaldor-Hicks criterion. Finally, we apply our formula to the compensation of automation: in the U.S., one additional robot per thousand workers requires a reduction (resp., increase) in the average tax rate at the 10th (resp., 90th) percentile of the income distribution equal to 2 percentage points (resp., 0.5 pp).

Keywords: income taxation, compensation principle, general equilibrium, wage disruption, robots
Jel codes: E62, H21, H22, I38

ł Yale University

ŧ Toulouse School of Economics

Horizon 2020
The Framework Programme for Research and Innovation

Acknowledgments

We thank Andy Atkeson, Don Brown, Ariel Burstein, Raj Chetty, Wolfgang Dauth, Georgy Gorov, Eduardo Faingold, Axelle Ferriere, Sebastian Findeisen, Jim Hines, Marek Kapicka, Louis Kaplow, Rohan Kekre, Nicolas Lambert, Tim Lee, Jesse Perla, Pascual Restrepo, Stefanie Stantcheva, Christopher Tonetti, Gianluca Violante, Dominik Sachs, and Kjetil Storesletten for comments. This research has received financial support from the ADEMU network grant, part of the EU H2020 program (grant agreement No 649396).

The ADEMU Working Paper Series is being supported by the European Commission Horizon 2020 European Union funding for Research & Innovation, grant agreement No 649396.

Horizon 2020
The Framework Programme for Research and Innovation

Introduction

In this paper we generalize the classic concept of compensating variation and the welfare compensation principle to a general equilibrium environment in which only distortionary taxes are available.

Consider a disruption in the economy, for example, an inflow of immigration or a change in technology, that impacts the distribution of workers' wages. This economic shock generally creates winners and losers, i.e., welfare gains for some individuals and welfare losses for others. The welfare compensation problem consists of designing a reform of the tax-and-transfer system that offsets these losses by redistributing the gains of the winners. The traditional public finance literature (Kaldor [1939], Hicks [1939, 1940]) gives a straightforward answer to the welfare compensation problem. In an economy where type-dependent lump-sum taxes are available, the tax reform that redistributes the welfare gains and losses from the economic shock simply consists of raising (resp., lowering) in a lump-sum way the tax liability of agents whose welfare increases (resp., decreases) from the disruption, up to the point where everyone is exactly as well off as before the change. This standard Kaldor-Hicks approach is flawed, however. First, because of asymmetric information, the only tax instrument at the disposal of the government, the income tax, is distortionary (Mirrlees [1971]), so that agents' labor supplies adjust in response to the tax change. Second, we argue that it is important to design the tax reform in an environment that explicitly accounts for the fact that wages are endogenously determined in general equilibrium.

Consider for example an immigration inflow, i.e., an exogenous (relative) increase in the total labor supply of a given skill. This disruption lowers the wage of agents with the same skill because the marginal product of labor is decreasing and raises the wage of those whose skills are complementary in production. In this situation, therefore, it is clear that immigration flows have non-trivial welfare consequences only because of the general equilibrium forces. Similarly, the impact of automation on inequality can be understood as a race between education and technology, whereby movements in relative wages are driven by the changes in the relative supply and demand of skills. Now suppose that the government implements a tax reform that aims at compensating the welfare of agents whose wage is adversely impacted by the disruption. Since the only available policy tools are distortionary taxes, such a reform affects the agents' labor supply choices. By the same general equilibrium forces as we just described, these labor supply adjustments impact in turn individuals' wages, and hence their utility. These welfare implications need to be themselves compensated using the distortionary tax code, leading to an a priori complex fixed point problem.

We start by analyzing the welfare compensation problem in a partial equilibrium environment where wages are exogenous. We show that the design of the compensating tax reform which brings every agent's utility back to its pre-disruption level is simple, even when only distortionary taxes are available. The key insight here is that individual utility is only affected by the *average* tax rates of the reform – that is, the changes in *marginal* tax rates do not impact welfare. This follows from an envelope theorem argument: the marginal tax rate that the individual faces affects his indirect utility only through his optimal labor supply decision, so that the corresponding welfare effect is second-order. As a consequence, it is straightforward to show (Proposition 1) that a suitably designed adjustment in the average tax rate – namely, one that exactly cancels out the income gain or loss caused by the exogenous disruption – is sufficient to achieve welfare compensation.

The analysis becomes significantly more complicated when distortionary taxes are coupled with the general equilibrium considerations. In this case, despite the envelope theorem, the endogenous changes in labor supply do matter for welfare, through their impact on wages that result from the decreasing returns and the complementarities in production. Therefore, in general equilibrium, because of the labor supply responses it generates, the *marginal* rates of the tax reform affect directly the agent's utility, even conditional on the *average* tax rate change. In other words, to determine the compensating tax reform, we need to simultaneously solve for the average and the marginal tax rate functions. This is the key difference with the partial equilibrium environment and the main technical challenge of our paper.

The main result of our paper is Theorem 1 that gives a closed-form solution and thus provides the complete analytical characterization of the compensating tax reform in response to any wage disruption in general equilibrium. This formula is valid for marginal wage disruptions; that is, our tax reform compensates the first-order effects on welfare caused by this shock. Corollary 2 also derives a closed-form formula for the fiscal surplus of the wage disruption and its compensation, i.e. the impact on government budget of the disruption and its associated compensation, which generalizes the traditional Kaldor-Hicks criterion and provides a simple test to determine whether economic shocks or policies are beneficial, in the sense that offseting their associated individual welfare gains and losses using only distortionary tax instruments is budget-feasible.

For ease of exposition, our theoretical analysis proceeds in two steps. We first analyze a simplified version of our model in Section 1, in which we make a number of assumptions ensuring that all of the relevant elasticity variables are constant. Specifically, we assume there that the utility function is quasilinear with isoelastic disutility of labor, that there are no labor force participation decisions, that the production function has a constant elasticity of substitution (CES) over labor inputs, and that the tax schedule in the initial (undisrupted) economy has a constant rate of progressivity. These functional-form assumptions allow us to derive in the simplest possible way the welfare compensating tax reform and analyze its economic implications. Second, in Section 2, we relax all of these assumptions: we allow for general individual-specific preferences with income effects, intensive (hours) and extensive (participation) labor supply decisions, a general production function over the labor inputs of all skills and capital, and an arbitrarily non-linear initial tax schedule. We derive a closed-form solution to the compensation problem in this environment that directly generalizes that obtained in the simpler framework.

Our theoretical analysis shows that there are three key elements, all given in closed-form, in the formula for the welfare compensating tax reform that depart from the simple partial equilibrium policy. First, the *modified wage disruption* variable properly defines the relevant disruption that needs to be compensated – namely, one that accounts for all of the labor demand spillovers induced by the initial shock.

Second, the *progressivity* variable accounts for the fact that a reform of the marginal tax rate of an agent distorts his labor supply, which in general equilibrium affects his wage because the marginal product of labor is decreasing. Therefore, the compensation needs to be designed in such a way that the welfare effects caused indirectly by the marginal tax rates of the reform counteract those induced by the average tax rates. This naturally leads to a differential equation for the compensation, and hence exponentially decreasing or increasing income tax rates. This implies, in response to a positive (resp., negative) disruption of a given wage, a progressive (resp., regressive) tax reform on incomes below that of the disrupted agent. The rate of progressivity is determined by the ratio of the labor supply and labor demand elasticities, net of the rate of progressivity of the initial tax code.

Third, the *compensation-of-compensation* variable accounts for the fact that a lower marginal tax rate at a given income, by distorting labor supply, also affects the entire wage distribution because of the cross-wage effects originating from the skill complementarities in production. The welfare impact of this indirect wage adjustments needs to be itself compensated using the tax schedule. However, the marginal tax rates of this second round of compensation generate in turn further wage and welfare changes for all of the agents, and so on. This leads to an a priori complex sequence of compensations – formally represented by an integro-differential equation. We show that we can generally solve this fixed point problem in closed-form by defining inductively a sequence of functions that each capture a given round of iterated compensation. Remarkably, if the production function is CES, we show that each round of iterated compensation is a constant fraction of the previous one. In this case, compensating the welfare gains and losses resulting from the skill complementarities in production simply requires a uniform shift of the marginal tax rates in addition to the progressive reform derived in the absence of cross-wage effects.

We finally propose in Section 3 a concrete application of our theory in the context of the robotization of the U.S. and the German economies between 1990 and 2007. We use Acemoglu and Restrepo [2017]'s data for the U.S., and Dauth et al (2017) data for Germany, which give the estimated impact of an additional robot per one thousand workers on the wages of different skills – roughly the amount of automation observed in the U.S. between these dates. The closed-form solution that we derive allows to immediately determine quantitatively the compensating reform. We find that in the U.S., an additional robot per thousand workers requires a progressive tax reform, where the tax payment of agents at the 10^{th} (resp., 90^{th}) percentile of the wage distribution decreases (resp., increases) by 110% of their income loss (resp., 125% of their income gain) from the disruption. This represents a 2 percentage point decrease (resp., a 0.5 pp increase) in their average tax rate, and generates a positive \$16 budget surplus for the government. In Germany, workers at the 10^{th} percentile should have their tax bill reduced by 310% of their income loss, while those at the 90^{th} percentile should have theirs reduced by 150% of their income loss.

Related literature. We now briefly describe the relationship to the literature. There are two main (and closely related) approaches in the theory of taxation. The first, represented by Saez and Stantcheva [2016], consists of assuming a particular social welfare function, or more generally of choosing the generalized social welfare weights that society assigns to different agents, and deriving optimal taxes given this criterion. The second approach, analyzed by Kaplow [2012, 2004] and Hendren [2014], consists of generalizing the Kaldor-Hicks principle in the partial equilibrium setting. Our results in Section 1.4, which consist of constructing the compensating tax reform in partial equilibrium, build on their work. Our main contribution (in particular, Proposition 2 and Theorem 1) is the analysis of the general equilibrium environment, where a disruption to the wage of an agent and a tax reform also directly impact the welfare of other individuals. We discuss the advantages of the compensation approach for the field of taxation in Section A.6.

Second, and most closely related to our general equilibrium framework, Itskhoki [2008] and Antras, de Gortari, and Itskhoki [2016] study compensating tax reforms and the welfare implications of trade liberalization in a general-equilibrium setting within a class of distortionary taxes. Itskhoki [2008] solves for optimal redistribution in a closed and open economy following trade liberalization within a class of distortionary taxes. Antras, de Gortari, and Itskhoki [2016] solve for the welfare and inequality correction following trade liberalization, restricting taxes (as well as tax reforms) to be of the CRP form (Bénabou [2002], Heathcote, Storesletten, and Violante [2016], Heathcote, Storesletten, Violante, et al. [2017]) and the production function to be CES. While we do not consider a sophisticated model of trade, we solve the compensation problem allowing for both general nonlinear tax schedules and tax reforms, and a general production function. More broadly, our model is within the class of Mirrleesean economies in general equilibrium. Stiglitz [1982a], Rothschild and Scheuer [2013, 2014, 2016], Ales, Kurnaz, and Sleet [2015a,b], Scheuer and Werning [2016], Sachs, Tsyvinski, and Werquin [2016] study optimal taxes in this environment. These papers do not address the compensation problem, which is our main contribution and leads to different economic insights, as well as a simpler implementation in practice (since it is known in closed-form).

Third, our application to automation relies on the results of Acemoglu and Restrepo [2017] (for the U.S.) and Dauth, Findeisen, Südekum, and Woessner [2017] (for Germany), who estimate the impact of robots on the wage distribution.

1 A Simple Model

We start by presenting a very simple version of our general framework, which allows us to derive most transparently our main result – namely, a closed-form formula for the tax reform that offsets the welfare gains and losses of an arbitrary disruption of the wage distribution in general equilibrium. Specifically, we make in this section the following assumptions: (i) the utility function is quasilinear with an isoelastic disutility of labor effort, and is the same for all agents; (ii) labor supply is chosen on the intensive margin only, i.e., there are no participation decisions; (iii) the production function has a constant elasticity of substitution (CES) over the labor inputs of all skill types, and there is no capital input; (iv) the labor income tax schedule in the initial (undisrupted) economy has a constant rate of progressivity (CRP). (Note, however, that we allow tax reforms to be arbitrary nonlinear functions of labor income.) The goal of these assumptions is to ensure that the relevant behavioral and price elasticities are constant.[1] We relax them and solve the fully general model in Section 2.

1.1 Initial equilibrium

There is a continuum of measure one of individuals indexed by $i \in [0,1]$. In this section, we assume for simplicity that all agents have quasilinear preferences over consumption c and labor supply l with isoelastic disutility of effort: $u(c,l) = c - \frac{l^{1+\frac{1}{e}}}{1+\frac{1}{e}}$. Agent i earns a wage $w_i \in \mathbb{R}_+$ which he takes as given. He chooses his labor supply l_i and earns pre-tax labor income $y_i = w_i l_i$. He pays a tax $T(y_i)$ on labor income, where the non-linear tax schedule $T : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable. Agent i maximizes his utility subject to the budget constraint $c = w_i l - T(w_i l)$. The agent's indirect utility is given by

$$U_i = w_i l_i - T(w_i l_i) - \frac{l_i^{1+\frac{1}{e}}}{1+\frac{1}{e}}, \tag{1}$$

[1] Specifically, Assumptions (i) and (ii) imply that the agents' elasticities of labor supply with respect to the marginal tax rate are constant, and that the income effects and the elasticities of participation are equal to zero. Assumption (iii) implies that the elasticities of labor demand with respect to the wage are constant, as well as the cross-wage elasticities with respect to the labor supply of a given skill, that is, an agent's labor supply has the same percentage impact on the wage of all other workers. Assumption (iv) ensures that the labor supply elasticities with respect to the wage, as well as the indirect adjustments of labor supply due to the agents' endogenous movements along the nonlinear tax schedule, are constant.

where his labor supply l_i is characterized by the first-order condition

$$l_i \;=\; \left[\left(1-T'\left(w_i l_i\right)\right) w_i\right]^e . \tag{2}$$

We denote by $\boldsymbol{w} \equiv \{w_i\}_{i\in[0,1]}$, $\boldsymbol{l} \equiv \{l_i\}_{i\in[0,1]}$, $\boldsymbol{U} \equiv \{U_i\}_{i\in[0,1]}$ and $\boldsymbol{L} \equiv \{L_i\}_{i\in[0,1]}$ the distributions of individual wages, labor supplies, indirect utilities and aggregate labor supplies in the initial economy with tax schedule T. Without loss of generality, we can order agents so that wages w_i are increasing in the index i, given the initial tax schedule T. Hence the agent's skill type i can be interpreted as his percentile in the wage distribution in the initial (undisrupted) economy.[2]

There is a continuum of mass one of identical firms that produce output using the aggregate labor supply[3] L_i of each type $i \in [0,1]$. In this section, we assume for simplicity that the aggregate production function has a CES functional form (as in, e.g., Heathcote et al. [2016]):

$$\mathscr{F}\left(\{L_i\}_{i\in[0,1]}\right) \;=\; \left[\int_0^1 \theta_i L_i^{1-\frac{1}{\varepsilon^D}} di\right]^{\frac{\varepsilon^D}{\varepsilon^D-1}}, \tag{3}$$

where $\theta_i > 0$ for all i. The parameter $\varepsilon^D > 0$ is the constant elasticity of substitution between any two labor inputs. If $\varepsilon^D \to \infty$, the environment converges to the partial equilibrium model where wages are exogenous and given by $w_i = \theta_i$ for all i.[4] In equilibrium, firms earn no profits, and the wage w_i is equal to the marginal product of type-i labor:

$$w_i \;=\; \mathscr{F}_i'(\boldsymbol{L}) \;=\; \theta_i \left(\frac{\bar{Y}}{L_i}\right)^{1/\varepsilon^D}, \tag{4}$$

where $\mathscr{F}_i' \equiv \partial \mathscr{F}/\partial L_i$ denotes the partial derivative of $\mathscr{F}$ with respect to its i^{th}

[2] Using (2) and (5), it is then easy to show that incomes $y_i = w_i l_i$ are then strictly increasing in skills i, so that there are one-to-one maps between skills, wages and incomes in the initial equilibrium. Importantly, we do not require that the tax reforms we consider keep this monotonicity property. We assume that incomes y belong to a compact interval $[\underline{y}, \bar{y}] \subset \mathbb{R}_+$ and have a continuous density $f_Y(\cdot)$.

[3] Since the distribution of agents on $[0,1]$ is uniform, we have $L_i = l_i$ in equilibrium. Note, however, that each individual agent is atomistic within his skill group, so that his wage changes only if all individuals with the same skill adjust their labor supply (e.g., in response to a tax change). In particular, each agent takes his wage as given and independent of his own choices.

[4] If $\varepsilon^D = 1$ (resp., $\varepsilon^D \to 0$), the production function is Cobb-Douglas (resp., Leontieff). Labor inputs are gross substitutes (resp., gross complements) if $\varepsilon^D > 1$ (resp., $\varepsilon^D < 1$).

variable L_i and $\bar{Y} \equiv \mathscr{F}(\boldsymbol{L})$ is the average income in the economy.

The government levies taxes on labor income. In this section, we assume for simplicity that in the initial equilibrium, i.e. before the disruption occurs, the tax schedule has a CRP functional form:[5]

$$T(y) = y - \frac{1-\tau}{1-p} y^{1-p}, \tag{5}$$

with $p \in (-\infty, 1)$ and $\tau \in \mathbb{R}$. The parameter p is the constant rate of progressivity of the tax schedule, defined as (minus) the elasticity of the retention rate $(1 - T'(y))$ with respect to gross income y. In particular, if $p = 0$ (resp., $p > 0$, $p < 0$) the initial tax schedule is linear (resp., progressive, regressive). Importantly, we allow the government to implement arbitrarily nonlinear reforms (i.e., not necessarily in the CRP class) in response to a given wage disruption.

1.2 Wage disruptions and the welfare compensation problem

In this section we start by defining a disruption of the economy's initial equilibrium, and then formally set up the welfare compensation problem. A disruption can be caused by various exogenous shocks: e.g., a perturbation of the production function $\mathscr{F}$ (due to, say, technological change) or of the distribution of aggregate labor supplies $\boldsymbol{L} = \{L_j\}_{j\in[0,1]}$ (due to, say, immigration flows).[6] Suppose that this shock affects the wage distribution $\boldsymbol{w}$ by $\mu\hat{\boldsymbol{w}}^E = \{\mu\hat{w}_i^E\}_{i\in[0,1]}$ for some $\mu > 0$, so that the wage of agent $i \in [0,1]$ changes from w_i to $w_i + \mu\hat{w}_i^E$. The disruptions we consider are continuous maps $i \mapsto \mu\hat{w}_i^E$ on $[0,1]$, and without loss of generality we normalize $\|\hat{\boldsymbol{w}}^E\| \equiv \max_{i\in[0,1]} |\hat{w}_i^E| = 1$. The function $\hat{\boldsymbol{w}}^E$ thus defines the *direction* of the wage disruption, while the scalar μ represents its *size*.

Definition 1. (Wage disruption.) *Consider an exogenous shock* $(\hat{\mathscr{F}}^E, \hat{\boldsymbol{L}}^E)$ *to the economy's production function* $\mathscr{F}$ *and the initial equilibrium distribution of aggregate labor supplies* $\boldsymbol{L}$. *We define the wage disruption* $\mu\hat{\boldsymbol{w}}^E$, *with* $\|\hat{\boldsymbol{w}}^E\| = 1$ *and* $\mu > 0$, *by the change in the wage distribution* $\boldsymbol{w}$ *caused by these shocks, keeping individual labor supplies fixed: for all* $i \in [0,1]$, $\mu\hat{w}_i^E = \tilde{\mathscr{F}}_i'(\{L_j + \hat{L}_j^E\}_{j\in[0,1]}) - \mathscr{F}_i'(\{L_j\}_{j\in[0,1]})$, *where* $\tilde{\mathscr{F}} \equiv \mathscr{F} + \hat{\mathscr{F}}^E$.

[5] See, e.g., Bénabou [2002], Heathcote, Storesletten, and Violante [2016].

[6] In Section 2, where capital is an input in production, a disruption can also be caused by an exogenous change in the aggregate capital stock.

The government can implement an arbitrarily nonlinear tax reform $\mu\hat{T}(\cdot)$ of the tax schedule,[7] so that the statutory tax payment at income level y changes from $T(y)$ to $T(y)+\mu\hat{T}(y)$.

In response to a wage disruption $\mu\hat{\boldsymbol{w}}^E$ and a tax reform $\mu\hat{T}$, individuals optimally adjust their labor supply. In general equilibrium, this further impacts their wage, which in turn alters their labor supply decisions, and so on. We denote by $\mu\hat{w}_i$ and $\mu\hat{l}_i$ the total endogenous changes in individual i's wage and labor supply following the perturbation $(\mu\hat{\boldsymbol{w}}^E, \mu\hat{T})$. That is, the wage and labor supply of an agent with skill i in the equilibrium of the disrupted economy are respectively equal to $\tilde{w}_i = w_i+\mu\hat{w}_i^E+\mu\hat{w}_i$ and $\tilde{l}_i = l_i + \mu\hat{l}_i$. We denote by $\tilde{U}_i = U_i + \mu\hat{U}_i$ the resulting indirect utility of agent i in the new equilibrium. Formally, the agent's welfare in the disrupted economy is given by

$$\tilde{U}_i \quad = \quad \tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) - \frac{\tilde{l}_i^{1+\frac{1}{e}}}{1+\frac{1}{e}}, \tag{6}$$

where $(\tilde{w}_i, \tilde{l}_i)$ are defined by the perturbed first-order condition

$$\tilde{l}_i \quad = \quad \left[\left(1 - T'(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}'(\tilde{w}_i\tilde{l}_i)\right)\tilde{w}_i\right]^e, \tag{7}$$

and the perturbed wage equation

$$\tilde{w}_i \quad = \quad \tilde{\mathscr{F}}_i'(\{\tilde{L}_j\}_{j\in[0,1]}). \tag{8}$$

with $\tilde{L}_j \equiv L_j + \hat{L}_j^E + \mu\hat{l}_j$.

A wage disruption $\mu\hat{\boldsymbol{w}}^E$ generally creates winners and losers, i.e., welfare gains for some individuals and welfare losses for others. The welfare compensation problem consists of designing a reform $\hat{T}$ of the existing tax code that offsets these losses by redistributing the income gains of the winners. Such a tax reform must be designed

[7] In Section 1.5 we assume that the tax reforms $\hat{T}$ that the government can implement are continuously differentiable, bounded, with bounded first derivative. This defines a Banach space on which the norm of a function $\hat{T}$ is given by $\|\hat{T}\| = \sup_{y\in\mathbb{R}_+} |\hat{T}(y)| + \sup_{y\in\mathbb{R}_+} |\hat{T}'(y)|$. Note that we do not impose $\|\hat{T}\| = 1$, so that the normalization of the tax reform by the same scalar $\mu > 0$ as the wage disruption is without loss of generality. The same holds for the endogenous wage and labor supply adjustments $\mu\hat{w}_i, \mu\hat{l}_i$ below.

such that each agent's compensating variation[8] $\tilde{U}_i - U_i$ is equal to zero, taking into account the endogenous wage and labor supply responses that it induces.

Definition 2. (Welfare compensation problem.) *A welfare compensating policy in response to a wage disruption $\mu\hat{\boldsymbol{w}}^E$ is a tax reform $\mu\hat{T}$ such that: (i) the utility $\tilde{U}_i$ of each agent i after the disruption, defined in (6), and the tax reform satisfies $\tilde{U}_i = U_i$; (ii) labor supply is chosen optimally, i.e. (7) holds; and (iii) the wage is equal to the marginal product of labor, i.e. (8) holds. Equation (41) in the Appendix defines the fiscal surplus, i.e., the change in government revenue induced by the wage disruption $\mu\hat{\boldsymbol{w}}^E$ and the compensating tax reform $\mu\hat{T}$.*

In what follows, we characterize analytically the solution to the welfare compensation problem for *marginal* wage disruptions, i.e., as $\mu \to 0$. Thus, our exercise consists of designing a tax reform $\hat{T}$ that compensates the first-order welfare effects of a small wage disruption in the direction $\hat{\boldsymbol{w}}^E$.

1.3 Elasticity concepts

We first define the elasticities $\varepsilon_i^{S,r}$ and $\varepsilon_i^{S,w}$ of labor supply l_i with respect to the retention rate $r_i \equiv 1 - T'(w_i l_i)$ and the wage w_i respectively, along the nonlinear budget constraint, as[9]

$$\varepsilon_i^{S,r} \equiv \frac{\partial \ln l_i}{\partial \ln r_i} = \frac{e}{1+pe}, \quad \text{and} \quad \varepsilon_i^{S,w} \equiv \frac{\partial \ln l_i}{\partial \ln w_i} = \frac{(1-p)\,e}{1+pe}. \tag{9}$$

The (constant) labor supply elasticity $\varepsilon^{S,r}$ differs from the structural parameter e as it takes into account that the initial labor supply response to a change in the retention rate impacts the marginal tax rate $T'(w_i l_i)$ faced by the agent, if the initial tax schedule is nonlinear, by an amount equal to the rate of progressivity p of the nonlinear tax schedule; this in turn causes a further endogenous labor supply adjustment given by the elasticity e, leading to the correction term $p \times e$ in the denominator of $\varepsilon^{S,r}$. Moreover, the elasticity with respect to the wage, $\varepsilon^{S,w}$, differs from that with respect

[8] See, e.g., Mas-Colell, Whinston, and Green [1995], p. 82. Since the utility function is quasilinear, it is the monetary amount that an agent i would be willing to pay, after the wage disruption $\mu\hat{\boldsymbol{w}}^E$ and the tax reform $\mu\hat{T}$, in order to be as well off as in the initial equilibrium. A positive (resp., negative) value implies that an individual i benefits (resp., loses) from these shocks.

[9] The assumptions we made in Section 1.1 ensure that the these elasticities are constant. This allows us to drop the subscripts i for the remainder of Section 1.

to the retention rate, $\varepsilon^{S,r}$, because a change in the wage affects $(1-T'(w_i l_i))\,w_i$, and hence labor supply, both directly as in the case of an exogenous perturbation in the retention rate, and indirectly through its effect on the marginal tax rate $T'(w_i l_i)$. The latter is accounted for by the correction $(1-p)$ in the numerator of $\varepsilon^{S,w}$.

Second, we define the elasticities of wages $\{w_i\}_{i\in[0,1]}$ with respect to the aggregate labor supply L_j. The labor supply of type j affects the wage of any other skill $i \neq j$ because different skills are imperfect substitutes in production, and the wage of skill j because the marginal product of labor is decreasing. We define the corresponding structural cross-wage and own-wage elasticities $\gamma_{i,j}$ and $1/\varepsilon_j^D$, respectively, by

$$\gamma_{i,j} \equiv \frac{\partial \ln w_i}{\partial \ln L_j} = \frac{1}{\varepsilon_D}\frac{y_j}{\bar{Y}}, \quad \text{and} \quad \frac{1}{\varepsilon_j^D} \equiv -[\frac{\partial \ln w_j}{\partial \ln L_j} - \lim_{i\to j}\frac{\partial \ln w_i}{\partial \ln L_j}] = \frac{1}{\varepsilon^D} \tag{10}$$

where both equalities are proved in the Appendix. The first expression shows that when the production function is CES, the cross-wage elasticity $\gamma_{i,j} \equiv \frac{\partial \ln w_i}{\partial \ln L_j}$ does not depend on i, implying that a change in the labor supply of type j has the same percentage impact on the wage of every type $i \neq j$; for the remainder of this section we thus simply denote $\gamma_{i,j}$ by γ_j. The second expression constructs the own-wage elasticity $1/\varepsilon_j^D$, or equivalently the inverse of the partial-equilibrium elasticity of labor demand, by subtracting from $\frac{\partial \ln w_j}{\partial \ln L_j}$ the complementarity $\lim_{i\to j}\frac{\partial \ln w_i}{\partial \ln L_j}$ between skill j and its neighboring skills $i \approx j$, thus capturing the impact of the labor effort L_j on the wage w_j arising purely from the fact that the marginal productivity of skill j is a decreasing function of the aggregate labor of its own type. With a CES production function with parameter ε^D, this elasticity is constant and equal to $1/\varepsilon^D$.

1.4 Compensation in Partial Equilibrium

In this section, we show that the solution to the compensation problem takes a simple form in partial equilibrium, even when when taxes are distortionary. Suppose that there is infinite substitutability between skills in production, i.e., $\varepsilon^D \to \infty$. The production function thus reads $\mathscr{F}(\boldsymbol{L}) = \int_0^1 \theta_i L_i di$, so that wages are exogenous and equal to $w_i = \theta_i$ for all i in the initial equilibrium.[10,11] In this case, the wage disruption $\mu\hat{\boldsymbol{w}}^E$ generates no further endogenous adjustment in the wage: $\hat{w}_i = 0$ for all $i \in [0,1]$,

[10] This is the standard partial-equilibrium assumption made by Mirrlees [1971].

[11] As will be clear in Section 2.4, none of the results of this section (except formula (12)) rely on the specific functional forms assumed for the utility function and the tax schedule.

so that $\tilde{w}_i$ is simply equal to $w_i + \mu \hat{w}_i^E$. We characterize analytically the solution to the welfare compensation problem, i.e., the compensating tax reform $\mu\hat{T}$ and its fiscal surplus $\mu\mathcal{R}(\hat{\boldsymbol{w}}^E)$, for marginal wage disruptions. The proofs are gathered in the Appendix.

A first-order Taylor expansion of equation (6) around the initial equilibrium (i.e., as $\mu \to 0$) implies that the change $\hat{U}_i$ in the indirect utility of agent i induced by the wage disruption and the tax reform is given by:

$$0 \;=\; \hat{U}_i \;=\; \left(1 - T'\left(y_i\right)\right) y_i \,\frac{\hat{w}_i^E}{w_i} - \hat{T}\left(y_i\right), \tag{11}$$

where the first equality imposes that agent i keeps the same level of welfare in the disrupted economy as in the initial equilibrium (i.e., $\tilde{U}_i = U_i$), once the new tax schedule is implemented. This equation shows that, in the partial equilibrium model, the change in the indirect utility of agent i is due to: (i) the exogenous change (say, increase) $\hat{w}_i^E$ in his wage, weighted by the share $(1 - T'(y_i))$ that he keeps after paying taxes on the implied income gain $l_i \hat{w}_i^E = \frac{y_i}{w_i}\hat{w}_i^E$ (the first term of (11)); (ii) the change in his tax liability $\hat{T}(y_i)$ (the second term of (11)), which makes him poorer (resp. richer) if $\hat{T}(y_i) > 0$ (resp. < 0).

Crucially, note that the change in the *marginal* tax rate, $\hat{T}'(y_i)$, does not enter equation (11), and therefore does not matter for welfare (conditional on the average tax rate $\hat{T}(y_i)$). This follows from the envelope theorem: the marginal tax rate that individuals face affect agents' indirect utility only through their labor supply decision (equation (2)); but since they choose labor supply optimally before the perturbation, their behavioral response to the marginal tax rate change induces no first-order effect on welfare.[12]

Next, a first-order Taylor expansion of equation (6), which imposes that the labor supply of agent i remains optimal in the disrupted economy, can be written in terms of the elasticity notations introduced in Section 1.1 as:

$$\frac{\hat{l}_i^{\mathbf{pe}}}{l_i} \;=\; \varepsilon^{S,w}\,\frac{\hat{w}_i^E}{w_i} - \varepsilon^{S,r}\,\frac{\hat{T}'\left(y_i\right)}{1 - T'\left(y_i\right)}. \tag{12}$$

This equation shows that the agent's labor supply adjusts because of the change in his wage $\hat{w}_i^E$, by an amount given by the labor supply elasticity with respect to the

[12]See details and discussion in the Appendix (equation (40)).

wage $\varepsilon^{S,w}$, and the change in his marginal tax rate $\hat{T}'(y_i)$, by an amount given by the elasticity with respect to the retention rate $\varepsilon^{S,r}$.

We now summarize the results obtained so far. Equation (11) immediately gives the tax reform $\mu\hat{T}$ which ensures that, after reoptimizing their behavior, individuals remain as well off as before the wage disruption $\mu\hat{\boldsymbol{w}}^E$. Equation (12) gives the corresponding change in the labor supply of agents following this wage disruption and compensating tax reform, and the impact on government budget is then straightforward to derive. We thus obtained the solution to the welfare compensation problem in closed form.

Proposition 1. *Suppose that there is infinite substitutability between skills in production, i.e., $\varepsilon^D \to \infty$. Consider a marginal disruption of the wage distribution $\boldsymbol{w}$ in the direction $\hat{\boldsymbol{w}}^E = \{\hat{w}_i^E\}_{i\in[0,1]}$. There exists a unique tax reform $\hat{T}$ that solves the welfare compensation problem, namely: for all $i \in [0,1]$,*

$$\hat{T}(y_i) \quad = \quad (1 - T'(y_i))\, y_i \frac{\hat{w}_i^E}{w_i}. \tag{13}$$

The fiscal surplus $\mathcal{R}(\hat{\boldsymbol{w}}^E)$ is given by expression (42) in the Appendix.

Proposition 1 is our first step in generalizing the standard Kaldor-Hicks criterion to the environment where type-specific lump-sum taxes are unavailable. It shows that if wages are exogenous, the compensating tax reform consists of increasing or decreasing the average tax rates $\frac{\hat{T}(y_i)}{y_i}$ by an amount equal to the net-of-tax income gain or loss of agents resulting from the economy's disruption, $(1 - T'(y_i))\frac{\hat{w}_i^E}{w_i}$.

1.5 Compensation in General Equilibrium

In this section we analyze the welfare compensation problem in the general equilibrium environment, that is, for any $\varepsilon^D > 0$. We show in the Appendix that the mathematical structure of this problem is a system of integro-differential algebraic equations (IDAE). We derive its solution in a closed-form for marginal wage disruptions, i.e., to a first-order as the size of the shock $\mu \to 0$. The proofs are gathered in the Appendix.

As discussed in Section 1.2, in general equilibrium, the initial wage disruption $\mu\hat{\boldsymbol{w}}^E$ generates further endogenous adjustments $\mu\hat{w}_i$ in the wage, which directly affect every agent's indirect utility and choice of labor supply. A first-order Taylor expansion of equation (8) implies that the endogenous wage changes $\hat{w}_i$ are given by

$$\frac{\hat{w}_i}{w_i} \;\; = \;\; -\frac{1}{\varepsilon^D}\frac{\hat{l}_i}{l_i} + \int_0^1 \gamma_j \frac{\hat{l}_j}{l_j} dj, \tag{14}$$

where $1/\varepsilon^D$ and γ_j are respectively the own-wage and cross-wage elasticities, defined in (10). This equation has the following economic interpretation: a one percent increase in the labor supply of an individual with skill i leads to a $-1/\varepsilon^D$ percent change in the wage of type i, because the marginal product of labor is decreasing; analogously, a one percent increase in the labor supply of an individual of type j, for any $j \in [0,1]$, leads to a γ_j percent change in the wage of type i, through the complementarities between skills in production.

Now, following the same steps as in Section 1.4, a first-order Taylor expansion of equation (6) around the initial equilibrium implies that the change $\hat{U}_i$ in the indirect utility of agent i induced by the wage disruption and the tax reform is given by:

$$0 \;\; = \;\; \hat{U}_i \;\; = \;\; (1 - T'(y_i))\, y_i \left[\frac{\hat{w}_i^E}{w_i} + \frac{\hat{w}_i}{w_i}\right] - \hat{T}(y_i), \tag{15}$$

where the first equality imposes that agent i keeps the same level of welfare in the disrupted economy as in the initial equilibrium. This expression generalizes equation (11) (replacing $\hat{w}_i^E$ with $\hat{w}_i^E + \hat{w}_i$) and implies that, in addition to the two partial-equilibrium forces described in Section 1.4, there is now the third channel through which the compensating variation of the agent changes, namely: (iii) the endogenous changes $\hat{l}_i$ and $\{\hat{l}_j\}_{j\in[0,1]}$ in the labor supplies of type-i and type-j agents, by impacting the wage of skill i by $\hat{w}_i$ (through equation (14)), have a first-order impact on the indirect utility of agent i.

Crucially, despite the envelope theorem, the endogenous changes in labor supply now matter for welfare, through their impact on wages resulting from the decreasing marginal productivities and the complementarities in production. As we demonstrate below, it follows that in general equilibrium and when only distortionary tax instruments are available the *marginal* tax rates of the reform now affect directly the agent's utility through the labor supply responses they induce. This is the key difference with the partial equilibrium environment.

Next, a first-order Taylor expansion of equation (6), which imposes that the labor supply of agent i remains optimal in the disrupted economy, can be written in terms of the elasticity notations introduced in Section 1.1 as:

$$\frac{\hat{l}_i}{l_i} \;=\; \varepsilon^{S,w}\left[\frac{\hat{w}_i^E}{w_i} + \frac{\hat{w}_i}{w_i}\right] - \varepsilon^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)}. \tag{16}$$

This expression generalizes equation (12) obtained in partial equilibrium (replacing $\hat{w}_i^E$ with $\hat{w}_i^E + \hat{w}_i$). The presence of the endogenous wage change $\hat{w}_i$ in the right hand side, along with equation (14), implies that, in addition to the direct effects caused by the exogenous wage and tax changes $\hat{w}_i^E$ and $\hat{T}'(y_i)$, the adjustment in labor supply of agent i, $\hat{l}_i$, is now also affected by those of all other agents j, $\{\hat{l}_j\}_{j\in[0,1]}$. Hence the labor supply changes of all of the agents now have to be solved for simultaneously as functions of the wage disruption function $\hat{\boldsymbol{w}}^E$ and the tax reform $\hat{T}$. The following lemma, which follows from Sachs, Tsyvinski, and Werquin [2016], derives the closed-form solution for $\hat{l}_i$, for all $i \in [0,1]$.

Lemma 1. *The solution to (16) is given by: for all i,*

$$\frac{\hat{l}_i}{l_i} \;=\; \frac{\delta \hat{l}_i^{\boldsymbol{pe}}}{l_i} + \delta\varepsilon^{S,w}\int_0^1 \Gamma_j\,\frac{\delta \hat{l}_j^{\boldsymbol{pe}}}{l_j}\,dj, \tag{17}$$

where $\hat{l}_i^{\boldsymbol{pe}}$ is defined in (12), $\delta \equiv 1/[1+\frac{\varepsilon^{S,w}}{\varepsilon^D}]$, and $\Gamma_j \equiv \gamma_j/\delta$.

Equation (17) shows that the percentage change in the labor supply of type i, $\hat{l}_i/l_i$, is the sum of two terms. The first, $\hat{l}_i^{\mathbf{pe}}/l_i$, is the partial-equilibrium expression (12), weighted by δ. This weight accounts for the fact that the marginal product of labor is decreasing, so that the agent's initial labor supply adjustment (say, increase) $\hat{l}_i^{\mathbf{pe}}$ lowers his wage by a factor $1/\varepsilon^D$, which in turn leads him to reduce his labor supply by a factor $\varepsilon^{S,w}/\varepsilon^D$, therefore dampening his initial response by $\delta \equiv 1/[1+\frac{\varepsilon^{S,w}}{\varepsilon^D}]$. The second term in (17) accounts for the fact that the wage disruption and the tax reform also lead to percentage increases $\delta\,\hat{l}_j^{\mathbf{pe}}/l_j$ in the labor supplies of agents of type $j \neq i$. These responses impact the wage of agent i by $\Gamma_j\,(\delta\,\hat{l}_j^{\mathbf{pe}}/l_j)$, where $\Gamma_i = \gamma_j/\delta$ can be thought of the total elasticity of the wage of skill i with respect to the labor supply of type j. This total cross-wage elasticity accounts for the direct effect γ_j, as well as all of the indirect effects occuring in general equilibrium – the wage change induces further labor supply responses, which in turn affect wages, etc. When the production function is CES these spillover effects are simply captured by the amplification factor

$1/\delta$.[13] Now, this total change in w_i implies a change in l_i given by the elasticity $\varepsilon^{S,w}$, again weighted by the factor δ to take into account the decreasing marginal product of labor. Summing over all types $j \in [0,1]$ leads to equation (17).

Taking stock. We gather and discuss the results obtained so far. In contrast to equation (11) in partial equilibrium, (15) does not yield directly the solution for the compensating tax change $\hat{T}(y_i)$ as a function of the exogenous disruption $\hat{w}_i^E$. This is because the agent's indirect utility is also affected by the endogenous adjustment in his wage, $\hat{w}_i$, which is determined by the labor supply responses of all agents, $\{\hat{l}_j\}_{j\in[0,1]}$, via equation (14). In turn, the labor supply change of any agent j, $\hat{l}_j$, depends on the changes in the marginal tax rates $\{\hat{T}'(y_k)\}_{k\in[0,1]}$ faced by everyone in the economy, via equation (17). Thus, in general equilibrium, both the average and the marginal tax rates of the reform have first-order welfare repercussions – this implies that the consequences of a given tax reform are much richer, and hence the design of the compensating policy much more complex, than in partial equilibrium.

Specifically, a higher *average* tax rate at a given income y^*, $\hat{T}(y^*) > 0$, implies a reduction in the welfare of agent y^*, by directly making him poorer, as in partial equilibrium (last term in equation (15)). Moreover, in general equilibrium, a higher *marginal* tax rate at income y^*, $\hat{T}'(y^*) > 0$, implies: (a) a higher average tax rate for all incomes $y > y^*$, which reduces the welfare of these agents; (b) an *increase* in the welfare of agent y^*, who works less (substitution effect, first term in equation (17)) and hence earns a higher wage (decreasing marginal product, first term in equation (14)); (c) a *decrease* in the welfare of all agents $y \neq y^*$, whose wage decreases due to the lower labor supply of agent y^* (production complementarities, second term in equation (14)).

Suppose that the planner implements the tax reform (13) that would compensate every agent's welfare in partial equilibrium. Through standard substitution effects, this tax reform affects individual labor supplies and hence, through decreasing returns and complementarities in production, the wage distribution. These lead to additional first-order welfare effects that need to be themselves compensated, by further reforming the tax-and-transfer system. Therefore, the combination of distortionary tax instruments and elastic labor supply (whereby marginal tax rates affect labor supply

[13] This is because each round of indirect general-equilibrium effect on the wage is a constant fraction of the direct effect. See Lemma 2 for the general expression of the elasticities $\Gamma_{i,j}$.

behavior) and general equilibrium (whereby labor supply decisions determine wages) leads to a fixed point problem for the compensating tax reform. Formally, the tax reform $\hat{T}$ is the solution to an integro-differential equation that we derive in Lemma 3 in the Appendix.

Main result. The next proposition gives a complete analytical characterization of the compensating tax reform in response to any wage disruption in general equilibrium. Note that since there is a one-to-one map between types i and incomes y_i, we can change variables and index by income the wages $w_{y_i} \equiv w_i$, labor supplies $l_{y_i} \equiv l_i$, wage disruptions $\hat{w}_{y_i}^E \equiv \hat{w}_i^E$, and elasticities $\gamma_{y_j} \equiv \gamma_j / y'(j)$, $\Gamma_{y_j} \equiv \Gamma_j / y'(j)$.

Proposition 2. *Suppose that that the utility function is quasilinear with isoelastic disutility of labor, the production function is CES, and the initial tax schedule is CRP. Consider a marginal disruption of the wage distribution $\boldsymbol{w}$ in the direction $\hat{\boldsymbol{w}}^E = \{\hat{w}_i^E\}_{i\in[0,1]}$. The following tax reform $\hat{T}$ solves the welfare compensation problem: for all i,*

$$\hat{T}(y_i) \quad = \quad (1 - T'(y_i))\, y_i \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i, y_j} [\hat{\Omega}_{y_j}^E + \lambda]\, dy_j, \tag{18}$$

where the modified wage disruption *variable $\hat{\boldsymbol{\Omega}}^E$ is defined for all $j \in [0,1]$ by*

$$\hat{\Omega}_{y_j}^E \quad \equiv \quad \frac{\delta \hat{w}_{y_j}^E}{w_{y_j}} + \delta \varepsilon^{S,w} \int_{\underline{y}}^{\bar{y}} \Gamma_{y_k} \frac{\delta \hat{w}_{y_k}^E}{w_{y_k}}\, dy_k, \tag{19}$$

the progressivity *variable $\mathscr{E}$ is defined by*

$$\mathscr{E}_{y_i, y_j} \quad \equiv \quad \frac{\varepsilon^D}{\delta\, \varepsilon^{S,r}} \frac{1}{y_j} \left(\frac{y_i}{y_j} \right)^{\varepsilon^D / \varepsilon^{S,r}}, \tag{20}$$

and the compensation-of-compensation *variable λ is a constant equal to*[14] $\mathbb{E}[\frac{y_j}{Y}(\hat{\Omega}_{y_j}^E - \int_{y_j}^{\bar{y}} \mathscr{E}_{y_j, y_k} \hat{\Omega}_{y_k}^E dy_k)]$.

We discuss and interpret formula (18) in Section 1.6. Note that this is a closed-form expression, as it depends only on the exogenous wage disruption $\hat{\boldsymbol{w}}^E$ and on variables that are all observed (or known in closed-form as a function of observables) in

[14] This expression for λ assumes $\bar{y} \to \infty$. The expression for finite $\bar{y}$ is given in the Appendix.

the pre-disruption economy: statutory marginal tax rates, elasticities of labor supply and labor demand, cross-wage elasticity between skills. Therefore it is straightforward to implement such a tax reform in practice.[15] In Appendix B.1 we provide a graphical representation and detailed discussion of this formula.

1.6 Analysis of the compensating tax reform

The compensating tax reform (18) features three important departures from the partial equilibrium compensation (13).

1. Modified disruption: Accounting for the incidence of the disruption

The modified wage disruption $\hat{\Omega}_j^E$ reflects the importance of correctly accounting for the incidence of a given economic shock in general equilibrium. The proof of Proposition 2 shows that $\hat{\Omega}_j^E$ is equal to the sum of the exogenous and endogenous wage adjustments, $\delta(\hat{w}_j^E + \hat{w}_j)/w_j$. Intuitively, (19) shows that for any k the initial shock $\frac{\hat{w}_k^E}{w_k}$ to the wage of any type k translates into a labor supply response of type k given by $\delta \varepsilon^{S,w} \frac{\hat{w}_k^E}{w_k}$, which in turn impacts the wage w_j of type j by the elasticity Γ_k. Therefore, the relevant disruption that the tax reform must compensate is $\hat{\Omega}_j^E$ rather than simply $\hat{w}_j^E/w_j$. Importantly, it is possible that empirical studies that evaluate the impact of a disruption on the wage distribution, capture not only the direct effect of the disruption, $\{\hat{w}_j^E\}_{j\in[0,1]}$, but also all of the indirect effects due to the labor demand spillovers in general equilibrium; this is the case, for instance, in our empirical application in Section 3. In this case, formula (18) can be applied directly using $\{\hat{\Omega}_j^E\}_{j\in[0,1]}$ as a primitive.[16]

2. Progressivity: Accounting for the decreasing marginal product of labor

To interpret the progressivity variable (19), we consider a slightly simpler production function than (3), with decreasing marginal product of labor but infinite substitutability between skills: $\mathscr{F}(\boldsymbol{L}) = \int_0^1 \theta_i L_i^{1-\frac{1}{\varepsilon^D}} di$.[17] We can easily show that in this

[15] See Section 3 for an application.

[16] Conversely, it is also straightforward to derive the exogenous disruption $\{\hat{w}_j^E\}_{j\in[0,1]}$ from the modified disruption $\{\hat{\Omega}_j^E\}_{j\in[0,1]}$.

[17] This reflects, for example, the downward-sloping demand curve for labor when there is a fixed factor of production, such as land or capital, for each type. We assume for simplicity that the government taxes firms' profits at 100%.

case, $\lambda = 0$ in formula (18), so that

$$\frac{\hat{T}(y_i)}{y_i} = (1 - T'(y_i)) \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i,y_j} \hat{\Omega}_{y_j}^E \, dy_j, \tag{21}$$

with $\hat{\Omega}_{y_j}^E = \delta \hat{w}_{y_j}^E / w_{y_j}$. To understand this expression, recall first that in partial equilibrium (i.e., as $\varepsilon^D \to \infty$), the tax reform that compensates a disruption $\{\hat{w}_i^E\}_{i \in [0,1]}$ is given by $\frac{\hat{T}(y_i)}{y_i} = (1 - T'(y_i)) \frac{\hat{w}_{y_i}^E}{w_{y_i}}$. That is, as we discussed in Section 1.4, the change in the average tax rate must exactly compensate the exogenous wage disruption, weighted by the retention rate of the initial tax schedule. Now, when the marginal product of labor is decreasing, instead, it is easy to show that equations (15) and (16) imply

$$\frac{\hat{T}(y_i)}{y_i} = (1 - T'(y_i)) \frac{\delta \hat{w}_{y_i}^E}{w_{y_i}} + \frac{\delta \varepsilon^{S,r}}{\varepsilon^D} \hat{T}'(y_i). \tag{22}$$

That is, the change in the average tax rate must now compensate both the (modified) wage disruption and, in addition, the wage correction generated endogenously by the marginal tax rate of the reform – recall that a change in the marginal tax rate by $\hat{T}'(y_i)$ impacts the labor supply of agents i by $\delta \varepsilon^{S,r} \hat{T}'(y_i)$, and hence their wage by $\frac{\delta \varepsilon^{S,r}}{\varepsilon^D} \hat{T}'(y_i)$. Solving this differential equation leads to the solution (21).

Now, consider in particular a disruption that raises the wage of skill i^* only, i.e. $\hat{w}_{i^*}^E > 0$ and $\hat{w}_i^E = 0$ for all $i \neq i^*$.[18] The partial-equilibrium compensation $\hat{T}(y_i)$ is then equal to zero for all incomes $y_i \neq y_{i^*}$that are not directly disrupted (i.e. $\hat{w}_i^E = 0$). In general equilibrium, instead, equation (22) shows that, for agents with income $y_i < y_{i^*}$ who are not initially disrupted, the compensating tax reform must satisfy $\frac{\hat{T}(y_i)}{y_i} = \frac{\delta \varepsilon^{S,r}}{\varepsilon^D} \hat{T}'(y_i)$. In order to raise the tax payment of agent i^* so as to redistribute his income gain, the government must raise the marginal tax rates on (at least some) incomes $y_i < y_{i^*}$, so that $\hat{T}'(y_i) > 0$.[19] But this generates a welfare gain for agent i – formally, an increase in the marginal tax rate of agent i by $\hat{T}'(y_i)$ lowers his labor supply by $\delta \varepsilon^{S,r} \hat{T}'(y_i)$ (by construction of the labor supply elasticity), so that his wage increases by $\frac{1}{\varepsilon^D} \delta \varepsilon^{S,r} \hat{T}'(y_i)$ (by construction of the labor demand elasticity). This benefit needs to be compensated counteracted by a welfare

[18]Formally, the disruption $\frac{\hat{w}_i^E}{w_i}$ is a Dirac delta function at skill i^*.

[19]For incomes $y > y_{i^*}$, we have $\hat{T}(y) = \hat{T}'(y) = 0$.

loss of equal magnitude via an increase in the average tax rate $\frac{\hat{T}(y_i)}{y_i} > 0$. Thus, the key insight is that in general equilibrium, the government impacts individual welfare through both the average and the marginal tax rates: an increase in the former (resp., the latter) lowers (resp., raises) the agent's utility. Therefore, a welfare compensating tax reform must be such that these two forces exactly cancel out, so that an agent that incurs a marginal tax rate increase must also incur an average tax rate increase. Crucially, notice that the key parameter is the ratio between the elasticity of labor supply and the elasticity of labor demand, which determines the extent to which an increase in the marginal tax rate raises the agent's welfare – by lowering his labor supply ($\varepsilon^{S,r}$) and raising his wage ($1/\varepsilon^D$).

The shape of the compensating tax reform $\hat{T}$ depends in particular on whether $\frac{\varepsilon^D}{\delta\varepsilon^{S,r}} \geq 1$, or equivalently $\frac{\varepsilon^D}{\varepsilon^{S,r}} \geq p$, where p is the local rate of progressivity of the initial tax schedule.[20] Suppose first that $\frac{\varepsilon^D}{\delta\varepsilon^{S,r}} = 1$, i.e., $\frac{\varepsilon^D}{\varepsilon^{S,r}} = p$. The relationship $\frac{\hat{T}(y_i)}{y_i} = \frac{\delta\varepsilon^{S,r}}{\varepsilon^D}\hat{T}'(y_i)$ then requires that the average and the marginal tax rates of the reform must coincide, so that the compensating tax schedule $\hat{T}$ must be linear for incomes $y_i \leq y_{i^*}$. More generally, the ratio between the marginal and the average tax rates must be equal to the constant $\frac{\varepsilon^D}{\delta\varepsilon^{S,r}} = 1 - p + \frac{\varepsilon^D}{\varepsilon^{S,r}}$, so that the tax reform that redistributes the wage gain of skill i^* is given by $\frac{\hat{T}(y_i)}{y_i} \propto y_i^{\varepsilon^D/\varepsilon^{S,r}-p}\,\mathbb{I}_{\{y_i \leq y_{i^*}\}}$.[21] Therefore the tax reform is *progressive* (resp., linear, regressive) on $[\underline{y}, y_{i^*})$, i.e. the change in the average tax rate $\hat{T}(y_i)/y_i$ is increasing (resp., constant, decreasing) with income, if and only if the ratio of the elasticity of labor demand and the elasticity of labor supply in the initial (undisrupted) economy, $\varepsilon^D/\varepsilon^{S,r}$, is larger than the rate of progressivity p of the pre-existing tax code. Empirically, the inequality $\frac{\varepsilon^D}{\varepsilon^{S,r}} > p$ is clearly satisfied since we have $p \approx 0.15$, $\varepsilon^{S,r} \approx 0.3$, and $\varepsilon^D \geq 0.5$.[22]

[20]This follows from the relationship $\frac{\varepsilon^D}{\delta\varepsilon^{S,r}} = 1 - p + \frac{\varepsilon^D}{\varepsilon^{S,r}}$, obtained using the definition of $\delta = 1/[1 + \frac{(1-p)\varepsilon^{S,r}}{\varepsilon^D}]$. Intuitively, the rate of progressivity of the initial tax schedule matters for the shape of the compensating tax reform, because if p is higher then a given increase in the marginal tax rate raises welfare by a larger amount – indeed, the wage increase that it induces leads to a smaller increase in labor supply, as $\varepsilon^{S,w} = (1-p)\,\varepsilon^{S,r}$. Thus the increase in the marginal tax rate that is necessary to compensate the welfare impact of a given rise in the average tax rate is smaller.

[21]Formally, this formula is obtained by letting $\hat{\Omega}^E_{y_j}$ be a Dirac delta function at y_{i^*} in formula (21).

[22]See our calibration for the U.S. in Section 3.

3. Compensation-of-compensation: Accounting for skill complementarities

The third novel effect in formula (18), the constant λ, is due to the cross-wage effects originating from the skill complementarities in production. Recall that expression (21) compensates both the individual welfare gains and losses generated by the initial wage disruption, and the own-wage effects induced endogenously by the compensation itself when the marginal product of labor is decreasing. Now, if the government implements this tax reform, a lower marginal tax rate at income y_j also affects all the other wages $\{w_k\}_{k\neq j}$ via the cross-wage elasticities Γ_{y_j}. The welfare impact of this indirect wage adjustment needs to be itself compensated using the tax schedule. In turn, the marginal tax rates of this second round of compensation generate further wage and welfare changes for every agent. These again must be compensated, and so on.

Formula (18) shows that when the production function is CES and the labor supply elasticities are constant, this series of "compensation-of-compensation" takes a strikingly simple form. Formally, the following argument explains why the uniform adjustment λ to the disruption is necessary and sufficient to compensate the cross-wage effects induced indirectly by the tax reform. We show in the Appendix that the compensating tax reform $\hat{T}$ satisfies the following equation, which generalizes the formula (21) obtained in the absence of cross-wage effects:

$$\frac{\hat{T}\left(y_i\right)}{y_i} = \left(1-T'\left(y_i\right)\right)\int_{y_i}^{\bar{y}}\mathscr{E}_{y_i,y_j}\left[\hat{\Omega}_{y_j}^{E}-\delta\int_{\underline{y}}^{\bar{y}}\Gamma_{y_k}\delta\varepsilon^{S,r}\frac{\hat{T}'\left(y_k\right)}{1-T'\left(y_k\right)}dy_k\right]dy_j. \quad (23)$$

Indeed, the average tax change at income y_i must now compensate both the (modified) exogenous wage disruption, as described above, but also the welfare effects induced by the changes in marginal tax rates at incomes $\{y_k\}_{k\in[0,1]}$. Specifically, an increase in the marginal tax rate at income y_k by $\hat{T}'\left(y_k\right)$ reduces the labor effort of skill k by $\delta\varepsilon^{S,r}\frac{\hat{T}'(y_k)}{1-T'(y_k)}$, by definition of the labor supply elasticity $\varepsilon^{S,r}$. This in turn reduces the wage of agent y_j by $\Gamma_{y_k}\times\delta\varepsilon^{S,r}\frac{\hat{T}'(y_k)}{1-T'(y_k)}$. Summing over all $k\in[0,1]$ leads to the second term in the square brackets of expression (23).

Notice that because of the terms $\hat{T}'\left(y_k\right)$ in the right hand side, equation (23) is a priori a non-trivial functional equation, and hence does not immediately deliver a closed form solution for the compensating tax reform – as opposed to the simpler equation (21). However, since the production function is CES and the labor supply elasticities are constant, the cross-wage elasticity Γ_{y_k} depends only on y_k, that is, a change in the labor supply of skill k affects the wage of all the other skills $j \in [0,1]$ by the same percentage amount. But this in turn implies that the marginal tax rate changes $\{\hat{T}'(y_k)\}_{k\in[0,1]}$ induce the same welfare effect $\lambda \equiv \delta \int_{\underline{y}}^{\bar{y}} \Gamma_{y_k} \delta\varepsilon^{S,r} \frac{\hat{T}'(y_k)}{1-T'(y_k)} dy_k$ on every agent $j \in [0,1]$. Therefore, the wage changes that must be compensated in addition to the exogenous disruption are simply a constant. Formula (18) follows immediately; the closed-form expression for λ given in Proposition 2 is obtained by solving the functional equation (23) explicitly.

Assuming for simplicity that $\bar{y} \to \infty$, the reform derived in (21) must be complemented by the following reform: $(1 - T'(y_i))\, y_i \int_{y_i}^{\infty} \mathscr{E}_{y_i,y_j} \lambda dy_j = \lambda (1 - T'(y_i))\, y_i$. This additional compensation consists of a constant change (in percentage terms) in the retention rate of the tax schedule: $\frac{\hat{T}'(y)}{1-T'(y)} = \lambda (1-p)$.[23] Therefore, compensating wage gain by agents with income y^* requires a uniform shift of the marginal tax rates in addition to the progressive reform on incomes $y < y^*$ already characterized in the absence of cross-wage effects.

1.7 Fiscal Surplus and other concepts of compensation

Once we know the compensating tax reform in closed-form, it is straightforward to obtain an expression for the fiscal surplus $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$ defined in (41), i.e., the budget impact of the wage disruption and its compensation (Corollary 2 in the Appendix).

The welfare gains of the wage disruption $\hat{\boldsymbol{w}}^E$ are redistributable if and only if $\mathcal{R}(\hat{\boldsymbol{w}}^E) \geq 0$. This, in turn, means that it is possible to use the tax system to obtain a Pareto improvement.[24] More generally than its sign, the value of the fiscal surplus is important: it provides a metric that allows to compare, in monetary units, different economic shocks. For example, suppose that a given disruption (say, an inflow of immigration) generates more revenue, after implementing the compensating tax reform, than another (say, automation). It follows that the government can achieve a strictly better Pareto improvement from the former shock. s.[25] Therefore, $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$ provides

[23] We can easily show that this is equivalent to an increase in the parameter τ of the baseline tax schedule $T(y) = y - \frac{1-\tau}{1-p} y^{1-p}$ by an amount $\hat{\tau}$ given by $\frac{\hat{\tau}}{1-\tau} = \lambda (1-p)$.

[24] For instance, the government can redistribute lump-sum the budget surplus, which creates no distortions since the utility is quasilinear and raises everybody's welfare.

[25] To see this, consider the best possible redistribution of the tax revenue in case B, say $\mu^B \hat{T}^B$. By implementing a tax reform that has the same direction $\hat{T}^B$ but a strictly higher magnitude $\mu^A > \mu^B$ (this is possible due to the larger amount of revenue that is available) in case A, the government

as a measure of the benefit or cost of a given economic shock.

Finally, it is natural to wonder what the compensating tax reform would be if the government's objective were to compensate every agent so that their welfare would be at least as large (rather than exactly as large) as in the initial economy – i.e., such that $\tilde{U}_i \geq U_i$ for all i in equation (6). In this case, equation (15) would be an inequality and there would be an obvious multiplicity of solutions to the compensation problem. The first way to address the issue is to implement the exact compensation (18), and in a second (simultaneous) stage use the extra revenue $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$, if any, to achieve a Pareto improvement. The second way to address it is to solve the compensation problem by replacing 0 with a given function $h(\cdot)$ in the left hand side of (15), with $h(y_i) \geq 0$ for all i. That is, we solve the compensation problem by directly specifying the positive welfare improvements that we want to achieve for every skill level. The corresponding compensating tax reform can then be derived following identical steps as in the proof of Proposition 2, and its solution would depend directly on the desired function h.

2 The General Model

In this section we relax all of the major assumptions we made in Section 1 and derive a closed-form generalization of formula (18) for the compensating tax reform.

2.1 Initial equilibrium

Agents differ along two dimensions: their skill $i \in [0,1]$, as in Section 1, and their fixed cost of participating in the labor force $\kappa \in \mathbb{R}_+$.[26] An agent with types (i, κ) has idiosyncratic preferences over consumption c and labor supply l described by $u_i(c,l) - \kappa \mathbb{I}_{\{l>0\}}$, where the utility function u_i is a general, twice continuously differentiable function that satisfies $u'_{i,c} > 0$, $u''_{i,cc} \leq 0$, $u'_{i,l}, u''_{i,ll} < 0$, and where $\mathbb{I}_{\{l>0\}}$ is an indicator function equal to 1 if the agent is employed. If the agent decides to work, he earns a wage w_i, chooses his labor supply (hours) l_i, earns pre-tax labor income $y_i = w_i l_i$, and

achieves a strictly higher welfare improvement, since the first-order welfare measures are linear in μ by construction.

[26] These two characteristics can be arbitrarily correlated in the population.

pays a labor income tax $T(y_i)$.[27] If he decides to stay non-employed, his labor supply is equal to zero and he earns the government-provided transfer $-T(0)$. Finally, he also owns an exogenous quantity k_i of the economy's capital stock, which earns a pre-tax return r.[28] Capital income is taxed at the constant rate τ.

The maximization problem of agent (i,κ) reads

$$U_{i,\kappa} \equiv \max \left\{ \max_{l>0} \; u_i\left(c_i\left(l\right), l\right) - \kappa \, ; \, u_i\left(c_i\left(0\right), 0\right) \right\}. \tag{24}$$

where $c_i(l)$ is defined by the budget constraint: $c_i(l) = w_i l - T(w_i l) + (1-\tau)\, r\, k_i$ for any $l \geq 0$. Conditional on working, agent (i,κ) chooses labor supply l_i that satisfies the first-order condition

$$-\frac{u'_{i,l}\left(c_i\left(l_i\right), l_i\right)}{u'_{i,c}\left(c_i\left(l_i\right), l_i\right)} = \left[1 - T'\left(w_i l_i\right)\right] w_i. \tag{25}$$

We assume that l_i is the unique solution to this problem. Moreover, the agent decides to participate if and only if his fixed cost of work κ is smaller than a threshold κ_i^*, given by

$$\kappa_i^* = u_i\left[w_i l_i - T\left(w_i l_i\right) + (1-\tau)\, r\, k_i \, , \, l_i\right] - u_i\left[-T\left(0\right) + (1-\tau)\, r\, k_i \, , \, 0\right]. \tag{26}$$

Denote by $f_i(\kappa)$ the density of κ conditional on skill i and by $L_i = l_i \int_0^{\kappa_i^*} f_i(\kappa)\, d\kappa$ the total amount of labor supplied by workers of skill i.

Firms produce output using the aggregate labor supply L_i of each type $i \in [0,1]$ and the aggregate capital stock K, which we assume to be in fixed supply. The aggregate production function is denoted by $\mathscr{F}(\{L_i\}_{i\in[0,1]}\, , K)$. We assume that $\mathscr{F}$ has constant returns to scale. In equilibrium, firms earn no profits and the wage w_i is equal to the marginal product of type-i labor, i.e.,

$$w_i = \mathscr{F}'_i(\{L_j\}_{j\in[0,1]}\, , K). \tag{27}$$

The equilibrium interest rate is equal to the marginal product of capital, i.e., $r =$

[27] As in Section 1, we order skills so that there is a one-to-one map between skills i and wages w_i in the initial equilibrium with tax schedule T. See Appendix A.2 for details.

[28] We impose that all agents with a given skill i, i.e. a given wage w_i, own the same amount of capital, which ensures that they all choose the same level of labor supply (conditional on working) l_i, independent of their fixed cost of working. We discuss this assumption in Appendix A.2.

$\mathscr{F}'_K(\{L_j\}_{j\in[0,1]}, K)$.

The government levies taxes on labor and capital incomes. The initial labor income tax schedule is twice continuously differentiable but is allowed to be arbitrarily nonlinear. We define the local rate of progressivity of the tax schedule as (minus) the elasticity of the retention rate $(1 - T'(y))$ with respect to gross income y: $p(y) = -\frac{\partial \ln(1-T'(y))}{\partial \ln y}$. Finally, we restrict the initial tax schedule and tax reforms on capital income to be linear.

2.2 The welfare compensation problem

We define an exogenous wage disruption analogously to Section 1.2, and denote by $\mu\hat{r}^E$ the corresponding disruption to the interest rate, i.e., the difference between the marginal productivities of capital before and after the shock, keeping individual labor supplies fixed at their pre-disruption level. The government can implement an arbitrarily nonlinear reform $\mu\hat{T}$ of the labor income tax schedule, and a reform of the capital income tax rate by $\mu\hat{\tau}$. In response to a disruption $(\mu\hat{\boldsymbol{w}}^E, \mu\hat{r}^E)$ and a tax reform $(\mu\hat{T}, \mu\hat{\tau})$, individuals optimally adjust their labor supply and participation decisions. In general equilibrium, this further impacts their wage and the interest rate, which in turn affects again their labor supply choices, and so on. We denote by $\mu\hat{w}_i$, $\mu\hat{r}$, $\mu\hat{l}_i$ and $\mu\hat{\kappa}_i^*$ the total endogenous changes in individual i's wage, interest rate, labor supply (conditional on working) and participation threshold, respectively, following the disruption and tax reform. That is, in the disrupted economy we have $\tilde{w}_i = w_i + \mu\hat{w}_i^E + \mu\hat{w}_i$, $\tilde{r} = r + \mu\hat{r}^E + \mu\hat{r}$, $\tilde{l}_i = l_i + \mu\hat{l}_i$ and $\tilde{\kappa}_i^* = \kappa_i^* + \mu\hat{\kappa}_i^*$. We finally denote by $\tilde{U}_{i,\kappa} = U_{i,\kappa} + \mu\hat{U}_{i,\kappa}$ the resulting indirect utility of agents with type (i, κ) in the final equilibrium. The welfare compensation problem consists of designing a reform $(\mu\hat{T}, \mu\hat{\tau})$ of the tax system such that the welfare of every agent is the same as it was before the wage disruption; that is, $\tilde{U}_{i,\kappa} = U_{i,\kappa}$ for all $(i, \kappa) \in [0, 1] \times \mathbb{R}_+$.

We start by proving that if the government implements the welfare compensating policy, then it must be the case that no agent switches participation status, i.e., $\hat{\kappa}_i^* = 0$ for all i. Indeed, first note that we can always choose to adjust the capital income tax rate by $\frac{\hat{\tau}}{1-\tau} = \frac{\hat{r}}{r}$, so that the net of tax return $(1 - \tau)\, r$, and hence the capital income of each agent, remains constant. Thus, we can leave unchanged the welfare $u_i[-T(0) + (1 - \tau)\, r\, k_i\,, 0]$ of agents who are non-employed both before and after the perturbation by keeping the unemployment transfer $-T(0)$ unaffected. Moreover, in order to leave unchanged the welfare of agents who are employed both before and after the perturbation, the combination of the wage disruption and the tax reform must make the utility $\tilde{U}_i \equiv u_i[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + (1-\tau)\ r\,k_i\ ,\ \tilde{l}_i]$ equal to its initial value U_i for all i. Now, since the participation decision (26) of an individual with skill i depends only on the difference between the utilities conditional on employment and on non-employment, we obtain that the participation threshold κ_i^* must also remain constant for all i. That is, in order to leave everyone's welfare unchanged, the compensating tax reform must ensure that the individuals who were employed (resp., non-employed) before the disruption remain so in the new equilibrium.[29]

The welfare compensation problem therefore consists of constructing a labor income tax reform $\hat{T}$ such that the welfare of each employed agent in the disrupted economy is equal to their welfare in the initial equilibrium:

$$U_i \;=\; \tilde{U}_i \;\equiv\; u_i[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + (1-\tau)\ r\,k_i\ ,\ \tilde{l}_i], \tag{28}$$

where $(\tilde{w}_i, \tilde{l}_i)$ are defined by the perturbed first-order condition

$$-\frac{u'_{i,l}[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + (1-\tau)\,rk_i, \tilde{l}_i]}{u'_{i,c}[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) + (1-\tau)\,rk_i, \tilde{l}_i]} = [1 - T'(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}'(\tilde{w}_i\tilde{l}_i)]\tilde{w}_i, \tag{29}$$

and the perturbed wage equation

$$\tilde{w}_i \;=\; \tilde{\mathscr{F}}'_i(\{L_j + \hat{L}_j^E + \mu\hat{l}_j\}_{j\in[0,1]}\ ,\ \tilde{K}). \tag{30}$$

As in Section 1, our goal is to characterize analytically the solution to the welfare compensation problem for *marginal* wage disruptions, i.e., as $\mu \to 0$. The proofs are gathered in the Appendix.

2.3 Elasticity concepts

We first define the elasticities $\varepsilon_i^{S,r}$, $\varepsilon_i^{S,w}$ and $\varepsilon_i^{S,n}$ of labor supply l_i with respect to the retention rate $r_i \equiv 1 - T'(w_il_i)$, the wage w_i, and the non-labor (lump-sum) income

[29] This implies in particular that the values of the elasticities of participation with respect to the tax rates (which otherwise would matter to determine the endogenous wage adjustments $\hat{w}_i$) are irrelevant for the construction of the compensating tax reform.

n_i, as

$$\varepsilon_i^{S,r} \equiv \frac{\partial \ln l_i}{\partial \ln r_i} = \frac{e_i^c}{1+p\left(y_i\right)e_i^c}, \ \varepsilon_i^{S,w} \equiv \frac{\partial \ln l_i}{\partial \ln w_i} = \frac{\left(1-p\left(y_i\right)\right)e_i^c+e_i^n}{1+p\left(y_i\right)e_i^c}, \ \varepsilon_i^{S,n} \equiv r_i \frac{\partial \ln l_i}{\partial n_i} = \frac{e_i^n}{1+p\left(y_i\right)e_i^c},$$

where e_i^c is the standard Hicksian (compensated) elasticity of labor supply, and e_i^n is the standard income effect parameter, defined by the Slutsky equation $e_i^n = e_i^u - e_i^c$, where e_i^u is the Marshallian (uncompensated) elasticity of labor supply. The interpretation of these variables is identical to those in Section 1.1, and their formal closed-form expressions are given by equations in the Appendix.

Second, as in Section 1.1, we define the cross-wage (resp., own-wage) elasticity of skill j with respect to the labor supply of skill i (resp., j) as

$$\gamma_{i,j} \equiv \frac{\partial \ln w_i}{\partial \ln L_j} = \frac{L_j \mathscr{F}_{ij}''\left(\boldsymbol{L},K\right)}{\mathscr{F}_i'\left(\boldsymbol{L},K\right)}, \quad -\frac{1}{\varepsilon_j^D} \equiv \frac{\partial \ln w_j}{\partial \ln L_j} - \lim_{i\to j} \frac{\partial \ln w_i}{\partial \ln L_j},$$

where $\mathscr{F}_{ij}''$ denotes the second partial derivative of the production function with respect to the variables (L_i, L_j). The interpretations of these expressions are identical to those in Section 1.1, except that for a general (non-CES) production function $\mathscr{F}$, the cross-wage elasticities $\gamma_{i,j}$ now depend on both skills i and j, and the own-wage elasticities $1/\varepsilon_j^D$ depend on the skill j and hence are no longer constant.

2.4 Compensation in Partial Equilibrium

As in Section 1.4, we start by assuming that wages are exogenous. Equation (11) holds in our more general environment with arbitrary preferences and an arbitrary initial tax system. As a result, the compensating tax reform in partial equilibrium is still given by $\hat{T}\left(y_i\right) = \left(1-T'\left(y_i\right)\right) y_i \frac{\hat{w}_i^E}{w_i}$. With preferences that are no longer quasilinear, however, the change in labor supply of agent i can now be expressed in terms of the elasticity notations introduced in Section 2.1 as:

$$\frac{\hat{l}_i^{\mathbf{pe}}}{l_i} = \varepsilon_i^{S,w} \frac{\hat{w}_i^E}{w_i} - \varepsilon_i^{S,r} \frac{\hat{T}'\left(y_i\right)}{1-T'\left(y_i\right)} + \varepsilon_i^{S,n} \frac{\hat{T}\left(y_i\right)}{\left(1-T'\left(y_i\right)\right)y_i}. \tag{31}$$

In addition to the changes in the exogenous wage $\hat{w}_i^E$ and in the marginal tax rate $\hat{T}'\left(y_i\right)$ already discussed in the context of equation (12), a third variable now causes an adjustment in the labor supply of agent i: namely, the change in his average tax rate $\hat{T}\left(y_i\right)/y_i$ induces a response of hours determined by the income effect parameter $\varepsilon_i^{S,n}$.

2.5 Compensation in General Equilibrium

We now turn to the general-equilibrium model and follow the same steps as in Section 1.5. A first-order Taylor expansion of the perturbed wage equation (30) leads to

$$\frac{\hat{w}_i}{w_i} = -\frac{1}{\varepsilon_i^D}\frac{\hat{l}_i}{l_i} + \int_0^1 \gamma_{i,j}\frac{\hat{l}_j}{l_j}dj. \tag{32}$$

This equation generalizes (to the case of non-constant own- and cross-wage elasticities) equation (14) in the simpler model of Section 1 and has the same economic interpretation.

A first-order Taylor expansion of equation (28) around the initial equilibrium implies that the change in the indirect utility of agent i induced by the wage disruption and the tax reform (weighted by the marginal utility of consumption to obtain a monetary measure of welfare), $\hat{U}_i/u'_{c,i}$, is given by:

$$0 = \frac{\hat{U}_i}{u'_{i,cs}} = \left(1-T'\left(y_i\right)\right)y_i\left[\frac{\hat{w}_i^E}{w_i}+\frac{\hat{w}_i}{w_i}\right]-\hat{T}\left(y_i\right). \tag{33}$$

This equation generalizes (to the case of non-quasilinear preferences) equation (15) in the simpler model of Section 1 and has the same economic interpretation.

A first-order Taylor expansion of equation (28), which imposes that the labor supply of agent i remains optimal in the disrupted economy, can be written as:

$$\frac{\hat{l}_i}{l_i} = \varepsilon_i^{S,w}\left[\frac{\hat{w}_i^E}{w_i}+\frac{\hat{w}_i}{w_i}\right]-\varepsilon_i^{S,r}\frac{\hat{T}'\left(y_i\right)}{1-T'\left(y_i\right)}+\varepsilon_i^{S,n}\frac{\hat{T}\left(y_i\right)}{\left(1-T'\left(y_i\right)\right)y_i}. \tag{34}$$

This equation generalizes (to the case of non-constant elasticities and non-quasilinear preferences) equation (16) in the simpler model of Section 1 and has the same economic interpretation, except that labor supply now also adjusts in response to a change in the average tax rate $\hat{T}\left(y_i\right)/y_i$ by an amount given by the income effect parameter $\varepsilon_i^{S,n}$.

Substituting for the endogenous wage adjustment $\hat{w}_i$ in equation (34) and using (32) leads to an integral equation for the labor supply changes of all agents, $\{\hat{l}_j\}_{j\in[0,1]}$. The following lemma, which generalizes Lemma 2 derived in the simpler model of Section 1 and which follows from Proposition 1 in Sachs, Tsyvinski, and Werquin [2016], gives the closed-form solution to this equation.

Lemma 2. *Assume that* $\int_{[0,1]^2} \left|\delta_i \varepsilon_i^{S,w} \gamma_{ij}\right|^2 didj < 1.$[30] *The solution to (34) is given by: for all* $i \in [0,1]$,

$$\frac{\hat{l}_i}{l_i} = \delta_i \frac{\hat{l}_i^{\boldsymbol{pe}}}{l_i} + \delta_i \, \varepsilon_i^{S,w} \int_0^1 \Gamma_{i,j} \, \delta_j \, \frac{\hat{l}_j^{\boldsymbol{pe}}}{l_j} \, dj, \tag{35}$$

where $\hat{l}_i^{\boldsymbol{pe}}$ *is defined by (31),* $\delta_i \equiv 1/[1 + \frac{\varepsilon_i^{S,w}}{\varepsilon_i^D}]$, *and* $\Gamma_{i,j} \equiv \sum_{n=0}^{\infty} \Gamma_{i,j}^{(n)}$ *with* $\Gamma_{i,j}^{(0)} = \gamma_{i,j}$ *and for all* $n \geq 1$, $\Gamma_{i,j}^{(n)} = \int_0^1 \Gamma_{i,k}^{(n-1)} \, \delta_k \varepsilon_k^{S,w} \, \gamma_{k,j} \, dk$.

Equation (35) has the same structure and interpretation as (17) in the simpler model of Section 1, except that the expression for the total cross-wage effect $\Gamma_{i,j}$ is now defined by a series $\sum_{n=0}^{\infty} \Gamma_{i,j}^{(n)}$. Recall that, in contrast to the structural elasticity $\gamma_{i,j}$, the variable $\Gamma_{i,j}$ captures the adjustment in the wage of type i caused by a change in the labor supply of type j, accounting for the infinite sequence of feedback cross-wage effects between different skills that occur in general equilibrium. First, the initial change in type-j labor supply, $\delta_j \, \hat{l}_j^{\mathbf{pe}}/l_j$, directly affects the wage of type i through the structural elasticity $\gamma_{i,j}$ – this is the first term $\Gamma_{i,j}^{(0)}$ in the series defining $\Gamma_{i,j}$. Second, the change in labor supply of type j affects the wage of every other type k by $\gamma_{k,j}$, hence the labor supply of type k by $\delta_k \varepsilon_k^{S,w} \gamma_{k,j}$, which in turn impacts the wage of type i by $\gamma_{i,k} \delta_k \varepsilon_k^{S,w} \gamma_{k,j}$ – this is the second term $\Gamma_{i,j}^{(1)}$. By induction, $\Gamma_{i,j}^{(n)}$ represents the adjustment in w_i through the behavior of $(n-1)$ intermediate types, e.g., for $n = 3$, $j \to k_1 \to k_2 \to i$.[31]

Taking stock. As in the simpler model of Section 1, individual welfare is now affected both by the average tax rates *and* the marginal tax rates, because by affecting agents' labor supplies the latter impact their wages and hence utilities. As a result, equation (33) does not directly lead to a formula for the compensating tax reform: we need to solve for the fixed point between the average and marginal tax rates of the compensation. However, because of the non-constant elasticities and the presence of

[30] This condition ensures that the series defining $\Gamma_{i,j}$ converges. The assumptions made in Section 1 provide sufficient conditions on primitives such that this condition is satisfied.

[31] Note that in the simpler version of the model of Section 1, we showed that the total cross-wage effect $\Gamma_{i,j}$ is equal to the structural elasticity $\gamma_{i,j}$ amplified by the factor $1/\delta_i$. This is because a CES production function, where $\gamma_{i,j}$ depends only on j, along with constant labor supply elasticities $\delta_k \varepsilon_k^{S,w} = \delta \varepsilon^{S,w}$, implies that each round of general equilibrium wage adjustment is a constant fraction of the first round.

income effects in preferences, constructing such a tax reform is more difficult than in the model of Section 1.

Before proceeding to the analytical solution to the compensation problem, we summarize the various ways through which taxes affect welfare in the general model. Suppose for simplicity that the cross-wage elasticities $\gamma_{i,j}$ are positive for all $i \neq j$. A higher average tax rate at income y^*, $\hat{T}(y^*) > 0$, implies:[32] (a) a reduction in welfare of agent y^*, by directly making him poorer, as in partial equilibrium (third term in equation (33)); (b) a reduction in welfare of agent y^*, by making him work more (income effect, third term in (31)) and hence earn a lower wage (decreasing marginal product, first term in (32));[33] (c) an increase in welfare of all agents $y \neq y^*$, whose wage increases due to the higher labor supply of agent y^* (production complementarities, second term in (32)). Moreover, a higher marginal tax rate at income y^*, $\hat{T}'(y^*) > 0$, implies: (a) a higher average tax rate for all incomes $y > y^*$, since $\hat{T}(y) = \int_0^y \hat{T}'(x)\,dx$, which has analogous welfare consequences to those we just described; (b) an increase in welfare of agent y^*, by making him work less, as in partial equilibrium (substitution effect, second term in (31)), and hence earn a higher wage (decreasing marginal product, first term in (32)); (c) a reduction in welfare of all agents $y \neq y^*$, whose wage decreases due to the lower labor supply of agent y^* (production complementarities, second term in (32)).

Main result. The next Theorem, which generalizes Proposition 2, gives a closed-form characterization of the compensating tax reform in response to any wage disruption in general equilibrium. This is the main result of the paper.[34]

Theorem 1. *Consider a marginal disruption in the direction* $\hat{\boldsymbol{w}}^E = \{\hat{w}_i^E\}_{i\in[0,1]}$ *of the wage distribution* $\boldsymbol{w}$. *The following tax reform* $\hat{T}$ *solves the welfare compensation*

[32] For simplicity we ignore the effects of the changes in the average and marginal tax rates on wages and welfare through agents' participation decisions, since we argued above that no agent switches participation status if the government implements the correct compensating reform.

[33] Because the cross-wage elasticities $\gamma_{i,j}$, and hence $\Gamma_{i,j}$, are positive, the wage and welfare of agent y^* are still reduced after taking into account the second, third, etc. rounds of general equilibrium spillovers. This follows from equation (43) in the Appendix. The same reasoning applies for the next bullet points.

[34] Note that since there is a one-to-one map between types i and incomes y_i, we can change variables and index by income the wages $w_{y_i} \equiv w_i$, labor supplies $l_{y_i} \equiv l_i$, wage disruptions $\hat{w}_{y_i}^E \equiv \hat{w}_i^E$, and elasticities $\varepsilon_{y_i}^{S,x} \equiv \varepsilon_i^{S,x}$ for $x \in \{r, w, n\}$, $\varepsilon_{y_i}^D \equiv \varepsilon_i^D$, and $\gamma_{y_i,y_j} \equiv \gamma_{i,j}/y'(j)$, $\Gamma_{y_i,y_j} \equiv \Gamma_{i,j}/y'(j)$.

problem: for all i,

$$\hat{T}(y_i) = (1-T'(y_i))\, y_i \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i,y_j} \left[\hat{\Omega}^E_{y_j} + \int_{\underline{y}}^{\bar{y}} \Lambda_{y_j,y_k} \hat{\Omega}^E_{y_k} dy_k\right] dy_j, \tag{36}$$

where the modified wage disruption *variable* $\hat{\Omega}^E$ *is defined by*

$$\hat{\Omega}^E_{y_j} \equiv \delta_{y_j} \frac{\hat{w}^E_{y_j}}{w_{y_j}} + \delta_{y_j} \int_{\underline{y}}^{\bar{y}} \Gamma_{y_j,y_k}\, \delta_{y_k} \varepsilon^{S,w}_{y_k} \frac{\hat{w}^E_{y_k}}{w_{y_k}} dy_k, \tag{37}$$

the progressivity *variable* $\mathscr{E}$ *is defined by*

$$\mathscr{E}_{y_i,y_j} \equiv \frac{\varepsilon^D_{y_j}}{\delta_{y_j} \varepsilon^{S,r}_{y_j} y_j} e^{-\int_{y_i}^{y_j} \left[\frac{\varepsilon^D_{y_k} + 2\varepsilon^{S,n}_{y_k}}{\varepsilon^{S,r}_{y_k}}\right] \frac{dy_k}{y_k}}, \tag{38}$$

and the compensation-of-compensation *variable* $\Lambda_{y_i,y_j} \equiv \sum_{n=0}^{\infty} \Lambda^{(n)}_{y_i,y_j}$ *is defined by*

$$\Lambda^{(0)}_{y_i,y_j} = \delta_{y_i} \Gamma_{y_i,y_j} \varepsilon^D_{y_j} - \delta_{y_i} \int_{\underline{y}}^{y_j} \Gamma_{y_i,y_k} \varepsilon^D_{y_k} \mathscr{E}_{y_k,y_j} dy_k, \tag{39}$$

and for all $n \geq 1$, $\Lambda^{(n)}_{y_i,y_j} = \int_{\underline{y}}^{\bar{y}} \Lambda^{(n-1)}_{y_i,y_k} \delta_{y_k} \Lambda^{(0)}_{y_k,y_j} dy_k$. *Corollary 2 in the Appendix gives the fiscal surplus* $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$.

Analogously to equation (18), formula (36) features three departures from the partial-equilibrium compensation (13). First, the modified wage disruption $\hat{\Omega}^E$ accounts for the full incidence on wages of the initial shock, and has the same interpretation as (19). Second, the progressivity variable $\mathscr{E}$ is a direct generalization of (20), and has the same interpretation.[35]

Third, the compensation-of-compensation term (the integral in the square brackets of (36)), which accounts for the cross-wage effects originating from the skill complementarities in production, is now more complex than in (18). Indeed, the functional equation (23) is more difficult to solve when the labor supply of type k does not have the same impact on the wage of two different skills j, j', so that Γ_{y_j,y_k} can depend arbitrarily on y_j. Our proof shows that for each k, the welfare impact of these indirect

[35]It is immediate to show that expression (38) reduces to (20) when the labor supply and demand elasticities are constant and there are no income effects , so that $\frac{\varepsilon^D_{y_k} + 2\varepsilon^{S,n}_{y_k}}{\varepsilon^{S,r}_{y_k}} = \frac{\varepsilon^D}{\varepsilon^{S,r}}$ is a constant.

wage adjustments is determined by the first term $\Lambda^{(0)}_{y_j,y_k}$ in the series Λ_{y_j,y_k} defined in (39), so that the total effect on type j is given by $\int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}_{y_j,y_k} \hat{\Omega}^E_{y_k} dy_k$. This welfare change needs to be itself compensated using the tax schedule, thus leading to the term $(1 - T'(y_i))\, y_i \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i,y_j} [\int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}_{y_j,y_k} \hat{\Omega}^E_{y_k} dy_k] dy_j$ in (36). In turn, the marginal tax rates of this second round of compensation generate further wage and welfare changes for all of the agents. These again must be compensated (third round of "compensating the compensation"), leading to the term $(1 - T'(y_i))\, y_i \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i,y_j} [\int_{\underline{y}}^{\bar{y}} \Lambda^{(1)}_{y_j,y_k} \hat{\Omega}^E_{y_k} dy_k] dy_j$ in (36). The full sequence of tax reforms that achieves the fixed point of the compensation problem is constructed by defining inductively the sequence of variables $\Lambda^{(n)}_{y_i,y_j}$ for all $n \geq 0$, where each $\Lambda^{(n)}_{y_i,y_j}$ captures one round of iterated compensation.

3 Compensating the Impact of Robots

In this final section, we show how our theoretical results can be straightforwardly implemented in an empirical application: compensating the welfare consequences of robotization in the U.S. and the German economies. For reasons of space the analysis of Germany is in Appendix B.1.

The data on the impact of robots in the U.S. are obtained from the 1990 and 2007 Censuses and were provided to us by Daron Acemoglu and Pascual Restrepo. Specifically, Acemoglu and Restrepo [2017] estimate the impact of industrial automation on different skill cells defined by age, gender, education and race. They give the baseline size and employment rate (share with salaried jobs), hourly wage, and hours worked per year of each group. Acemoglu and Restrepo [2017] then estimate the impact of one additional robot per thousand workers in each skill cell on log-wages, hours and employment rate.[36] These estimates are obtained by comparing two people in the same cell but who reside in commuting zones with different exposure to industrial automation. They include both the direct effects of robots on employment and wages and any indirect spillover effects that might arise because of a resulting decline in local demand. In other words, they estimate the modified disruption $\hat{\Omega}^E$ rather than $\hat{\boldsymbol{w}}^E$.

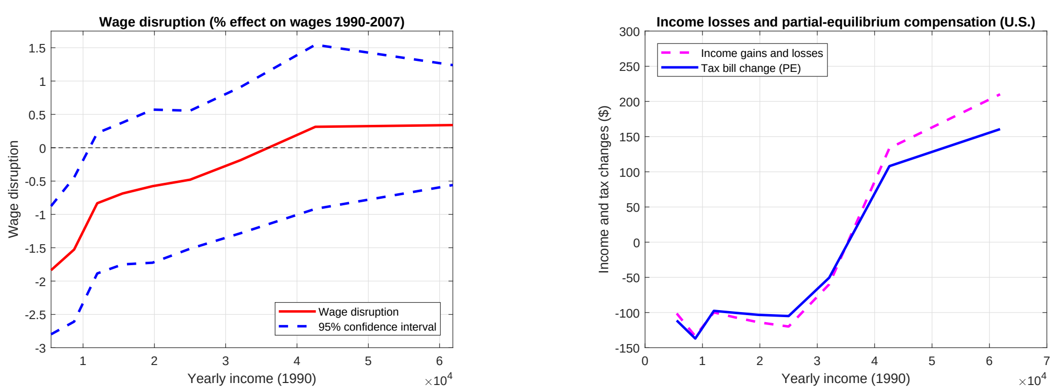
We assume that the economy is described by the model of Section 1. The initial tax schedule is CRP with $p = 0.156$ and $\tau = -3$ (Heathcote, Storesletten, and Violante [2016]). The production function is CES with $\varepsilon^D = 0.6$ (Dustmann, Frattini, and

[36]This corresponds to the increase in robots observed in the US between 1990 and 2007.

Preston [2013]), $\varepsilon^D = 1.5$, or $\varepsilon^D = \infty$ (partial equilibrium). We estimate the labor supply elasticity in their data[37] and find $\varepsilon^{S,r} = 0.47$, which is in the range of the empirical estimates. The left panel of Figure 1 plots the wage disruption (i.e., the percentage change in the wage) along the baseline (1990) earnings distribution, as well as the standard errors. Consistent with our theoretical analysis, we group agents by wage deciles, so that the values of the wage disruption $100 \times \frac{\hat{w}_i}{w_i}$ (in the y-axis) are those reported in Figure 13 of Acemoglu and Restrepo [2017]. This figure shows that the change in the wage due to automation is increasing with the agent's position in the income distribution. The lowest wages in 1990 are reduced by 1.84%, while the 80th and 90th percentiles experience an estimated increase in their wage of 0.31% and 0.34%.

We can now apply our theoretical formulas (13) and (18) to this disruption. In the right panel of Figure 1, we plot the implied income losses (dashed magenta curve), as well as the compensating tax reform $\hat{T}$ (solid blue curve) obtained in the partial equilibrium environment (formula (13)). The partial-equilibrium compensation corrects for the fact that the initial tax schedule is progressive but otherwise tracks one-for-one the shape of the income gains and losses. The 10th income percentile ($5,500 per year) have their tax bill reduced by $100 (i.e., 110% of their income loss), while the 90th income percentile ($62,000 per year) face a tax increase of $160 per year (i.e., 76% of their income gain).[38]

Figure 1: Wage disruption (left) and Partial-equilibrium compensation (right)

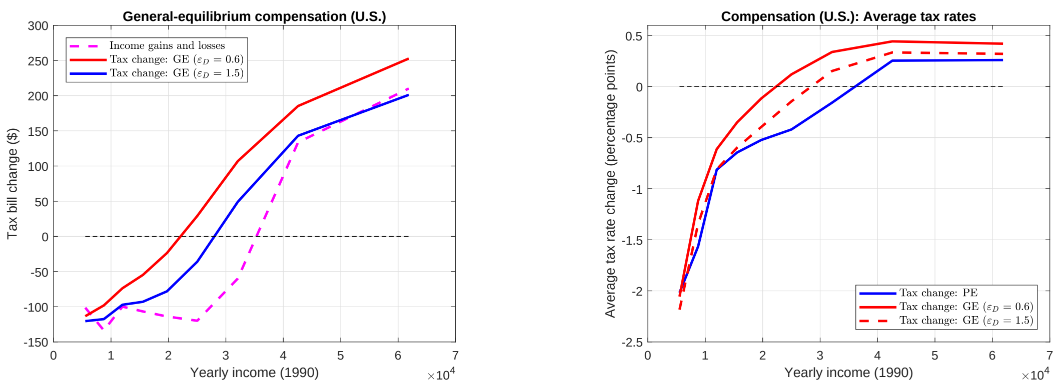


[37] See Appendix B.1 for details.

[38] Formula (13) implies that the ratio of the tax change to the income disruption is larger than 100% for low-income agents and then decreasing, because the marginal tax rate $T'(y_i)$ of the initial tax schedule is negative at the bottom and then increasing.

The left panel of Figure 2 plots the compensation in general equilibrium. The dashed magenta curve gives the gross income losses caused by the exogenous disruption, and the solid red (resp., blue) curve gives the tax changes $\hat{T}(y_i)$ for $\varepsilon^D = 0.6$ (resp., $\varepsilon^D = 1.5$). Importantly, since the tax change at a given income y in formula (18) depends on the disruption affecting agents with incomes larger than y and up to the top of the income distribution $\bar{y}$, we need to make an assumption about the disruption on incomes higher than the largest in our dataset (about \$60,000). In Figure 2 we make the conservative assumption that incomes above \$60,000 incur the same wage disruption as those who earn \$60,000, i.e., their wage increases by 0.34%. In the Appendix (Figure 4), we assume that the size of the wage disruption continues to increase linearly above \$60,000, albeit at a slow rate – agents with an income 20 times larger face a disruption that is 3 times larger, i.e., their wage increases by 1.02% due to robots.

Figure 2: General-equilibrium compensation (left) and Average tax changes (right)



To compensate their income loss, low-income agents get a tax rebate equal to \$113 if $\varepsilon^D = 0.6$ (i.e., 111.9% of their income loss) or \$120 if $\varepsilon^D = 1.5$ (i.e., 118.9% of their income loss). To redistribute their income gain, high-income agents face an increase in their tax payment equal to \$260 if $\varepsilon^D = 0.6$ (i.e., 124% of their income gain) or \$198 if $\varepsilon^D = 1.5$ (i.e., 94% of their income gain). The right panel of Figure 2 plots the changes in the average tax rates induced by the reform, i.e. $\hat{T}(y_i)/y_i$. The average tax rate on low incomes is reduced by 2.1 percentage points (resp., 2.18 pp) if $\varepsilon^D = 0.6$ (resp., $\varepsilon^D = 1.5$), while that on high incomes is increased by 0.42 pp (resp., 0.32 pp). Recall that these numbers are for one additional robot per thousand workers; when more robots are introduced, the compensation should be scaled accordingly. Finally, applying the formula of Corollary 2, we obtain that the the robot-induced disruption generates a small fiscal surplus equal to \$16 if $\varepsilon^D = 0.6$, but a small fiscal deficit of \$11.5 if $\varepsilon^D = 1.5$ and a deficit of \$37.3 in partial equilibrium.

Note that the trade-off between compensating the welfare gains and losses using average versus marginal tax rates implies that the tax increases must be front-loaded in general equilibrium, in order to not generate increases in tax rates as steep as the income disruption (e.g., between \$25,000 and \$45,000). Note moreover that at the top, the increase in the tax payment is larger than the increase in income caused by the disruption, and larger than the partial-equilibrium compensation. This is because these agents also face an increase in their marginal tax rate, which raises their welfare and compensates for the difference between their larger tax bill and their benefit from automation. Therefore, while optimal taxation analyses typically suggest that "trickle-down" forces imply lower marginal tax rates at the top in general equilibrium (Stiglitz [1982b], Rothschild and Scheuer [2013], Ales, Kurnaz, and Sleet [2015a], Sachs, Tsyvinski, and Werquin [2016]), the compensation exercise by contrast requires higher marginal (and average) tax increases on high incomes than in partial equilibrium in response to a positive wage disruption: the compensation at the 90^{th} percentile is 1.6 times higher once the general-equilibrium forces (progressivity variable and compensation-of-compensation in formula (18)) are taken into account.

Finally, using the data and empirical estimates of Dauth, Findeisen, Südekum, and Woessner [2017] for Germany between 1994 and 2014, we find in Appendix B.1 that the wage of all the workers decreased from being exposed to robots, and that the exposure and hence the corresponding income losses were larger for higher-income agents. As a result, we show that workers at the 10^{th} percentile of the distribution should have their tax bill reduced by 310% of their income loss, while those at the 90^{th} percentile should have theirs reduced by 150% of their income loss, representing in both cases a 1.4 percentage point reduction in the average tax rate.

Conclusion

A classic policy question of compensating winners and losers from an economic disruption becomes quite involved when the environment features both distortionary taxes and general equilibrium. At the same time, both of these considerations are important in many applied and policy questions. We provide a general closed-form formula for the design of the welfare-compensating tax reform in general equilibrium.

This equation has a clear economic meaning and is easy to implement in practical applications. In Appendix A.6 we discuss the advantages of the compensation approach over the traditional optimal taxation approach.

References

Daron Acemoglu and Pascual Restrepo. Robots and jobs: Evidence from us labor markets. 2017.

Laurence Ales, Musab Kurnaz, and Christopher Sleet. Technical change, wage inequality, and taxes. *The American Economic Review*, 105(10):3061–3101, 2015a.

Laurence Ales, Musab Kurnaz, and Christopher M Sleet. Tasks, talents and taxes. *American Economic Review (forthcoming)*, 2015b.

Pol Antras, Alonso de Gortari, and Oleg Itskhoki. Globalization, inequality and welfare. *Working Paper*, 2016.

Uri M Ascher and Linda R Petzold. *Computer methods for ordinary differential equations and differential-algebraic equations*, volume 61. Siam, 1998.

R. Bénabou. Tax and education policy in a heterogenous-agent economy: What of levels of redistribution maximize growth and efficiency? *Econometrica*, 70(2): 481–517, 2002.

Hermann Brunner. *Collocation methods for Volterra integral and related functional differential equations*, volume 15. Cambridge University Press, 2004.

Stephen L Campbell. Linearization of daes along trajectories. *Zeitschrift für angewandte Mathematik und Physik ZAMP*, 46(1):70–84, 1995.

Stephen L Campbell and Eberhard Griepentrog. Solvability of general differential algebraic equations. *SIAM Journal on Scientific Computing*, 16(2):257–270, 1995.

Raj Chetty. Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply. *Econometrica*, 80(3):969–1018, 2012.

Wolfgang Dauth, Sebastian Findeisen, Jens Südekum, and Nicole Woessner. German robots-the impact of industrial robots on workers. 2017.

Christian Dustmann, Tommaso Frattini, and Ian P Preston. The effect of immigration along the distribution of wages. *The Review of Economic Studies*, 80(1):145–173, 2013.

Ernst Hairer and Gerhard Wanner. *Solving ordinary differential equations. II, volume 14 of Springer Series in Computational Mathematics*. Springer-Verlag, Berlin, 1996.

Jonathan Heathcote, Kjetil Storesletten, and Giovanni L Violante. Optimal tax progressivity: An analytical framework. *NBER Working Paper No. 19899*, 2016.

Jonathan Heathcote, Kjetil Storesletten, Giovanni L Violante, et al. Optimal progressivity with age-dependent taxation. Technical report, Federal Reserve Bank of Minneapolis, 2017.

Nathaniel Hendren. The inequality deflator: Interpersonal comparisons without a social welfare function. *NBER Working Paper 20351*, 2014.

John R Hicks. The foundations of welfare economics. *The Economic Journal*, 49 (196):696–712, 1939.

John R Hicks. The valuation of the social income. *Economica*, 7(26):105–124, 1940.

Oleg Itskhoki. Optimal redistribution in an open economy. *Working Paper*, 2008.

Nicholas Kaldor. Welfare propositions of economics and interpersonal comparisons of utility. *The Economic Journal*, pages 549–552, 1939.

By Louis Kaplow. Optimal control of externalities in the presence of income taxation. *International Economic Review*, 53(2):487–509, 2012.

Louis Kaplow. On the (ir) relevance of distribution and labor supply distortion to government policy. *Journal of Economic Perspectives*, 18(4):159–175, 2004.

Fabian Kindermann, Lukas Mayr, and Dominik Sachs. Inheritance taxation and wealth effects on the labor supply of heirs. 2017.

Peter Kunkel and Volker Mehrmann. *Differential-algebraic equations: analysis and numerical solution*. European Mathematical Society, 2006.

René Lamour, Roswitha März, and Caren Tischendorf. *Differential-algebraic equations: a projector based analysis*. Springer Science & Business Media, 2013.

Roswitha März. On linear differential-algebraic equations and linearizations. *Applied Numerical Mathematics*, 18(1-3):267–292, 1995.

Roswitha März. Notes on linearization of differential-algebraic equations and on optimization with differential-algebraic constraints. 2011.

Andreu Mas-Colell, Michael Dennis Whinston, and Jerry R Green. *Microeconomic theory*, volume 1. Oxford university press New York, 1995.

J. A. Mirrlees. An exploration in the theory of optimum income taxation. *The Review of Economic Studies*, 38(2):175–208, 1971.

Patrick J Rabier and Werner C Rheinboldt. A general existence and uniqueness theorem for implicit differential-algebraic equations. Technical report, DTIC Document, 1990.

Patrick J Rabier and Werner C Rheinboldt. A geometric treatment of implicit differential-algebraic equations. *Journal of Differential Equations*, 109(1):110–146, 1994.

Werner C Rheinboldt. Differential-algebraic systems as differential equations on manifolds. *Mathematics of computation*, 43(168):473–482, 1984.

Casey Rothschild and Florian Scheuer. Redistributive taxation in the roy model. *The Quarterly Journal of Economics*, 128(2):623–668, 2013.

Casey Rothschild and Florian Scheuer. A theory of income taxation under multidimensional skill heterogeneity. *NBER Working Paper No. 19822*, 2014.

Casey Rothschild and Florian Scheuer. Optimal taxation with rent-seeking. *Review of Economic Studies (forthcoming)*, 2016.

D. Sachs, A. Tsyvinski, and N. Werquin. Nonlinear tax incidence and optimal taxation in general equilibrium. *NBER Working Paper 22646*, 2016.

E. Saez. Using Elasticities to Derive Optimal Income Tax Rates. *Review of Economic Studies*, 68(1):205–229, 2001. ISSN 1467-937X.

Emmanuel Saez and Stefanie Stantcheva. Generalized social marginal welfare weights for optimal tax theory. *American Economic Review*, 106(1):24–45, 2016.

Florian Scheuer and Iván Werning. Mirrlees meets diamond-mirrlees. *NBER Working Paper No. 22076*, 2016.

Gennady A. Shishkin. *[In Russian] Linear Integrodifferential equations of Fredholm*. Izdatelstvo Buryatskogo Universiteta, Ulan Ude, 2007.

Joseph E. Stiglitz. Self-selection and pareto efficient taxation. *Journal of Public Economics*, 17(2):213–240, 1982a.

Joseph E Stiglitz. Self-selection and pareto efficient taxation. *Journal of Public Economics*, 17(2):213–240, 1982b.

Mordukhai M. Vainberg. *[In Russian] Integro-differential equations*. Itogi Nauki i Tehniki. Serija "Teorija verojatnostej. Matematicheskaja statistika. Tehnicheskaja kibernetika", 1964.

Stephen M Zemyan. *The classical theory of integral equations: a concise treatment*. Springer Science & Business Media, 2012.

Appendix

A Proofs

A.1 Partial equilibrium

We start by characterizing the welfare compensating tax reform and its fiscal surplus in partial equilibrium.

Proof of Proposition 1. Equation (11), and hence the formula (13) for the compensating tax reform, follow immediately from the proof of equations (15) and (33) below. Equation (12) follows from the proof of equations (16) and (34) below. Here we only expand on the first-order Taylor expansion of equation (6) around the initial equilibrium (i.e., as $\mu \to 0$). It reads

$$\begin{aligned} \tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \frac{\tilde{l}_i^{1+\frac{1}{e}}}{1+\frac{1}{e}} - \mu\hat{T}(\tilde{w}_i\tilde{l}_i) \quad &= \quad w_i l_i - T\left(w_i l_i\right) - \frac{l_i^{1+\frac{1}{e}}}{1+\frac{1}{e}} - \mu\hat{T}\left(w_i l_i\right) \\ &\quad + \mu\left(1 - T'\left(w_i l_i\right)\right)\left(w_i\hat{l}_i + l_i\hat{w}_i^E\right) - \mu l_i^{\frac{1}{e}}\,\hat{l}_i. \end{aligned} \tag{40}$$

This equation can be simplified by recognizing that $\left(1 - T'\left(w_i l_i\right)\right) w_i\hat{l}_i - l_i^{\frac{1}{e}}\,\hat{l}_i = 0$, which follows immediately from the first-order condition (2), or from the envelope theorem: since individuals choose their labor supply optimally before the perturbation, their labor supply adjustment $\hat{l}_i$ induces no first-order effect on welfare. Therefore, imposing that agent i keeps the same level of welfare in the disrupted economy (once the new tax schedule is implemented) as in the initial equilibrium, i.e. $\hat{U}_i \equiv \tilde{U}_i - U_i = 0$, leads to equation (11).

□

We now derive the budget impact of the wage disruption and its compensation. We define the fiscal surplus as

$$\mu\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) \quad = \quad \int_0^1 \left[T(\tilde{w}_i\tilde{l}_i) + \mu\hat{T}(\tilde{w}_i\tilde{l}_i) - T(w_i l_i)\right] di. \tag{41}$$

Corollary 1. *Suppose that there is infinite substitutability between skills in production, i.e., $\varepsilon^D \to \infty$. Consider a marginal disruption of the wage distribution $\boldsymbol{w}$ in the direction $\hat{\boldsymbol{w}}^E = \{\hat{w}_i^E\}_{i\in[0,1]}$. The fiscal surplus of the wage disruption $\hat{\boldsymbol{w}}^E$ and the compensating tax reform $\hat{T}$, defined by (13), is given by*

$$\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) \quad = \quad \int_0^1 \left[\frac{\hat{w}_i^E}{w_i} + T'\left(y_i\right)\frac{\hat{l}_i}{l_i}\right] y_i\, di, \tag{42}$$

where the labor supply change $\hat{l}_i$ of agent i following the wage disruption and the tax reform is given by (12).

Proof. The effect of the wage disruption and the corresponding compensating tax reform on government budget is given by

$$\begin{aligned}
\mathcal{R}(\hat{\boldsymbol{w}}^E) &= \lim_{\mu\to 0}\frac{1}{\mu}\left\{\int_0^1 \left[T(w_i\tilde{l}_i)+\mu\hat{T}(w_i\tilde{l}_i)\right] f(i)\,di - \int_0^1 T(w_i l_i) f(i)\,di\right\}\\
&= \int_0^1 \hat{T}(y_i) f(i)\,di + \int_0^1 \left(\frac{\hat{w}_i^E}{w_i}+\frac{\hat{l}_i}{l_i}\right) w_i l_i T'(w_i l_i) f(i)\,di,
\end{aligned}$$

where f is the (uniform) density of skills in the initial economy. Using equation (13), we can rewrite this expression as

$$\mathcal{R}(\hat{\boldsymbol{w}}^E) = \int_0^1 (1-T'(y_i))\, y_i \frac{\hat{w}_i^E}{w_i} f(i)\,di + \int_0^1 \left(\frac{\hat{w}_i^E}{w_i}+\frac{\hat{l}_i}{l_i}\right) y_i T'(y_i) f(i)\,di,$$

which easily leads to equation (42).

$\square$

The interpretation of formula (42) is as follows. Consider an individual i who earns income $y_i = w_i l_i \in [\underline{y}, \bar{y}]$ before the disruption. His wage changes by $\hat{w}_i^E$, so that his income (absent any labor supply responses) changes by $l_i\hat{w}_i^E = \frac{y_i}{w_i}\hat{w}_i^E$. The government keeps a share $T'(y_i)$ of this income change. Moreover, by equation (13), the government raises the agent's tax liability by $\hat{T}(y_i) = (1-T'(y_i))\frac{y_i}{w_i}\hat{w}_i^E$. Therefore tax revenue increases by $T'(y_i)\frac{y_i}{w_i}\hat{w}_i^E + (1-T'(y_i))\frac{y_i}{w_i}\hat{w}_i^E = \frac{y_i}{w_i}\hat{w}_i^E$. This is the first term in the square brackets of expression (42). Second, the wage disruption and the compensating tax reform lead the agent to adjust (say, reduce) his labor supply by $\hat{l}_i$, given by (12). This lowers government revenue by a fraction $T'(y_i)$ of the corresponding income loss $w_i\hat{l}_i = y_i\frac{\hat{l}_i}{l_i}$. This yields the second term in the square brackets of (42). Summing over all agents $i \in [0,1]$ leads to the total change in government revenue $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$. Note that (42) is a closed-form expression, since it depends only on the exogenous wage disruption $\hat{\boldsymbol{w}}^E$ and on the characteristics of the undisrupted economy.

Note that the government is able to compensate the gains and losses from the wage disruption in a budget-neutral way, taking into account the labor supply distortions that such redistribution will induce, if and only if $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) \geq 0$. It is thus possible that a shock to the economy (say, technological change, immigration inflow, or opening to international trade) generates strictly positive aggregate gains, both in terms of gross incomes for the agents (i.e., $\mathbb{E}[\frac{y_i}{w_i}\hat{w}_i^E] > 0$), and in terms of government revenue (i.e., $\mathbb{E}[T'(y_i)\frac{y_i}{w_i}\hat{w}_i^E] > 0$), but that these gains are not redistributable (i.e., $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) < 0$), because the labor supply distortions generated by the disruption and/or by the compensating tax reform $\hat{T}$ outweigh the aggregate gains of the initial economic shock.

A.2 General equilibrium: Linearization of the equilibrium conditions

We now analyze the welfare compensation problem in general equilibrium. Without loss of generality we order skills so that there is a one-to-one map between skills i and wages w_i in the initial equilibrium with tax schedule T. We assume in addition that there is a one-to-one map between wages w_i and earnings $y_i = w_i \times l_i$, i.e. that incomes y_i are increasing in skills i. This is satisfied in particular if the agents' utility functions u_i are the same for all agents i and the Spence-Mirrlees single-crossing condition holds (that is, the marginal rate of substitution between consumption and pre-tax income $-u_l'/(wu_c')$ is decreasing in w). See Sachs, Tsyvinski, and Werquin [2016] for details.

Note moreover that we impose that all agents with a given skill i, i.e. a given wage w_i, own the same amount of capital, which ensures that they all choose the same level of labor supply (conditional on working) l_i, independent of their fixed cost of working. We can easily relax this restriction by assuming that agents i who are employed in the initial equilibrium own a different amount of capital than agents with the same skill i but who are not employed. However, if we allowed the level of capital (and hence labor supply) to vary more generally with the fixed cost of working κ, a tax system that consists of a labor income tax schedule and a constant capital tax rate would not be sufficient to compensate the impact of arbitrary wage disruptions, unless individual preferences have no income effects on labor supply.

Integro-Differential Algebraic Equations (IDAE). We first show that if the production function is CES, the welfare compensation problem (6)-(7)-(8) with $\tilde{U}_i = U_i$ in (6) has a particular mathematical structure. The unknown functions to solve for are $(\hat{T}, \hat{\boldsymbol{l}})$. Ignoring for now the wage equation (8), the system consists of (i) a differential equation (7), which involves the derivative $\hat{T}'$ of the tax function, arising from the requirement that the first order conditions of the agents remain satisfied following the disruption and the tax reform; (ii) an algebraic component (6), which features only the unknown functions $(\hat{T}, \hat{\boldsymbol{l}})$ but not their derivatives, arising from the requirement that the indirect utility of agents remains at the level defined by their pre-disruption utility (the level set constraint). This is a system of Differential Algebraic Equations (DAE).[39]

The difficulty in the analysis of such a system, relative to a standard system of differential equations, is that the Jacobian of the implicit ODE is singular due to the presence of the algebraic constraint that does not include the derivatives of the unknown function.[40] The DAEs can be viewed as differential equations on manifolds.[41] The algebraic constraint forms a manifold, and the literature proceeds by analyzing the behavior of the suitably projected differential equation.

Now note that there is in addition (iii) an integral component to the system: both equations (6) and (7) depend on an integral of the function $\hat{\boldsymbol{l}}$ via the wage equation (8) along with the definition of the CES production function. This implies that (6)-(7)-(8) is a system of Integro-Differential Algebraic Equations (IDAE).[42]

[39]The DAE theory is recent (Ascher and Petzold [1998], p. 231). See Kunkel and Mehrmann [2006] for the first textbook treatment of this topic.

[40]See Ascher and Petzold [1998], p. 231.

[41]Rheinboldt [1984], Hairer and Wanner [1996], Chapters VI and VII, Brunner [2004], Chapter 8

[42]Lamour, März, and Tischendorf [2013] (p. xxi) argue that the IDAEs are a special case of abstract differential-algebraic equations (ADAE), so that the methods of analysis of the DAEs are applicable.

The main technical challenge of the paper is to solve the resulting system of IDAE. We leave the details to the appendix and briefly outline the strategy of the proofs here. First, we follow Hairer and Wanner [1996] (Chapter VI) and Kunkel and Mehrmann [2006] (Chapter 4) to linearize the system of nonlinear IDAEs (i.e., we focus on marginal wage disruptions). This allows us to to transform the system of nonlinear IDAE into one integro-differential equation.[43] Second, we then follow Vainberg [1964] and Shishkin [2007] to derive the analytical solution to the resulting integro-differential equation.

Linearization. We first derive the impact of any exogenous disruption and tax reform on wages.

Proof of equations (14) and (32). Consider an exogenous disruption $\mu\hat{\mathscr{F}}^E$ of the initial economy's production function and a tax reform $\mu\hat{T}$, with $\mu > 0$ (the proof extends immediately to a disruption of the aggregate labor supply distribution or the aggregate capital stock). The corresponding wage disruption is defined by

$$\hat{w}_i^E \quad = \quad \frac{\partial \hat{\mathscr{F}}^E}{\partial L_i}(\{L_j\}_{j\in[0,1]}, K).$$

Denote by $\mu\hat{w}_i$ and $\mu\hat{l}_i$ the first-order endogenous changes as $\mu \to 0$ in the wage and labor supply of type i, and let $\tilde{w}_i = w_i + \mu\hat{w}_i^E + \mu\hat{w}_i$ and $\tilde{l}_i = l_i + \mu\hat{l}_i$. In the perturbed equilibrium, the wage is equal to the marginal product of the labor of the corresponding type:

$$\tilde{w}_i \quad = \quad \frac{\partial[\mathscr{F} + \mu\hat{\mathscr{F}}^E]}{\partial L_i}(\{L_j + \mu\hat{l}_j\}_{j\in[0,1]}, K).$$

A first-order Taylor expansion in $\mu \to 0$ of this equation around the initial equilibrium yields the following expression for the Gateaux derivative of the wage functional:

$$\begin{aligned}
\hat{w}_i \quad &\equiv \quad \lim_{\mu\to 0} \frac{1}{\mu}\left(\tilde{w}_i - w_i - \mu\hat{w}_i^E\right) \\
&= \quad \lim_{\mu\to 0} \frac{1}{\mu}\left[\frac{\partial[\mathscr{F} + \mu\hat{\mathscr{F}}^E]}{\partial L_i}(\boldsymbol{L} + \mu\hat{\boldsymbol{l}}, K) - \frac{\partial\mathscr{F}}{\partial L_i}(\boldsymbol{L}, K) - \mu\frac{\partial\hat{\mathscr{F}}^E}{\partial L_i}(\boldsymbol{L}, K)\right] \\
&= \quad \left[\hat{l}_i\frac{\partial^2\mathscr{F}(\boldsymbol{L}, K)}{\partial L_i^2} - \hat{l}_i \lim_{j\to i}\frac{\partial^2\mathscr{F}(\boldsymbol{L}, K)}{\partial L_i\partial L_j}\right] + \int_0^1 \hat{l}_j\frac{\partial^2\mathscr{F}(\boldsymbol{L}, K)}{\partial L_i\partial L_j}dj.
\end{aligned}$$

Therefore, using the definitions of the structural cross-wage and own-wage elasticities (10), we obtain

[43] Rabier and Rheinboldt [1990, 1994] provide conditions for the local existence and uniqueness of solutions of DAEs. März [2011] is perhaps the most comprehensive recent analysis of the conditions under which linearizations are valid (see also Campbell [1995]). Campbell and Griepentrog [1995] discuss the computational verification of solutions. However, complications primarily arise in complex systems of higher indices (Campbell and Griepentrog [1995]), while our linearized system is a Hessenberg index-1 DAE (see Hairer and Wanner [1996], p. 374) which poses fewer challenges (see, e.g., a discussion in März [1995]).

(since $j \mapsto w_j$ is continuous as $j \to i$)

$$\begin{aligned}
\hat{w}_i &= w_i \frac{\hat{l}_i}{l_i} \left[\frac{L_i}{w_i} \frac{\partial^2 \mathscr{F}(\boldsymbol{L}, K)}{\partial L_i^2} - \lim_{j \to i} \frac{L_i}{w_j} \frac{\partial^2 \mathscr{F}(\boldsymbol{L}, K)}{\partial L_i \partial L_j} \right] + w_i \int_0^1 \frac{\hat{l}_j}{l_j} \frac{L_j}{w_i} \frac{\partial^2 \mathscr{F}(\boldsymbol{L}, K)}{\partial L_i \partial L_j} dj \\
&= -w_i \frac{\hat{l}_i}{l_i} \frac{1}{\varepsilon_i^D} + w_i \int_0^1 \frac{\hat{l}_j}{l_j} \gamma_{ij} dj.
\end{aligned}$$

This leads to equation (14). In the model of Section 1, the own wage elasticity ε_i^D is constant and cross-wage elasticity γ_{ij} does not depend on i, which implies equation (14).

□

Next we derive the impact of any exogenous disruption and tax reform on indirect utilities and impose that the agent's welfare is unchanged.

Proof of equations (15) and (33). Imposing that every employed agent's welfare is the same after the disruption and the tax reform as in the initial equilibrium reads: for all $i \in [0,1]$,

$$U_i = u_i[\tilde{w}_i \tilde{l}_i - T(\tilde{w}_i \tilde{l}_i) - \mu \hat{T}(\tilde{w}_i \tilde{l}_i) + (1 - \tau - \mu \hat{\tau}) \tilde{r} k_i, \tilde{l}_i].$$

Recall that the reform of the capital income tax rate ensures that $(1 - \tau - \mu\hat{\tau}) \tilde{r} k_i = (1 - \tau) r k_i \equiv Rk_i$. A first-order Taylor expansion in $\mu \to 0$ of this equation yields

$$\begin{aligned}
0 = \hat{U}_i &= u_i \left[w_i l_i - T(w_i l_i) + \mu (1 - T'(w_i l_i)) \left(w_i \hat{l}_i + l_i \hat{w}_i^E + l_i \hat{w}_i \right) - \mu \hat{T}(w_i l_i) + Rk_i, l_i + \mu \hat{l}_i \right] \\
&\quad - u_i [w_i l_i - T(w_i l_i) + Rk_i, l_i] \\
&= \mu \left[(1 - T'(w_i l_i)) \left(w_i \hat{l}_i + l_i \hat{w}_i^E + l_i \hat{w}_i \right) - \hat{T}(w_i l_i) \right] u'_{i,c} + \mu \hat{l}_i u'_{i,l} \\
&= \mu \left[(1 - T'(w_i l_i)) y_i \left(\frac{\hat{w}_i^E}{w_i} + \frac{\hat{w}_i}{w_i} \right) - \hat{T}(w_i l_i) \right] u'_{i,c},
\end{aligned}$$

where the last equality follows from the first order condition (2). This leads to equation (15).

□

Finally we derive the impact of any exogenous disruption and tax reform on individual intensive-margin labor supplies, and express it in terms of the behavioral elasticities.

Proof of equations (16) and (34). The perturbed first-order condition of skill i reads

$$\begin{aligned}
0 &= [1 - T'(\tilde{w}_i \tilde{l}_i) - \mu \hat{T}'(\tilde{w}_i \tilde{l}_i)] \tilde{w}_i u'_{i,c} \left[\tilde{w}_i \tilde{l}_i - T\left(\tilde{w}_i \tilde{l}_i\right) - \mu \hat{T}\left(\tilde{w}_i \tilde{l}_i\right) + Rk_i, \tilde{l}_i \right] \\
&\quad + u'_{i,l} \left[\tilde{w}_i \tilde{l}_i - T\left(\tilde{w}_i \tilde{l}_i\right) - \mu \hat{T}\left(\tilde{w}_i \tilde{l}_i\right) + Rk_i, \tilde{l}_i \right],
\end{aligned}$$

where $\tilde{w}_i = w_i + \mu \hat{w}_i^E + \mu \hat{w}_i$ and $\tilde{l}_i = l_i + \mu \hat{l}_i$. Tedious algebra, the details of which can be found in Appendix D.1.1 of Sachs, Tsyvinski, and Werquin [2016] implies that the first-order Taylor expansion in $\mu \to 0$ of this equation reads

$$\begin{aligned} 0 \;=\; & \left[\left(1 - T'(y_i)\right) w_i y_i u'_{i,cc} + y_i u'_{i,cl} + w_i u'_{i,c} - w_i y_i \frac{T''(y_i)}{1 - T'(y_i)} u'_{i,c} \right] \frac{\hat{w}_i^E + \hat{w}_i}{w_i} \\ & + \left[\left(1 - T'(y_i)\right) w_i^2 u'_{i,cc} + w_i u'_{i,cl} + w_i u'_{i,cl} + \frac{u_{ll,i}}{1 - T'(y_i)} - w_i^2 \frac{T''(y_i)}{1 - T'(y_i)} u'_{i,c} \right] \hat{l}_i \\ & - \; w_i u'_{i,c} \frac{\hat{T}'(y_i)}{1 - T'(y_i)} \; - \; \left[\left(1 - T'(y_i)\right) w_i u'_{i,cc} + u'_{i,cl} \right] \frac{\hat{T}(y_i)}{1 - T'(y_i)}. \end{aligned}$$

Solving for $\hat{l}_i$ and rearranging implies

$$\begin{aligned} \hat{l}_i \;=\; & \frac{\dfrac{-\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 l_i u''_{i,cc} + \left(\frac{u'_{i,l}}{u'_{i,c}}\right) l_i u''_{i,cl} - \left(1 - \frac{y_i T''(y_i)}{1 - T'(y_i)}\right)\left(1 - T'(y_i)\right) w_i u''_{i,c}}{\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\frac{u'_{i,l}}{u'_{i,c}}\right) u''_{i,cl} + u''_{i,ll}}}{1 + \frac{y_i T''(y_i)}{1 - T'(y_i)} \dfrac{\frac{u'_{i,l}}{l_i}}{\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\frac{u'_{i,l}}{u'_{i,c}}\right) u''_{i,cl} + u''_{i,ll}}} \frac{\hat{w}_i^E + \hat{w}_i}{w_i} \\ & + \frac{\dfrac{\frac{u'_{i,l}}{l_i}}{\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\frac{u'_{i,l}}{u'_{i,c}}\right) u''_{i,cl} + u''_{i,ll}}}{1 + \frac{y_i T''(y_i)}{1 - T'(y_i)} \dfrac{\frac{u'_{i,l}}{l_i}}{\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\frac{u'_{i,l}}{u'_{i,c}}\right) u''_{i,cl} + u''_{i,ll}}} \frac{1}{\left(\frac{u_{l,i}}{u_{c,i}}\right)} y_i \hat{T}'(y_i) \\ & + \frac{\dfrac{-\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} + \left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cl}}{\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\frac{u'_{i,l}}{u'_{i,c}}\right) u''_{i,cl} + u''_{i,ll}}}{1 + \frac{y_i T''(y_i)}{1 - T'(y_i)} \dfrac{\frac{u'_{i,l}}{l_i}}{\left(\frac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\frac{u'_{i,l}}{u'_{i,c}}\right) u''_{i,cl} + u''_{i,ll}}} \frac{1}{\left(\frac{u_{l,i}}{u_{c,i}}\right)} \hat{T}(y_i), \end{aligned}$$

and hence, from the standard expressions for the labor supply elasticities e_i^c, e_i^u and income effect parameter $e_i^n = e_i^u - e_i^c$ (see, e.g., Saez [2001] p. 227), and the elasticities with respect to the non-linear budget constraint $\varepsilon_i^{S,w}, \varepsilon_i^{S,r}, \varepsilon_i^{S,n}$,

$$\begin{aligned} \frac{\hat{l}_i}{l_i} \;=\; & \frac{e_i^n + \left(1 - p(y_i)\right) e_i^c}{1 + p(y_i) e_i^c} \frac{\hat{w}_i^E + \hat{w}_i}{w_i} - \frac{e_i^c}{1 + p(y_i) e_i^c} \frac{\hat{T}'(y_i)}{1 - T'(y_i)} - \frac{e_i^n}{1 + p(y_i) e_i^c} \frac{\hat{T}(y_i)}{\left(1 - T'(y_i)\right) y_i} \\ \equiv\; & \varepsilon_i^{S,w} \frac{\hat{w}_i^E + \hat{w}_i}{w_i} - \varepsilon_i^{S,r} \frac{\hat{T}'(y_i)}{1 - T'(y_i)} - \varepsilon_i^{S,n} \frac{\hat{T}(y_i)}{\left(1 - T'(y_i)\right) y_i}, \end{aligned}$$

which leads to equation (16).

□

We now solve in closed-form for the labor supply changes $\{\hat{l}_i\}_{i \in [0,1]}$ in the simple version of the model of Section 1.

Proof of Lemma 1. Using equations (14) and (16), we obtain that the labor supply adjustments

$\{\hat{l}_i\}_{i\in[0,1]}$ satisfy the following linear Fredholm integral equation:

$$\begin{aligned}\frac{\hat{l}_i}{l_i} &= \varepsilon^{S,w}\left[\frac{\hat{w}_i^E}{w_i} - \frac{1}{\varepsilon^D}\frac{\hat{l}_i}{l_i} + \int_0^1 \gamma_j \frac{\hat{l}_j}{l_j} dj\right] - \varepsilon^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)} \\ &= \delta\varepsilon^{S,w}\frac{\hat{w}_i^E}{w_i} - \delta\varepsilon^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)} + \delta\varepsilon^{S,w}\int_0^1 \gamma_j \frac{\hat{l}_j}{l_j} dj.\end{aligned}$$

Multiplying both sides by γ_i and integrating over $i \in [0,1]$ implies

$$\begin{aligned}\int_0^1 \gamma_i \frac{\hat{l}_i}{l_i} di &= \delta\int_0^1 \gamma_i \left[\varepsilon^{S,w}\frac{\hat{w}_i^E}{w_i} - \varepsilon^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)}\right] di + \delta\varepsilon^{S,w}\left[\int_0^1 \gamma_j \frac{\hat{l}_j}{l_j} dj\right]\int_0^1 \gamma_i di \\ &= \frac{\delta}{1-\delta\varepsilon^{S,w}\int_0^1 \gamma_i di}\int_0^1 \gamma_i \left[\varepsilon^{S,w}\frac{\hat{w}_i^E}{w_i} - \varepsilon^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)}\right] di.\end{aligned}$$

Now note that $\int_0^1 \gamma_i di = \frac{1}{\varepsilon_D}\int_0^1 \frac{y_i}{Y} di = \frac{1}{\varepsilon_D}$, so that $1-\delta\varepsilon^{S,w}\int_0^1 \gamma_i di = \delta$. Substituting the previous equation into the integral equation for $\hat{l}_i/l_i$ leads to

$$\frac{\hat{l}_i}{l_i} = \delta\left[\varepsilon^{S,w}\frac{\hat{w}_i^E}{w_i} - \varepsilon^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)}\right] + \delta\varepsilon^{S,w}\int_0^1 \gamma_i \left[\varepsilon^{S,w}\frac{\hat{w}_i^E}{w_i} - \varepsilon^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)}\right] di.$$

This immediately implies equation (17).

□

Next we solve in closed-form for the labor supply changes $\{\hat{l}_i\}_{i\in[0,1]}$ in the general model of Section 2. Note that we assume from the outset that the extensive margin responses to the exogenous disruption and the compensating tax reform are equal to zero.

Proof of Lemma 2. Using equations (32) and (34), we obtain that the labor supply adjustments $\{\hat{l}_i\}_{i\in[0,1]}$ satisfy the following linear Fredholm integral equation:

$$\begin{aligned}\frac{\hat{l}_i}{l_i} &= \varepsilon_i^{S,w}\left[\frac{\hat{w}_i^E}{w_i} - \frac{1}{\varepsilon_i^D}\frac{\hat{l}_i}{l_i} + \int_0^1 \gamma_{ij} \frac{\hat{l}_j}{l_j} dj\right] - \varepsilon_i^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)} + \varepsilon_i^{S,n}\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i} \\ &= \delta_i\left[\varepsilon_i^{S,w}\frac{\hat{w}_i^E}{w_i} - \varepsilon_i^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)} + \varepsilon_i^{S,n}\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i}\right] + \delta_i\varepsilon_i^{S,w}\int_0^1 \gamma_{ij}\frac{\hat{l}_j}{l_j} dj.\end{aligned}$$

Denoting the expression in square brackets by $\hat{l}_i^{\mathbf{pe}}/l_i$, and substituting for $\hat{l}_j/l_j$ in the integral leads to

$$
\begin{aligned}
\frac{\hat{l}_i}{l_i} &= \frac{\delta_i \hat{l}_i^{\mathbf{pe}}}{l_i} + \delta_i \varepsilon_i^{S,w} \int_0^1 \gamma_{ij} \left[\frac{\delta_j \hat{l}_j^{\mathbf{pe}}}{l_j} + \delta_j \varepsilon_j^{S,w} \int_0^1 \gamma_{jk} \frac{\hat{l}_k}{l_k} dk \right] dj \\
&= \left[\frac{\delta_i \hat{l}_i^{\mathbf{pe}}}{l_i} + \delta_i \varepsilon_i^{S,w} \int_0^1 \gamma_{ij} \frac{\delta_j \hat{l}_j^{\mathbf{pe}}}{l_j} dj \right] + \delta_i \varepsilon_i^{S,w} \int_0^1 \left[\int_0^1 \gamma_{ik} \delta_k \varepsilon_k^{S,w} \gamma_{kj} dk \right] \frac{\hat{l}_j}{l_j} dj \\
&\equiv \left[\frac{\delta_i \hat{l}_i^{\mathbf{pe}}}{l_i} + \delta_i \varepsilon_i^{S,w} \int_0^1 \Gamma_{ij}^{(0)} \frac{\delta_j \hat{l}_j^{\mathbf{pe}}}{l_j} dj \right] + \delta_i \varepsilon_i^{S,w} \int_0^1 \Gamma_{ij}^{(1)} \frac{\hat{l}_j}{l_j} dj,
\end{aligned}
$$

where $\Gamma_{ij}^{(0)} = \gamma_{ij}$ and $\Gamma_{ij}^{(1)} = \int_0^1 \Gamma_{ik}^{(0)} \delta_k \varepsilon_k^{S,w} \gamma_{kj} dk$. By induction, it easy to show that for all $N \geq 0$,

$$
\frac{\hat{l}_i}{l_i} = \left[\frac{\delta_i \hat{l}_i^{\mathbf{pe}}}{l_i} + \delta_i \varepsilon_i^{S,w} \int_0^1 \left\{ \sum_{n=0}^{N} \Gamma_{ij}^{(n)} \right\} \frac{\delta_j \hat{l}_j^{\mathbf{pe}}}{l_j} dj \right] + \delta_i \varepsilon_i^{S,w} \int_0^1 \Gamma_{ij}^{(N+1)} \frac{\hat{l}_j}{l_j} dj
$$

where for all $n \geq 0$, $\Gamma_{ij}^{(n+1)} = \int_0^1 \Gamma_{ik}^{(n)} \delta_k \varepsilon_k^{S,w} \gamma_{kj} dk$. The condition $\int_0^1 \int_0^1 \left| \delta_i \varepsilon_i^{S,w} \gamma_{ij} \right|^2 di dj < 1$ ensures that the series $\sum_{n=0}^{N} \Gamma_{ij}^{(n)}$ converges as $N \to \infty$ (see the proof of Proposition 1 in Sachs, Tsyvinski, and Werquin [2016] for details). This implies equation (35). Finally, note that we can write the endogenous wage changes as

$$
\begin{aligned}
\frac{\hat{w}_i}{w_i} &= -\frac{\delta_i \varepsilon_i^{S,w}}{\varepsilon_i^D} \frac{\hat{w}_i^E}{w_i} + \frac{\delta_i \varepsilon_i^{S,r}}{\varepsilon_i^D} \frac{\hat{T}'(y_i)}{1 - T'(y_i)} - \frac{\delta_i \varepsilon_i^{S,n}}{\varepsilon_i^D} \frac{\hat{T}(y_i)}{(1 - T'(y_i)) y_i} \\
&\quad + \delta_i \int_0^1 \Gamma_{ij} \delta_j \left[\varepsilon_j^{S,w} \frac{\hat{w}_j^E}{w_j} - \varepsilon_j^{S,r} \frac{\hat{T}'(y_j)}{1 - T'(y_j)} + \varepsilon_j^{S,n} \frac{\hat{T}(y_j)}{(1 - T'(y_j)) y_j} \right] dj, \qquad (43)
\end{aligned}
$$

which follows from equations (31), (34) and (35).

□

A.3 General equilibrium: Compensating tax reform

We now proceed to the proof of Proposition 2 and Theorem 1. We start by deriving a lemma that gives a functional equation that the compensating tax reform must satisfy.

Lemma 3. *The compensating tax reform $\hat{T}$ satisfies the following functional equation: for all $i \in [0, 1]$,*

$$
\begin{aligned}
(1 - T'(y_i)) y_i \hat{\Omega}_i^E &= -\delta_i \frac{\varepsilon_i^{S,r}}{\varepsilon_i^D} y_i \hat{T}'(y_i) + \delta_i \left(1 + \frac{\varepsilon_i^{S,w}}{\varepsilon_i^D} + \frac{\varepsilon_i^{S,n}}{\varepsilon_i^D} \right) \hat{T}(y_i) \qquad (44) \\
&\quad + \delta_i \int_0^1 \Gamma_{ij} \tau_{ij} \delta_j \left[\varepsilon_j^{S,r} y_j \hat{T}'(y_j) - \varepsilon_j^{S,n} \hat{T}(y_j) \right] dj.
\end{aligned}
$$

where $\hat{\Omega}_i^E \equiv \delta_i \frac{\hat{w}_i^E}{w_i} + \delta_i \int_0^1 \Gamma_{ij} \delta_j \varepsilon_j^{S,w} \frac{\hat{w}_j^E}{w_j} dj$ *and* $\tau_{ij} \equiv \frac{(1 - T'(y_i)) y_i}{(1 - T'(y_j)) y_j}$.

Proof. Equations (34) and (35) imply that the wage adjustments $\{\hat{w}_i\}_{i\in[0,1]}$ are given by

$$\begin{aligned}
\frac{\hat{w}_i^E}{w_i}+\frac{\hat{w}_i}{w_i} &= \frac{1}{\varepsilon_i^{S,w}}\frac{\hat{l}_i}{l_i}+\frac{\varepsilon_i^{S,r}}{\varepsilon_i^{S,w}}\frac{\hat{T}'(y_i)}{1-T'(y_i)}-\frac{\varepsilon_i^{S,n}}{\varepsilon_i^{S,w}}\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i} \\
&= \delta_i\frac{\hat{w}_i^E}{w_i}-\frac{(\delta_i-1)\,\varepsilon_i^{S,r}}{\varepsilon_i^{S,w}}\frac{\hat{T}'(y_i)}{1-T'(y_i)}+\frac{(\delta_i-1)\,\varepsilon_i^{S,n}}{\varepsilon_i^{S,w}}\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i} \\
&\quad +\delta_i\int_0^1\Gamma_{ij}\left[\delta_j\varepsilon_j^{S,w}\frac{\hat{w}_j^E}{w_j}-\delta_j\varepsilon_j^{S,r}\frac{\hat{T}'(y_j)}{1-T'(y_j)}+\delta_j\varepsilon_j^{S,n}\frac{\hat{T}(y_j)}{(1-T'(y_j))\,y_j}\right]dj.
\end{aligned}$$

Using this equation with $\frac{\delta_i-1}{\varepsilon_i^{S,w}}=-\frac{\delta_i}{\varepsilon_i^{D}}$, we can substitute for $\frac{\hat{w}_i^E}{w_i}+\frac{\hat{w}_i}{w_i}$ in the level set constraint (33) to rewrite it as

$$\begin{aligned}
0 &= (1-T'(y_i))\,y_i\delta_i\left[\frac{\hat{w}_i^E}{w_i}+\frac{\varepsilon_i^{S,r}}{\varepsilon_i^{D}}\frac{\hat{T}'(y_i)}{1-T'(y_i)}-\frac{\varepsilon_i^{S,n}}{\varepsilon_i^{D}}\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i}\right] \\
&\quad +(1-T'(y_i))\,y_i\delta_i\int_0^1\Gamma_{ij}\delta_j\left[\varepsilon_j^{S,w}\frac{\hat{w}_j^E}{w_j}-\varepsilon_j^{S,r}\frac{\hat{T}'(y_j)}{1-T'(y_j)}+\varepsilon_j^{S,n}\frac{\hat{T}(y_j)}{(1-T'(y_j))\,y_j}\right]dj-\hat{T}(y_i),
\end{aligned}$$

which implies

$$\begin{aligned}
0 &= (1-T'(y_i))\,y_i\delta_i\left[\frac{\hat{w}_i^E}{w_i}+\int_0^1\Gamma_{ij}\delta_j\varepsilon_j^{S,w}\frac{\hat{w}_j^E}{w_j}dj\right] \\
&\quad +\delta_i\frac{\varepsilon_i^{S,r}}{\varepsilon_i^{D}}y_i\hat{T}'(y_i)-\delta_i\left(1+\frac{\varepsilon_i^{S,w}}{\varepsilon_i^{D}}+\frac{\varepsilon_i^{S,n}}{\varepsilon_i^{D}}\right)\hat{T}(y_i) \\
&\quad -\delta_i\int_0^1\Gamma_{ij}\delta_j\frac{(1-T'(y_i))\,y_i}{(1-T'(y_j))\,y_j}\left[\varepsilon_j^{S,r}y_j\hat{T}'(y_j)-\varepsilon_j^{S,n}\hat{T}(y_j)\right]dj.
\end{aligned}$$

This concludes the proof.

□

We now derive the closed-form solution to equation (44).

Proof of Theorem 1. There is a one-to-one map $i\mapsto y_i$, so that we can define $\varepsilon_{y_i}^{D}=\varepsilon_i^{D}$, $\varepsilon_{y_i}^{S,x}=\varepsilon_i^{S,x}$ for $x\in\{r,w,n\}$, $\delta_{y_i}=\delta_i$, $\tau_{y_i,y_j}=\tau_{ij}$, $\hat{\Omega}_{y_i}^{E}=\hat{\Omega}_i^{E}$, and $\gamma_{y_i,y_j}=\frac{\gamma_{ij}}{dy_j/dj}$, $\Gamma_{y_i,y_j}=\frac{\Gamma_{ij}}{dy_j/dj}$. Changing variables from i to y_i in equation (44) then leads to

$$\hat{T}'(y_i)-\left(\frac{\varepsilon_{y_i}^{S,w}+\varepsilon_{y_i}^{S,n}+\varepsilon_{y_i}^{D}}{\varepsilon_{y_i}^{S,r}y_i}\right)\hat{T}(y_i) = -\frac{\varepsilon_{y_i}^{D}}{\delta_{y_i}\varepsilon_{y_i}^{S,r}y_i}(1-T'(y_i))\,y_i\mathcal{A}(y_i), \tag{45}$$

where

$$\mathcal{A}(y_i) \equiv \hat{\Omega}_{y_i}^{E}-\delta_{y_i}\int_{\underline{y}}^{\bar{y}}\frac{\Gamma_{y_i,y_j}\delta_{y_j}}{(1-T'(y_j))\,y_j}\left[\varepsilon_{y_j}^{S,r}y_j\hat{T}'(y_j)-\varepsilon_{y_j}^{S,n}\hat{T}(y_j)\right]dy_j. \tag{46}$$

Equation (45) is a first-order ordinary differential equation. Using standard techniques and using the definition $\varepsilon_{y_i}^{S,w} = (1-p(y_i))\,\varepsilon_{y_i}^{S,r} + \varepsilon_{y_i}^{S,n}$, we can express its general solution (up to a constant c_0) as

$$\begin{aligned}
\hat{T}(y_i) &= \int_{y_i}^{\bar{y}} \frac{\varepsilon_{y_j}^{D}}{\delta_{y_j}\varepsilon_{y_j}^{S,r} y_j} e^{-\int_{y_i}^{y_j}\left(1-p(y_k)+\frac{\varepsilon_{y_k}^{D}+2\varepsilon_{y_k}^{S,n}}{\varepsilon_{y_k}^{S,r}}\right)\frac{dy_k}{y_k}} \left(1-T'(y_j)\right) y_j \mathcal{A}(y_j)\, dy_j \\
&= \int_{y_i}^{\bar{y}} \frac{\varepsilon_{y_j}^{D}}{\delta_{y_j}\varepsilon_{y_j}^{S,r} y_j} e^{-\int_{y_i}^{y_j}\frac{\varepsilon_{y_k}^{D}+2\varepsilon_{y_k}^{S,n}}{\varepsilon_{y_k}^{S,r}}\frac{dy_k}{y_k}} \frac{\left(1-T'(y_i)\right) y_i}{\left(1-T'(y_j)\right) y_j} \left(1-T'(y_j)\right) y_j \mathcal{A}(y_j)\, dy_j \\
&\equiv \left(1-T'(y_i)\right) y_i \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i,y_j} \mathcal{A}(y_j)\, dy_j. \qquad (47)
\end{aligned}$$

where the second equality uses the definition $\frac{p(y_k)}{y_k} = \frac{T''(y_k)}{1-T'(y_k)}$ and integrates this expression. (We can show that, if the baseline tax schedule is Pareto efficient, all of the compensating reforms indexed by the constant c_0 have the same impact on the government budget, so that we can pick $c_0 = 0$.) Using (45) and (47), we can rewrite that auxiliary function $\mathcal{A}(y)$ as

$$\begin{aligned}
\mathcal{A}(y_i) &= \hat{\Omega}_{y_i}^{E} - \delta_{y_i}\int_{\underline{y}}^{\bar{y}} \frac{\Gamma_{y_i,y_j}\delta_{y_j}}{\left(1-T'(y_j)\right) y_j}\left[-\frac{\varepsilon_{y_j}^{D}}{\delta_{y_j}}\left(1-T'(y_j)\right) y_j \mathcal{A}(y_j) + \left(\varepsilon_{y_j}^{S,w}+\varepsilon_{y_j}^{D}\right)\hat{T}(y_j)\right] dy_j \\
&= \hat{\Omega}_{i}^{E} + \delta_{y_i}\int_{\underline{y}}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^{D}\mathcal{A}(y_j)\, dy_j - \delta_{y_i}\int_{\underline{y}}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^{D}\frac{\hat{T}(y_j)}{\left(1-T'(y_j)\right) y_j} dy_j \\
&= \hat{\Omega}_{i}^{E} + \delta_{y_i}\int_{\underline{y}}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^{D}\mathcal{A}(y_j)\, dy_j - \delta_{y_i}\int_{y_j=\underline{y}}^{\bar{y}}\int_{y_k=y_j}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^{D}\mathscr{E}_{y_j,y_k}\mathcal{A}(y_k)\, dy_k dy_j
\end{aligned}$$

where the second equality uses the fact that $\delta_{y_j}\left(\varepsilon_{y_j}^{S,w}+\varepsilon_{y_j}^{D}\right) = \varepsilon_{y_j}^{D}$. Inverting the order of the two integrals in the last line implies that this expression can be rewritten as

$$\begin{aligned}
\mathcal{A}(y_i) &= \hat{\Omega}_{i}^{E} + \delta_{y_i}\int_{\underline{y}}^{\bar{y}} \Gamma_{y_i,y_j}\varepsilon_{y_j}^{D}\mathcal{A}(y_j)\, dy_j - \delta_{y_i}\int_{y_k=\underline{y}}^{\bar{y}}\left\{\int_{y_j=\underline{y}}^{y_k} \Gamma_{y_i,y_j}\varepsilon_{y_j}^{D}\mathscr{E}_{y_j,y_k}\, dy_j\right\}\mathcal{A}(y_k)\, dy_k \\
&= \hat{\Omega}_{i}^{E} + \delta_{y_i}\int_{\underline{y}}^{\bar{y}}\left\{\Gamma_{y_i,y_j}\varepsilon_{y_j}^{D} - \int_{\underline{y}}^{y_j}\Gamma_{y_i,y_k}\varepsilon_{y_k}^{D}\mathscr{E}_{y_k,y_j}\, dy_k\right\}\mathcal{A}(y_j)\, dy_j.
\end{aligned}$$

But this is a standard linear Fredholm integral equation, with kernel given by

$$\delta_{y_i}\Lambda_{y_i,y_j}^{(0)} \equiv \delta_{y_i}\left[\Gamma_{y_i,y_j}\varepsilon_{y_j}^{D} - \int_{\underline{y}}^{y_j}\Gamma_{y_i,y_k}\varepsilon_{y_k}^{D}\mathscr{E}_{y_k,y_j}\, dy_k\right].$$

Its solution is therefore known in closed form (see, e.g., Zemyan [2012]). Assume that

$$\int_{[0,1]^2}\left|\delta_{y_i}\Lambda_{y_i,y_j}^{(0)}\right|^2 di dj < 1,$$

which ensures the convergence of the series $\sum_{n=0}^{\infty}\Lambda_{y_i,y_j}^{(n)}$ defined in Theorem 1. We show below that this condition is satisfied in the case under the assumptions of Section 1. Following analogous steps as in the proof of Lemma 2, we get

$$\mathcal{A}(y_i) \quad = \quad \hat{\Omega}^E_{y_i} + \delta_{y_i} \int_{\underline{y}}^{\bar{y}} \left\{ \sum_{n=0}^{\infty} \Lambda^{(n)}_{y_i,y_j} \right\} \hat{\Omega}^E_{y_j} dy_j. \tag{48}$$

From equations (47) and (48), we obtain the solution to the compensating tax reform problem $\hat{T}(y_i) = (1 - T'(y_i))\, y_i \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i y_j} \mathcal{A}(y_j)\, dy_j$, leading to formula (36).

□

A.4 General equilibrium: Fiscal surplus

We finally derive the budget impact (fiscal surplus) of the wage disruption and its compensation.

Corollary 2. *The fiscal surplus generated by the disruption and the compensating tax reform is given by*

$$\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) \quad = \quad \int_{\underline{y}}^{\bar{y}} \boldsymbol{\rho}(y_i) \left[\hat{\Omega}^E_{y_i} + \int_{\underline{y}}^{\bar{y}} \Lambda_{y_i,y_j} \hat{\Omega}^E_{y_j} dy_j \right] dy_i \tag{49}$$

where

$$\boldsymbol{\rho}(y_i) \quad \equiv \quad \left(\varepsilon^{S,w}_{y_i} + \varepsilon^D_{y_i}\right) T'(y_i)\; y_i\, f_Y(y_i) + \int_{\underline{y}}^{y_i} \mathscr{E}_{y_j,y_i} \left(1 - \varepsilon^D_{y_j} T'(y_j)\right) y_j\, f_Y(y_j)\; dy_j.$$

Like equation (36), formula (49) is a closed-form expression: it only depends on variables that are observed in the current, pre-disruption, economy. The welfare gains of the wage disruption $\hat{\boldsymbol{w}}^E$ are redistributable if and only if $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E) \geq 0$. Moreover, since $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E)$ gives the impact on government revenue of the exogenous disruption $\hat{\boldsymbol{w}}^E$ and the corresponding compensating tax reform, it is a useful metric for comparing the benefits of different disruptions: as argued in Section 1, we can rank two disruptions $\hat{\boldsymbol{w}}^E_A$ and $\hat{\boldsymbol{w}}^E_B$ by their respective fiscal surpluses $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E_A)$ and $\hat{\mathcal{R}}(\hat{\boldsymbol{w}}^E_B)$.

Proof of Corollary 2. The effect of the wage disruption and the corresponding compensating tax reform on government budget is given by

$$\begin{aligned} \mathcal{R}(\hat{\boldsymbol{w}}^E) &= \lim_{\mu \to 0} \frac{1}{\mu} \left\{ \int_0^1 \left[T(\tilde{w}_i \tilde{l}_i) + \mu \hat{T}(\tilde{w}_i \tilde{l}_i) \right] f(i)\, di - \int_0^1 T(w_i l_i)\, f(i)\, di \right\} \\ &= \int_0^1 \hat{T}(y_i)\, f(i)\, di + \int_0^1 \left(\frac{\hat{w}^E_i}{w_i} + \frac{\hat{w}_i}{w_i} + \frac{\hat{l}_i}{l_i} \right) w_i l_i T'(w_i l_i)\, f(i)\, di, \end{aligned}$$

where f is the (uniform) density of skills in the initial economy. Using equations (34) and (35), the second integral in the right hand side can be rewritten as

$$\begin{aligned}
& \int_0^1 T'(y_i)\, y_i \left[\left(\frac{1}{\varepsilon_i^{S,w}}+1\right)\frac{\hat{l}_i}{l_i}+\frac{\varepsilon_i^{S,r}}{\varepsilon_i^{S,w}}\frac{\hat{T}'(y_i)}{1-T'(y_i)}-\frac{\varepsilon_i^{S,n}}{\varepsilon_i^{S,w}}\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i}\right] f(i)\, di \\
= & \int_0^1 T'(y_i)\, y_i \left(\frac{1}{\varepsilon_i^{S,w}}+1\right)\delta_i\varepsilon_i^{S,w}\left(\frac{\hat{w}_i^E}{w_i}+\int_0^1 \Gamma_{ij}\delta_j\varepsilon_j^{S,w}\frac{\hat{w}_j^E}{w_j}dj\right) f(i)\, di \\
& -\int_0^1 T'(y_i)\, y_i \left\{\left[\left(\frac{1}{\varepsilon_i^{S,w}}+1\right)\delta_i\varepsilon_i^{S,r}-\frac{\varepsilon_i^{S,r}}{\varepsilon_i^{S,w}}\right]\frac{\hat{T}'(y_i)}{1-T'(y_i)}\right. \\
& \qquad\qquad \left.+\left(\frac{1}{\varepsilon_i^{S,w}}+1\right)\delta_i\varepsilon_i^{S,w}\int_0^1 \Gamma_{ij}\delta_j\varepsilon_j^{S,r}\frac{\hat{T}'(y_j)}{1-T'(y_j)}dj\right\} f(i)\, di \\
& +\int_0^1 T'(y_i)\, y_i \left\{\left[\left(\frac{1}{\varepsilon_i^{S,w}}+1\right)\delta_i\varepsilon_i^{S,n}-\frac{\varepsilon_i^{S,n}}{\varepsilon_i^{S,w}}\right]\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i}\right. \\
& \qquad\qquad \left.+\left(\frac{1}{\varepsilon_i^{S,w}}+1\right)\delta_i\varepsilon_i^{S,w}\int_0^1 \Gamma_{ij}\delta_j\varepsilon_j^{S,n}\frac{\hat{T}(y_j)}{(1-T'(y_j))\,y_j}dj\right\} f(i)\, di.
\end{aligned}$$

This expression can be rewritten as

$$\begin{aligned}
& \int_0^1 T'(y_i)\, y_i \left(1+\varepsilon_i^{S,w}\right)\hat{\Omega}_i^E f(i)\, di \\
& -\int_0^1 T'(y_i)\, y_i \left\{\left(1-\frac{1}{\varepsilon_i^D}\right)\delta_i\varepsilon_i^{S,r}\frac{\hat{T}'(y_i)}{1-T'(y_i)}+\delta_i\left(1+\varepsilon_i^{S,w}\right)\int_0^1 \Gamma_{ij}\delta_j\varepsilon_j^{S,r}\frac{\hat{T}'(y_j)}{1-T'(y_j)}dj\right\} f(i)\, di \\
& +\int_0^1 T'(y_i)\, y_i \left\{\left(1-\frac{1}{\varepsilon_i^D}\right)\delta_i\varepsilon_i^{S,n}\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i}+\delta_i\left(1+\varepsilon_i^{S,w}\right)\int_0^1 \Gamma_{ij}\delta_j\varepsilon_j^{S,n}\frac{\hat{T}(y_j)}{(1-T'(y_j))\,y_j}dj\right\} f(i)\, di.
\end{aligned}$$

Equation (44) implies that

$$\begin{aligned}
& -\delta_i\int_{\underline{y}}^{\bar{y}} \Gamma_{ij}\delta_j\varepsilon_j^{S,r}\frac{\hat{T}'(y_j)}{1-T'(y_j)}dj+\delta_i\int_{\underline{y}}^{\bar{y}} \Gamma_{ij}\delta_j\varepsilon_j^{S,n}\frac{\hat{T}(y_j)}{(1-T'(y_j))\,y_j}dj \\
= & -\hat{\Omega}_i^E-\delta_i\frac{\varepsilon_i^{S,r}}{\varepsilon_i^D}\frac{\hat{T}'(y_i)}{1-T'(y_i)}+\delta_i\left(1+\frac{\varepsilon_i^{S,w}}{\varepsilon_i^D}+\frac{\varepsilon_i^{S,n}}{\varepsilon_i^D}\right)\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i}.
\end{aligned}$$

The previous expression can thus be rewritten as

$$\begin{aligned}
& -\int_0^1 T'(y_i)\, y_i \left\{\left(1-\frac{1}{\varepsilon_i^D}\right)\delta_i\varepsilon_i^{S,r}+\delta_i\left(1+\varepsilon_i^{S,w}\right)\frac{\varepsilon_i^{S,r}}{\varepsilon_i^D}\right\}\frac{\hat{T}'(y_i)}{1-T'(y_i)} f(i)\, di \\
& +\int_0^1 T'(y_i)\, y_i \left\{\left(1-\frac{1}{\varepsilon_i^D}\right)\delta_i\varepsilon_i^{S,n}+\delta_i\left(1+\varepsilon_i^{S,w}\right)\left(1+\frac{\varepsilon_i^{S,w}}{\varepsilon_i^D}+\frac{\varepsilon_i^{S,n}}{\varepsilon_i^D}\right)\right\}\frac{\hat{T}(y_i)}{(1-T'(y_i))\,y_i} f(i)\, di.
\end{aligned}$$

Tedious but straightforward algebra, using in particular the equality $\delta_i\left(1+\varepsilon_i^{S,w}\right)=1+\left(1-\frac{1}{\varepsilon_i^D}\right)\delta_i\varepsilon_i^{S,w}$, implies that this is in turn equal to

$$
\begin{aligned}
& -\int_0^1 T'(y_i)\, y_i \left[\varepsilon_i^{S,r} \frac{\hat{T}'(y_i)}{1-T'(y_i)} - \left(1+\varepsilon_i^{S,w}+\varepsilon_i^{S,n}\right) \frac{\hat{T}(y_i)}{(1-T'(y_i))\, y_i}\right] f(i)\, di \\
= & \int_0^1 T'(y_i)\, y_i \left[\frac{\varepsilon_i^D}{\delta_i} \mathcal{A}(y_i) + \left(1-\varepsilon_i^D\right) \frac{\hat{T}(y_i)}{(1-T'(y_i))\, y_i}\right] f(i)\, di,
\end{aligned}
$$

where the second equality uses equation (45). Using the solution for $\hat{T}$ derived in (47) as a function of the auxiliary function $\mathcal{A}$, and changing variables from skills to incomes, allows us to rewrite this expression as

$$
\begin{aligned}
& \int_{\underline{y}}^{\bar{y}} T'(y_i)\, y_i \left[\frac{\varepsilon_{y_i}^D}{\delta_{y_i}} \mathcal{A}(y_i) + \left(1-\varepsilon_{y_i}^D\right) \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i y_j} \mathcal{A}(y_j)\, dy_j\right] f_Y(y_i)\, dy_i \\
= & \int_{\underline{y}}^{\bar{y}} \left[T'(y_i) \frac{\varepsilon_{y_i}^D}{\delta_{y_i}} y_i f_Y(y_i)\right] \mathcal{A}(y_i)\, dy_i + \int_{\underline{y}}^{\bar{y}} \left[\int_{\underline{y}}^{y_i} T'(y_j) \left(1-\varepsilon_{y_j}^D\right) \mathscr{E}_{y_j y_i} y_j f_Y(y_j)\, dy_j\right] \mathcal{A}(y_i)\, dy_i
\end{aligned}
$$

where the second equality inverts the order of the two integrals. Finally, using (36), we can rewrite the mechanical effect of the tax reform on government revenue as

$$
\begin{aligned}
& \int_{\underline{y}}^{\bar{y}} \hat{T}(y_i)\, f_Y(y_i)\, dy_i \\
= \; & \int_{\underline{y}}^{\bar{y}} (1-T'(y_i))\, y_i \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i,y_j} \left[\hat{\Omega}_{y_j}^E + \int_{\underline{y}}^{\bar{y}} \Lambda_{y_j,y_k} \hat{\Omega}_{y_k}^E\, dy_k\right] dy_j f_Y(y_i)\, dy_i \\
= \; & \int_{\underline{y}}^{\bar{y}} \int_{y_i}^{\bar{y}} (1-T'(y_i))\, y_i \mathscr{E}_{y_i,y_j} f_Y(y_i)\, \hat{\Omega}_{y_j}^E\, dy_j dy_i \\
& + \int_{\underline{y}}^{\bar{y}} \int_{y_i}^{\bar{y}} (1-T'(y_i))\, y_i \mathscr{E}_{y_i,y_j} \left[\int_{\underline{y}}^{\bar{y}} \Lambda_{y_j,y_k} \hat{\Omega}_{y_k}^E\, dy_k\right] f_Y(y_i)\, dy_j dy_i \\
= \; & \int_{\underline{y}}^{\bar{y}} \int_{\underline{y}}^{y_i} (1-T'(y_j))\, y_j \mathscr{E}_{y_j,y_i} f_Y(y_j)\, \hat{\Omega}_{y_i}^E\, dy_j dy_i \\
& + \int_{\underline{y}}^{\bar{y}} \int_{\underline{y}}^{y_i} (1-T'(y_j))\, y_j \mathscr{E}_{y_j,y_i} \left[\int_{\underline{y}}^{\bar{y}} \Lambda_{y_i,y_k} \hat{\Omega}_{y_k}^E\, dy_k\right] f_Y(y_j)\, dy_j dy_i \\
= \; & \int_{\underline{y}}^{\bar{y}} \left[\int_{\underline{y}}^{y_i} (1-T'(y_j))\, y_j \mathscr{E}_{y_j,y_i} f_Y(y_j)\, dy_j\right] \left\{\hat{\Omega}_{y_i}^E + \int_{\underline{y}}^{\bar{y}} \Lambda_{y_i,y_k} \hat{\Omega}_{y_k}^E\, dy_k\right\} dy_i.
\end{aligned}
$$

Collecting the terms leads to equation (49).

□

A.5 General equilibrium: CES production

Next we derive formula (18), i.e., the compensating tax reform in the simpler version of the model of Section 1. We start by stating several useful properties of this environment.

Formulas for the CES technology. All of the following properties are derived formally in Sachs, Tsyvinski, and Werquin [2016]. The CES production function implies that wages are equal to

$$w_i \quad = \quad \theta_i L_i^{-1/\varepsilon^D} \left[\int_0^1 \theta_j L_j^{1-1/\varepsilon^D} dj \right]^{\frac{1}{\varepsilon^D - 1}}.$$

The labor demand and cross-wage elasticities (10) are respectively equal to $\varepsilon_i^D = \varepsilon^D$ and

$$\gamma_{ij} \quad = \quad \frac{1}{\varepsilon^D} \frac{\theta_j L_j^{1-1/\varepsilon^D}}{\int_0^1 \theta_k L_k^{1-1/\varepsilon^D} dk} \quad = \quad \frac{1}{\varepsilon^D} \frac{w_j L_j}{\mathscr{F}(\boldsymbol{L})},$$

for all $i, j \in [0,1]$, or after a change of variables,

$$\gamma_{y_j, y_j} \quad = \quad \frac{\gamma_{ij}}{dy_j / dj} \quad = \quad \frac{1}{\varepsilon^D} \frac{y_j f_Y(y_j)}{\bar{Y}} \quad \equiv \quad \gamma_{y_j}.$$

where $\bar{Y}$ denotes the average income in the economy. Moreover, we then have

$$\Gamma_{ij} \quad = \quad \frac{\gamma_{ij}}{1 - \int_0^1 \gamma_{kk} \delta_k \varepsilon_k^{S,w} dk},$$

with $\delta_k = \frac{1}{1 + \varepsilon_k^{S,w} / \varepsilon^D}$, so that the cross-wage elasticities γ_{ij}, Γ_{ij} (resp., $\gamma_{y_i,y_j}, \Gamma_{y_i,y_j}$) depend only on j (resp., y_j). Suppose moreover the disutility of labor is isoelastic with parameter e and that the tax schedule is CRP with parameter p, i.e., it has the functional form

$$1 - T'(y) \quad = \quad (1 - \tau) y^{-p}.$$

All of the labor supply elasticities are then constant:

$$\varepsilon^{S,r} = \frac{e}{1 + pe}, \quad \varepsilon^{S,w} = \frac{(1-p)e}{1+pe},$$

and

$$\begin{aligned} \Gamma_{ij} \quad &= \quad \frac{\gamma_{ij}}{1 - \delta \varepsilon^{S,w} / \varepsilon^D} \quad = \quad \frac{\gamma_{ij}}{\delta} \\ \Gamma_{y_j, y_j} \quad &= \quad \frac{1}{\delta \varepsilon^D} \frac{y_j f_Y(y_j)}{\bar{Y}}, \end{aligned}$$

Finally, under these functional form assumptions the progressivity term (20) is equal to

$$\mathscr{E}_{y_i, y_j} \quad = \quad \frac{\varepsilon^D}{\delta \varepsilon^{S,r} y_j} e^{-\int_{y_i}^{y_j} \frac{\varepsilon^D}{\varepsilon^{S,r}} \frac{dy_k}{y_k}} \quad = \quad \frac{\varepsilon^D}{\delta \varepsilon^{S,r}} \frac{y_i^{\varepsilon^D / \varepsilon^{S,r}}}{y_j^{1 + \varepsilon^D / \varepsilon^{S,r}}}.$$

$\square$

We now give the proof of formula (18).[44]

Proof of Proporision 2. Since all the elasticities are constant and, the cross-wage elasticities $\gamma_{y_i,y_j}, \Gamma_{y_i,y_j}$ do not depend on y_i, we obtain, following the same steps as in the proof of Proposition 2 above, that the kernel $\Lambda^{(0)}_{y_i,y_j}$ of the integral equation satisfied by the auxiliary function $\mathcal{A}$ is multiplicatively separable, i.e.,

$$\mathcal{A}(y_i) \quad = \quad \hat{\Omega}^E_{y_i} + \delta_{y_i} \int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}_{y_i,y_j} \mathcal{A}(y_j)\, dy_j \tag{50}$$

where the kernel $\delta_{y_i}\Lambda^{(0)}_{y_i,y_j}$ depends only on y_j:

$$\begin{aligned}
\delta_{y_i}\Lambda^{(0)}_{y_i,y_j} \quad &\equiv \quad \delta_{y_i}\left[\Gamma_{y_i,y_j}\varepsilon^D_{y_j} - \int_{\underline{y}}^{y_j} \Gamma_{y_i,y_k}\varepsilon^D_{y_k}\mathscr{E}_{y_k,y_j}\, dy_k\right] \\
&= \quad \delta\left[\frac{\gamma_{y_j}}{\delta}\varepsilon^D - \int_{\underline{y}}^{y_j} \frac{\gamma_{y_k}}{\delta}\varepsilon^D \mathscr{E}_{y_k,y_j}\, dy_k\right] \quad \equiv \quad \delta\Lambda^{(0)}_{y_j}.
\end{aligned}$$

The solution to this integral equation is then straightforward to obtain, and moreover, the convergence conditions assumed in the proof of Proposition 2 are satisfied in this case. Indeed, multiplying both sides of (50) by $\tilde{\Lambda}^{(0)}_{y_i}$ and integrating leads to

$$\begin{aligned}
\int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}_{y_i}\mathcal{A}(y_i)\, dy_i \quad &= \quad \int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}_{y_i}\hat{\Omega}^E_{y_i} dy_i + \delta\left(\int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}_{y_i} dy_i\right)\left(\int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}_{y_j}\mathcal{A}(y_j)\, dy_j\right) \\
&= \quad \frac{\int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}_{y_i}\hat{\Omega}^E_{y_i} dy_i}{1 - \delta\int_{\underline{y}}^{\bar{y}} \Lambda^{(0)}_{y_i} dy_i} \quad \equiv \quad \lambda,
\end{aligned}$$

where λ is a constant. We thus obtain

$$\mathcal{A}(y_i) \quad = \quad \hat{\Omega}^E_{y_i} + \delta\lambda,$$

where, using the expressions for the cross-wage elasticities Γ_{y_i,y_j} and the progressivity term $\mathscr{E}_{y_i,y_j}$

[44] An alternative proof consists of differentiating the functional equation (44) with respect to y_i; since with a CES production function and CRP tax code Γ_{y_i,y_j} does not depend on y_i, this leads to a second-order ordinary differential equation that can be easily integrated to lead to the same result as Proposition 2. See the NBER Working Paper version of this paper for details.

derived above, we get:

$$\begin{aligned}
\lambda &= \frac{\int_{\underline{y}}^{\bar{y}} \left[\frac{\gamma_{y_j}}{\delta}\varepsilon^D - \int_{\underline{y}}^{y_j} \frac{\gamma_{y_k}}{\delta}\varepsilon^D \mathscr{E}_{y_k,y_j} dy_k\right] \hat{\Omega}^E_{y_j} dy_j}{1 - \int_{\underline{y}}^{\bar{y}} \left[\gamma_{y_j}\varepsilon^D - \int_{\underline{y}}^{y_j} \gamma_{y_k}\varepsilon^D \mathscr{E}_{y_k,y_j} dy_k\right] dy_j} \\
&= \frac{\frac{1}{\delta}\int_{\underline{y}}^{\bar{y}} \left[\frac{y_j f_Y(y_j)}{\bar{Y}} - \int_{\underline{y}}^{y_j} \frac{y_k f_Y(y_k)}{\bar{Y}} \frac{\varepsilon^D}{\delta\varepsilon^{S,r}} \frac{y_k^{\varepsilon^D/\varepsilon^{S,r}}}{y_j^{1+\varepsilon^D/\varepsilon^{S,r}}} dy_k\right] \hat{\Omega}^E_{y_j} dy_j}{1 - \int_{\underline{y}}^{\bar{y}} \left[\frac{y_j f_Y(y_j)}{\bar{Y}} - \int_{\underline{y}}^{y_j} \frac{y_k f_Y(y_k)}{\bar{Y}} \frac{\varepsilon^D}{\delta\varepsilon^{S,r}} \frac{y_k^{\varepsilon^D/\varepsilon^{S,r}}}{y_j^{1+\varepsilon^D/\varepsilon^{S,r}}} dy_k\right] dy_j} \\
&= \frac{\frac{1}{\delta}\int_{\underline{y}}^{\bar{y}} y_j \hat{\Omega}^E_{y_j} f_Y(y_j)\, dy_j - \frac{\varepsilon^D}{\delta\varepsilon^{S,r}} \int_{\underline{y}}^{\bar{y}} \left[\int_{y_j}^{\bar{y}} \left(\frac{y_j}{y_k}\right)^{1+\varepsilon^D/\varepsilon^{S,r}} \hat{\Omega}^E_{y_k} dy_k\right] f_Y(y_j)\, dy_j}{\frac{\varepsilon^D}{\delta\varepsilon^{S,r}} \int_{\underline{y}}^{\bar{y}} \left[\int_{y_j}^{\bar{y}} \left(\frac{y_j}{y_k}\right)^{1+\varepsilon^D/\varepsilon^{S,r}} dy_k\right] f_Y(y_j)\, dy_j} \\
&= \frac{\int_{\underline{y}}^{\bar{y}} \left[y_j \hat{\Omega}^E_{y_j} - \frac{\varepsilon^D}{\delta\varepsilon^{S,r}} \int_{y_j}^{\bar{y}} \left(\frac{y_j}{y_k}\right)^{1+\varepsilon^D/\varepsilon^{S,r}} \hat{\Omega}^E_{y_k} dy_k\right] f_Y(y_j)\, dy_j}{\int_{\underline{y}}^{\bar{y}} \left[1 - \left(\frac{y_i}{\bar{y}}\right)^{\varepsilon^D/\varepsilon^{S,r}}\right] y_i f_Y(y_i)\, dy_i},
\end{aligned}$$

where the third equality exchanges the order of the integrals in the numerator and the denominator. Therefore the compensating tax reform is given by

$$\begin{aligned}
\frac{\hat{T}(y_i)}{y_i} &= (1 - T'(y_i)) \int_{y_i}^{\bar{y}} \mathscr{E}_{y_i y_j} \left[\hat{\Omega}^E_{y_j} + \delta\lambda\right] dy_j \\
&= (1 - T'(y_i)) \left[\int_{y_i}^{\bar{y}} \mathscr{E}_{y_i y_j} \hat{\Omega}^E_{y_j} dy_j + \lambda \frac{\varepsilon^D}{\varepsilon^{S,r}} \int_{y_i}^{\bar{y}} \frac{y_i^{\varepsilon^D/\varepsilon^{S,r}}}{y_j^{1+\varepsilon^D/\varepsilon^{S,r}}} dy_j\right] \\
&= (1 - T'(y_i)) \left[\int_{y_i}^{\bar{y}} \mathscr{E}_{y_i y_j} \hat{\Omega}^E_{y_j} dy_j + \lambda \left\{1 - \left(\frac{y_i}{\bar{y}}\right)^{\varepsilon^D/\varepsilon^{S,r}}\right\}\right].
\end{aligned}$$

Letting $\bar{y} \to \infty$ finally leads to:

$$\frac{\hat{T}(y_i)}{y_i} = (1 - T'(y_i)) \left[\int_{y_i}^{\bar{y}} \mathscr{E}_{y_i y_j} \hat{\Omega}^E_{y_j} dy_j + \lambda\right],$$

where the constant λ is equal to

$$\begin{aligned}
\lambda &= \frac{1}{\bar{Y}} \int_{\underline{y}}^{\bar{y}} \left[y_j \hat{\Omega}^E_{y_j} - \frac{\varepsilon^D}{\delta\varepsilon^{S,r}} \int_{y_j}^{\bar{y}} \left(\frac{y_j}{y_k}\right)^{1+\varepsilon^D/\varepsilon^{S,r}} \hat{\Omega}^E_{y_k} dy_k\right] f_Y(y_j)\, dy_j \\
&= \mathbb{E}\left[\frac{y_j}{\bar{Y}} \left(\hat{\Omega}^E_{y_j} - \int_{y_j}^{\bar{y}} \mathscr{E}_{y_j,y_k} \hat{\Omega}^E_{y_k} dy_k\right)\right].
\end{aligned}$$

This concludes the proof.

$\square$

A.6 Advantages of the compensation approach

We now discuss the advantages of the compensation approach taken in this paper over the more standard optimal taxation approach taken in most of the literature (see in particular Stiglitz [1982a], Rothschild and Scheuer [2013, 2014, 2016], Ales, Kurnaz, and Sleet [2015a,b], Scheuer and Werning [2016], Sachs, Tsyvinski, and Werquin [2016]).

The first advantage of the compensation approach is that we are able to derive a closed-form formula in a very general environment, while the optimal tax formula is generally very complex and must be solved numerically even in simple models. In particular, our formula depends only on the evaluation of sufficient statistics (elasticities, income distribution) in the *current*, pre-disruption, economy rather than in a fictional economy where the optimal tax schedule would already be implemented; it can thus be directly applied using actual data. Moreover, the response to a given disruption (e.g., automation) is given by a reform of the *actual* (e.g., U.S.) tax schedule, rather than of the optimal one, which was not implemented in the first place – this makes the insights from our analysis more directly policy-relevant.

The second main advantage of the compensation approach over the traditional optimal tax approach is that it does not rely on a particular social welfare function, and is thus robust to the choice of welfare criteria – our formula depends only on variables that are observable or measurable in the data.

B Numerical simulations

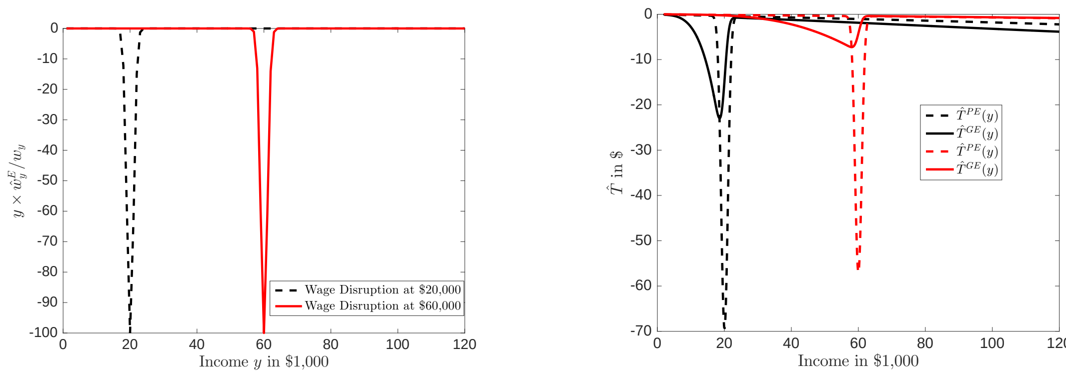
B.1 Graphical representation of formulas (13) and (18)

We calibrate the elasticity of labor supply to $e = 0.33$ (Chetty [2012]), the rate of progressivity of the initial tax schedule to $p = 0.156$ (Heathcote, Storesletten, and Violante [2016]), and the elasticity of substitution between skills to $\varepsilon^D = \infty$ (partial equilibrium) or $\varepsilon^D = 0.6$ (Dustmann, Frattini, and Preston [2013]). For illustrative purposes, we construct smooth wage disruptions that are normally distributed and centered around income y_{i^*} (or, equivalently, around the percentile i^* of the wage distribution), where $y_{i^*} = \$20,000$ or $y_{i^*} = \$60,000$.[45] We assume that at this point the wage decreases by an amount $\hat{w}_{i^*}^E$ that implies a decrease in pre-tax income of $y_{i^*} \times \frac{\hat{w}_{i^*}^E}{w_{i^*}} = \$100$. The resulting pre-tax income disruption is illustrated in the left panel of Figure 3. The right panel of Figure 3 plots the respective compensating tax reforms in partial equilibrium (dashed lines, formally derived in Proposition 1) and in general equilibrium (solid lines, formally derived in Proposition 2).

The partial equilibrium compensation shows that the decrease in the agent's average tax rate implied by the tax reform mirrors the income loss due to the wage disruption. Recall that the initial (pre-reform) marginal tax rates matter for the shape of the compensation via the term $(1 - T'(y_i))$ in formula (13). This explains why the compensation is larger for lower incomes, because the marginal

[45] This approximates a Dirac disruption at income y^*. The tax reform that compensates a general, non-Dirac, disruption is equal to the sum of the reforms that compensate the corresponding Dirac perturbations at each income level.

Figure 3: Wage disruptions centered at \$20,000 and \$60,000 (left panel) and respective compensating tax reforms in partial and general equilibrium (right panel)



tax rate in our calibration is increasing with income – it is equal to 10% at \$20,000 and 22% at \$60,000. As a consequence, the same pre-tax income loss of \$100 translates into an after-tax income loss of \$90 and \$78 respectively, so that the compensating tax reform requires respective reductions in tax payment of \$90 and \$78.

This insight is also present in general equilibrium, as reflected by the term $(1 - T'(y_i))$ in formula (18) – this implies that the peak of the general-equilibrium compensation (solid curves in the right panel of Figure 3) is lower when the disruption affects higher-income agents. However, the general-equilibrium compensation no longer mirrors the shape of the income losses. The first and main difference is that the reduction in the tax payment of the disrupted agent i^* is much smaller than in partial equilibrium, while at the same time agents who earn an income lower than y_{i^*} now also face substantial tax rebates, even though they were not initially hurt by the exogenous disruption. These features reflect the progressivity term (20) in formula (18). As discussed above, the compensation is exponentially decreasing at a rate determined by the ratio of the labor demand and labor supply elasticities, up to the income level at which the tax reform peaks.

To understand why this is the case, suppose that the government implements the partial-equilibrium compensation (represented by the dashed curves). Since agents with undisrupted incomes are not compensated, this tax reform creates large movements in the marginal tax rates around income y_{i^*}, which in general equilibrium has large unintended welfare consequences. Consider for example an agent with income just below y_{i^*}. His average tax rate is reduced by the after-tax income loss that he incurs. But his marginal tax rate is also reduced, which causes an additional welfare gain. As a result, this agent is strictly better off after the compensation than he was in the initial equilibrium, and this gain can be redistributed. To correct this, the government lowers the magnitude of his tax rebate and implements a reduction in marginal tax rates to achieve exact compensation. Agents with incomes y lower than y_{i^*} who are not initially disrupted have both their marginal and average tax rates reduced, with a zero net welfare effect.

This reasoning leads to an exponential reduction in the tax rates – indeed, if the average tax rate of agent y is lowered by $\frac{\hat{T}(y)}{y}$, his marginal tax rate must be reduced by a factor $\frac{\varepsilon^D}{\delta \varepsilon^{S,r}} > 1$

times $\frac{\hat{T}(y)}{y}$; but this in turn lowers further the tax bill of agents $y' > y$, whose marginal tax rate must therefore be reduced, and so on. Note finally that the compensation peaks at a skill i^{**} that is strictly below that of the agent i^* who incurs the largest disruption. Indeed, by definition the agent with the highest tax reduction (i^{**}) has a zero marginal tax rate change. Thus, an agent with a slightly higher skill gets almost the same total tax rebate (the difference between the two is second-order since $\hat{T}'(y_{i^{**}}) = 0$) and a strictly higher marginal tax rate change (the difference is first-order if $\hat{T}''(y_{i^{**}}) > 0$), and hence a strictly higher compensation, explaining why we must have $i^{**} < i^*$.

Finally, as we discussed above, the compensation-of-compensation effect implies an additional uniform (in percentage terms) downward shift of the marginal tax rates. This additional correction implies in particular a reduction in the tax rates on incomes strictly larger than that of the perturbed agent, while these were left unchanged both in partial equilibrium and in the absence of cross wage effects. This effect involves further progressivity in the tax reform, since initially the marginal rates of the tax schedule T are increasing with income.

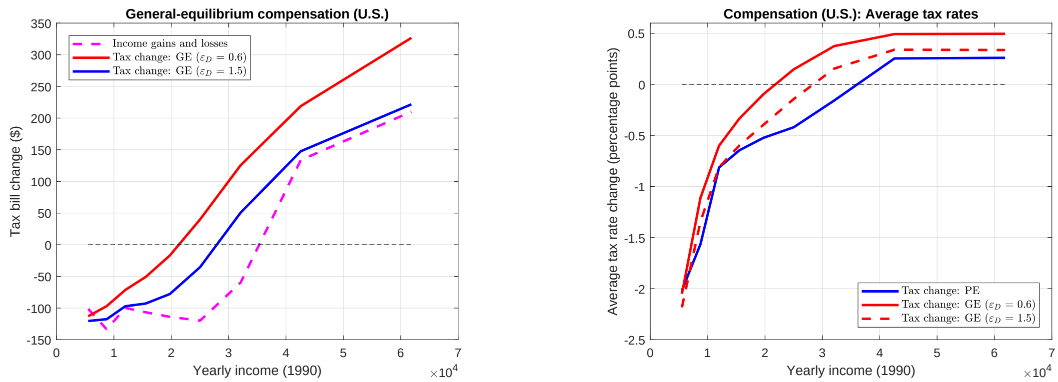
B.2 Details for Section 3

Evidence from the U.S.

We order skills i from the data by ascending wage w_i in 1990. Since there is not an exact one-to-one map between wages w_i and earnings $y_i = w_i l_i$ in the data, we replace the estimated hours series l_i with a simulated series $\bar{l}_i$ constructed as follows. Since in our model the elasticity of labor supply with respect to the wage is constant and equal to $\varepsilon^{S,w}$, there is a log-linear relationship between wages and hours, namely $\ln l_i = \alpha + \beta \ln w_i$ with $\beta = \varepsilon^{S,w}$. An OLS estimation of the parameters $(\hat{\alpha}, \hat{\beta})$ in the data gives us: (i) a calibration for the elasticity parameter $\varepsilon^{S,w} \equiv \hat{\beta}$; and (ii) a strictly increasing relationship between (constructed) hours $\bar{l}_i = e^{\hat{\alpha}} w_i^{\hat{\beta}}$ and (actual) wages w_i. We find that the elasticity of labor supply with respect to the retention rate, $\varepsilon^{S,r} = \varepsilon^{S,w} / (1-p)$ is equal to 0.47, and the correlation between the hours l_i reported in the dataset and our constructed variable $\bar{l}_i$ is 0.97.

Figure 4 plots the compensating tax reform (left panel) and average tax rate changes (right panel) in the case where we assume that the size of the wage disruption continues to increase linearly above \$60,000 (rather than staying constant), such that agents with an income 20 times larger face a disruption that is 3 times larger, i.e., their wage increases by 1.02% due to robots. Compared to Figure 2, the compensating tax increases are now higher at the top of the distribution. For individuals with incomes equal to \$60,000 per year, the tax increase on agents with income \$60,000 in 1990 is equal to \$327 ($\varepsilon^D = 0.6$) or \$222 ($\varepsilon^D = 1.5$). This is because, as explained in detail in Section B.1, when the wage disruption is larger at the top, the compensating tax bill below those incomes is larger – the tax increases necessary to redistribute the income gains experienced by the richest agents must be frontloaded via larger marginal tax rates at the bottom.

Figure 4: General-equilibrium compensation (Scenario 2)



Finally, when the compensating tax changes have been computed by can use the expression:

$$
\begin{aligned}
\mathcal{R}(\hat{\boldsymbol{w}}^E) &= \int_0^1 \hat{T}\left(y_i\right) di + \int_0^1 \left(\frac{\hat{w}_i^E}{w_i} + \frac{\hat{w}_i}{w_i} + \frac{\hat{l}_i}{l_i}\right) y_i T'\left(y_i\right) di \\
&= \int_0^1 \hat{T}\left(y_i\right) di + \int_0^1 \left(\left(1+\varepsilon_i^{S,w}\right)\left[\frac{\hat{w}_i^E}{w_i} + \frac{\hat{w}_i}{w_i}\right] - \varepsilon_i^{S,r}\frac{\hat{T}'\left(y_i\right)}{1-T'\left(y_i\right)}\right) y_i T'\left(y_i\right) di \\
&= \int_0^1 \hat{T}\left(y_i\right) di + \int_0^1 \left[\left(1+\varepsilon_i^{S,w}\right)\frac{\hat{T}\left(y_i\right)}{y_i} - \varepsilon_i^{S,r}\hat{T}'\left(y_i\right)\right] y_i \frac{T'\left(y_i\right)}{1-T'\left(y_i\right)} di
\end{aligned}
$$

to compute the fiscal surplus.

Evidence from Germany

We now exploit the data provided to us by Dauth, Findeisen, Südekum, and Woessner [2017] for Germany, who estimate the impact of automation on the wages of manufacturing workers between 1994 and 2014. They find in Column 6 of Table 7 that an increase of an additional robot per worker reduces earnings by 1.0822%. We then multiply this number by the average change in robot exposure over the twenty-year period (defined in their equation (1)) at each decile of the earnings distribution to obtain the total income losses at each decile. These are represented in the left panel of Figure 5 (magenta curve). Since higher deciles have been more exposed to robots than lower deciles in Germany, their income loss is higher. Finally, we use the labor income tax schedule reported in Kindermann, Mayr, and Sachs [2017], who calibrate a CRP functional form to German data and find a rate of progressivity equal to $p = 0.128$.

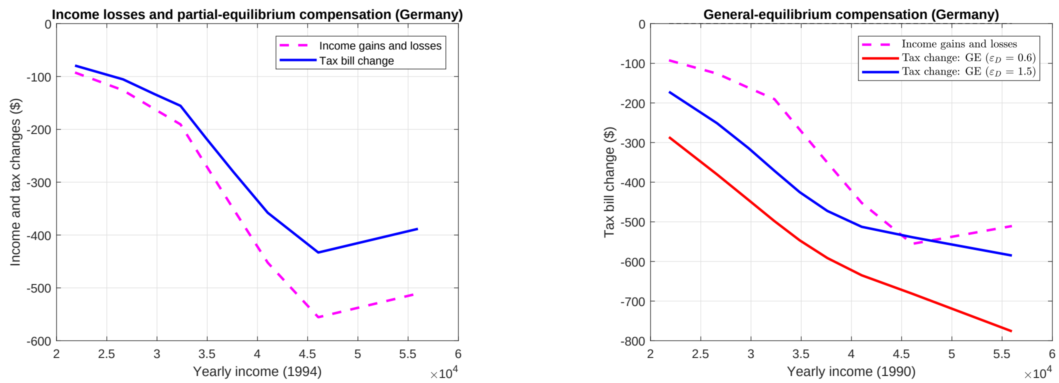
Note that Dauth, Findeisen, Südekum, and Woessner [2017] do not report hourly wages and yearly hours (they can only estimate daily wages and yearly earnings). This prevents us from estimating the labor supply elasticity from the data directly. We therefore assume that, as in Acemoglu and Restrepo [2017], that $\varepsilon^{S,r} = 0.5$. This then allows us to back out the hourly wage disruption from the estimated earnings dispersion and apply our formulas. Indeed, using $\frac{\hat{l}_i}{l_i} =$

$\varepsilon^{S,w}[\frac{\hat{w}_i^E}{w_i} + \frac{\hat{w}_i}{w_i}]$, it is straightforward to show that the modified wage disruption is given by

$$\hat{\Omega}_i^E = \frac{1}{1 + \frac{\varepsilon^{S,w}}{\delta}} \times \frac{\hat{y}_i}{y_i},$$

where $\frac{\hat{y}_i}{y_i} = \frac{\hat{w}_i^E}{w_i} + \frac{\hat{w}_i}{w_i} + \frac{\hat{l}_i}{l_i}$ is the earnings disruption. Note finally that Dauth, Findeisen, Südekum, and Woessner [2017] are able to estimate the robot exposure at the 10^{th} decile (about \$500,000), so that we do not have to make an assumption about the evolution of the disruption beyond the top income of our dataset (exposure to robots is roughly constant beyond the 9^{th} decile).

Figure 5: General-equilibrium compensation: Germany



The solid blue curve in the left panel of Figure 5 plots the compensation in partial equilibrium, which mirrors the income loss induced by exposure to robots. In the right panel of Figure 5, we plot the compensating tax reform in general equilibrium for $\varepsilon^D \in \{0.6, 1.5\}$. The tax rebate is larger than in partial equilibrium (and almost everywhere larger than the income loss due to the disruption). If $\varepsilon^D = 0.6$, the bottom incomes should have their tax payment reduced by \$286 per year (i.e., 310% of their income loss!), while the top incomes should have theirs reduced by \$776 per year (i.e., 152% of their income loss). Finally, these figures imply reductions in the tax rates equal to 1.3 percentage points at the bottom and 1.4 pp at the top. If $\varepsilon^D = 1.5$, the tax rebates are \$172 (186% of the income loss) and \$585 (115% of the income loss) at the bottom and the top, respectively.