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The Maturity Structure of Sovereign Debts within a Solidarity Zone

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Abstract

This paper characterizes the optimal bailout maturity structure for a sovereign on the verge of a default. I find that buying back long-term debt is strictly optimal when it can prevent a default today and in the future. Otherwise, buying back short-term debt is optimal and can prevent a default only today. The paper also investigates the choice of debt maturity structure of the sovereign in the presence of bailouts. I find that potential bailouts extend the sovereign's borrowing capacity and make it rely more on debt with shorter maturities on average. As short-term debt is vulnerable to rollover crises, it generates more default risk. Eventually, the paper analyses how potential bailouts affect ex post welfare and studies ex ante welfare-improving policies.

Keywords: sovereign debt, bailouts, maturity structure, risk
Jel codes: E43, E61, F30, F34, G15, H63

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1 Introduction

The ongoing Eurozone crisis has raised the debate about whether and how to stand by a country close to default. Before the crisis, a common belief that the Eurozone would not let one of its countries default had a significant impact on the countries' borrowing decisions. The example of the 2010 Greek debt crisis is edifying: the average maturity of its portfolio started to shorten a few years before the bailout. Furthermore, the joint intervention of the Eurozone and the IMF took place when the sovereign was unable to rollover its maturing debt, and was aimed at helping Greece both rollover its debt and pay its public expenditures. This first bailout did not put off default risk enough as it was rapidly followed by a second one in 2012. In this context, I want first to understand how potential bailouts affect the sovereign's choice of debt maturity structure. Second, I analyze how a bailout should optimally deal with the various maturities of the troubled sovereign. Third, I characterize *ex ante* policies that could improve total welfare.

More generally, I study the case of a sovereign that receives an exogenous liquidity need, and needs to borrow from the market. It expects to receive bailouts from deep-pocket guarantors if the country finds itself on the verge of a default. The guarantors can be thought of as a group of countries that would be affected by a default or the international community. In the framework, the country cannot commit to repay its debt, and potential guarantors cannot commit not to bailout the country. More precisely, I link the debtor country's incentive to repay (and therefore its ability to borrow) to its cost of default, and the guarantors' incentive to help to the spillover cost they incur if the country defaults. In fact, the country repays its debt if it is less costly than a default, and the guarantors bail out the country if it is less costly than to let the country default. The country's cost of default can include reputation costs, a reduced access to international financial markets, or the threat of sanctions. The spillover cost guarantors incur can include economical as well as political considerations such as a reduced trade, banking exposures, the fear of a run, or the end of the European construction in the case of the EU.
Hence, a country with a large cost of default is able to reimburse a large amount of debt and can therefore borrow extensively. Potential guarantors who would be very hurt by a default are likely to contribute a lot to bail out the country (nuisance power). In this situation, the country's borrowing capacity is determined by both its own cost of default and the collateral damage its default creates on potential guarantors.

I consider an endowment economy with a finite horizon where the country's income realization is random. There are two sources of market incompleteness: (i) the debt issued by the country is non-contingent and (ii) there is lack of commitment from both the country and potential guarantors. As a result, there are states of nature in which the country cannot or does not want to repay/rollover its debt, so that a default can happen along the equilibrium path. Yet, potential guarantors can decide to bail out the country by repurchasing some of its claims. In this case, they do so to prevent a default in the current period and avoid the spillover damage of a default. The debtor country borrows from a large number of foreign lenders. They are competitive, risk-neutral and do not incur any spillover cost in case of a default, on top of the forgone repayment of the debt.

I first study the optimal choice of debt maturity structure of the sovereign that internalizes the potential bailouts from the guarantors. I find three main zones depending on the sovereign's liquidity need: in the low liquidity need zone, the country chooses to borrow a low level of long-term debt only, in order to hedge against a risk of default in the short-term while remaining under the umbrella of the potential guarantors in the long-term. In the intermediate liquidity need zone, the country borrows more short-term debt, and takes a risk of default in the long-term where the size of the debt, possibly increased by rollovers, may discourage the potential guarantors. Eventually, in the high liquidity need zone, the country over-borrows on the short maturity, and takes a risk of default in the short-term where the size of the debt may be too large for a rollover as well as for a bailout.
Simple insights can be derived. The larger the liquidity need the sovereign faces, the more it increases its level of borrowings and the more it relies on short-term debt. Indeed, short-term debt presents a borrowing capacity advantage relative to long-term debt, as there are less options to default over a short period than over a long one and as a consequence the default risk is lower. This explains why short-term debt is more valuable in situations of high liquidity needs. In the presence of potential bailouts, the sovereign's borrowing capacity is increased because it not only pledges expected future revenues but also potential bailouts. Therefore, the reliance on short-term debt is reinforced in situations of high liquidity needs because the sovereign is able to pledge a potential bailout in the short-term to maximize its borrowing capacity. Interestingly, the set of liquidity needs where the sovereign relies more on short-term debt is larger in the presence of bailouts. Hence, as short-term debt is associated with rollover risk, there are more default risks in this situation.

Second, I study the form of the optimal bailout from potential guarantors. When they want to prevent a default in the current period, they have the choice between repurchasing short-term debt, long-term debt, or both, in order to make it incentive-compatible for the sovereign to rollover and/or repay the rest of its maturing debt. On the one hand, repurchasing short-term debt alleviates the net present value of the debt burden in the current period without changing it in the following ones. On the other hand, repurchasing long-term debt alleviates the net present value of the debt burden both in the current period and in the following ones. I find that repurchasing long-term debt is strictly optimal when it makes the sovereign rollover and/or repay its debt in the current period and in the following ones. In this case, it achieves two goals at the same time, i.e. prevent a default today and in the future. It corresponds to a *commitment effect*: by committing to a bailout in the future, guarantors are able to make the country repay its debt today (backloading result). Otherwise, repurchasing short-term debt is optimal. It corresponds to an *option value effect*: guarantors choose to repurchase only short-term debt to make the country rollover and/or repay the rest of its maturing debt. This achieves only one goal, i.e. prevent a default today without taking care of what can happen in the future.

Third, potential bailouts affect negatively the guarantors' ex post welfare while raising the sovereign's one on average. I study several instruments to alleviate this inefficiency ex ante. On the one hand, a regulatory instrument consisting of a limit ratio on the maturity structure contains the risk-taking behavior of the sovereign. Yet, total welfare is lowered as it reduces the sovereign's borrowing capacity in times of high liquidity needs by limiting its ability to borrow short-term debt. On the other hand, an initial contractual agreement between the country and guarantors where the country borrows from both the market and guarantors can improve welfare and prevent the occurrence of any default at the equilibrium. Intuitively, such a contract addresses directly the divergence of interests between the country which aims at maximizing transfers and guarantors who would like the country to minimize default risk.

To sum up, the presence of potential bailouts extends the initial borrowing capacity of the sovereign and makes it rely more on short-term debt, which is at the origin of more default risk on average. Short-term debt is used in times of high liquidity needs by the sovereign to extend as much as possible its borrowing capacity by pledging both its income in the short-term and the bailout it may receive as well at this same maturity. As for the guarantors, repurchasing long-term debt in a bailout is strictly optimal when they can prevent a default both today and in the future. Otherwise, repurchasing short-term debt is optimal to prevent a default only today. Eventually, only an initial contractual agreement between the country and the guarantors can be welfare-improving because it addresses directly the divergence of interests between the two parties.

Related literature This paper is related to several bodies of the literature. Papers such as Kehoe and Levine (1993), Kocherlakota (1996) and Alvarez and Jermann (2000) study the optimal debt contract under lack of commitment in an environment where markets are complete. Yet, in such a framework, a default cannot happen at the equilibrium. This paper thus follows the literature of sovereign debt under lack of commitment and incomplete markets, initiated by the seminal paper of Eaton and Gersovitz (1986). As pointed out by Arellano (2008), a default can happen at the equilibrium in such an environment. In Calvo (1988), the government auctions off its borrowing level and lets the market determine the interest rates. This leaves room for a multiplicity of equilibria driven by expectations. Cole and Kehoe (2000) also analyze multiple equilibria to study self-fulfilling crises brought about by a loss of confidence. In this paper, there is no multiplicity of equilibria as the country first sets the level of its repayments and lets the market determine the interest rates and its initial borrowing level.

This paper builds on the existence of exogenous costs of default. These costs are essential in explaining sovereign repayments. They can be external, such as trade embargoes, seizure of assets or military intervention (Sachs (1983) and Eaton, Gersovitz and Stiglitz (1986)), or internal as emphasized by recent research (Mengus (2014)).

Tirole (2015) investigates ex ante and ex post forms of solidarity towards an indebted sovereign in the presence of default. Here, I carry out a similar analysis in a dynamic setup where the maturity of debt is a choice of the sovereign. Several papers study the debt maturity structure of a sovereign that can decide to default, without the ingredient of solidarity. Arellano and Ramanarayanan (2012) build on the observation that the government interest rate spreads are low in normal times with the short-term spread being lower than the long-term one. During emerging market crises, both spreads rise and the short-term spread rises more than the long-term one. In parallel, the debt maturity shortens significantly. By analyzing the tradeoff between the *incentive benefit* of short-term debt and the *hedging benefit* of long-term debt, the paper is able to make sense of these observations. Broner, Lorenzoni and Schmuckler (2013) argue that lenders' risk aversion makes short-term borrowing cheaper. It explains why emerging economies tend to borrow short-term and why they rely even more on short-term debt in periods of financial turmoil. This paper builds on these results with the new element of potential bailouts from guarantors.

I present the setup of the model in section 2. Then, I study the Principal's optimal bailout strategy in section 3. I can then solve for the country's optimal borrowing strategy in section 4. In section 5, I analyze the impact of potential bailouts. Section 6 studies some simple extensions and discusses the results. In section 7, I carry out a welfare analysis and study some ex ante policies that could be Pareto-improving. Section 8 concludes.

2 The model

2.1 Framework

This section develops a model of sovereign debt with three risk neutral economic agents. The country borrows from the private sector (the market), which is competitive. Potential guarantors (the Principal) can intervene to help the country if it cannot or does not want to repay its debt.

There are 3 periods with a discount factor $\delta < 1$ at each period. At dates $t = 1, 2$, the country receives a random income according to a positively correlated process, with $\underline{\rho} \leq \alpha \leq \overline{\rho}$[1]. For simplicity, I assume that the income is $\overline{y_t} > 0$ in the good state of nature for $t = 1, 2$, and 0 otherwise (see Figure 1). A zero income is to be interpreted as some minimal level of consumption below which the country is not willing to go. At date 0, the country has no money, can decide to borrow b_0 against future incomes, and values this borrowing at Rb_0 with $R \geq 1$. At date 1, the country may have some debt to repay, and chooses whether to rollover and repay its maturing debt or to default. At date 2, the country may have some debt to repay and its only decision is whether to repay or to default.

There is no commitment in the sense that the country pays back its debt only if it finds it privately optimal to do so, and the Principal bails out the country only if he finds it privately optimal to do so. If the country chooses to default at date t, it defaults on all its debt and incurs the default cost $\Phi_t > 0$, which is the net present value of all the costs implied by a default at date t. In turn, the Principal is affected by a spillover cost $\phi_t > 0$, which is the net present value of the costs he incurs when the country defaults at date t. There is perfect information.

[1] This assumption corresponds to the persistence of sovereign incomes that is observed empirically (in either good or bad states).

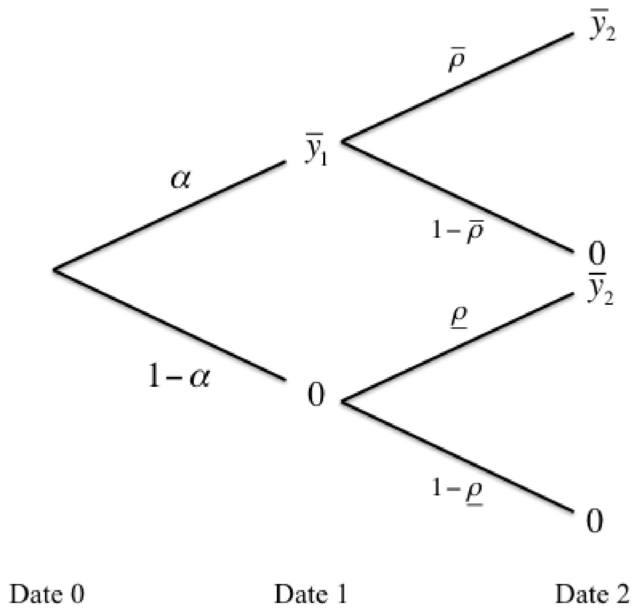


Figure 1: Income Realizations

I assume that both the default cost and the spillover cost are weakly decreasing over time, so that $\Phi_1 \geq \Phi_2$ and $\phi_1 \geq \phi_2$, and that the country's default cost is larger than the Principal's spillover cost $\Phi_t >> \phi_t$ for $t = 1, 2$.

The model is an extensive form game that can be solved by backward induction. I am therefore looking for subgame perfect Nash equilibria where the optimal borrowing and rollover strategies of the country, the optimal bailout of the Principal, and the interest rates paid on debt are endogenously determined in the model. The timing of the model is described in Figure 2.

Date 0: *Borrowing.* At date 0, the country borrows b_0 from the market against claims (d_1^0, d_2^0) to be repaid at dates 1 and 2 respectively. As for the notation, d_t^s is the amount of debt the country has at the end of date s to repay at date t (after the bailout and rollover decisions of date s took place). The additional upper script P denotes the level of debt remaining after the bailout offer from the Principal and the upper script R the additional amount of debt to be reimbursed at a given maturity after a rollover. I denote d_1^0 the short-term debt and d_2^0 the long-term debt. The country values this borrowing at Rb_0 (private benefit) where the parameter R measures the intensity of the country's liquidity needs. The higher the liquidity need R, the more the country is willing to borrow at date 0.

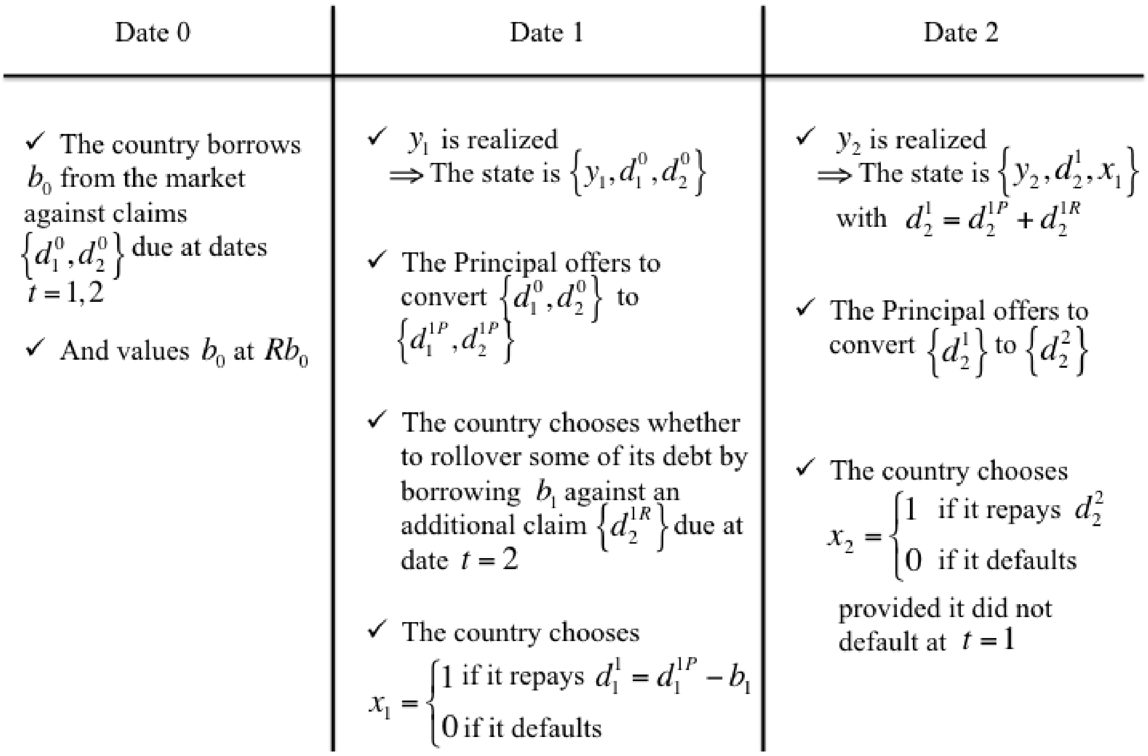


Figure 2: Timing of the model

Given its liquidity need R, the country chooses (d_1^0, d_2^0) to maximize its expected utility of date 0:

$$U_0^C(R) = \max_{\{d_1^0, d_2^0\}} \left\{ Rb_0 + \mathbb{E}_0 \left(\sum_{t=1}^{2} \delta^t (y_t - x_t d_t^t - (1 - x_t) x_{t-1} \Phi_t) \right) \right\}$$

such that:

$$b_0 = \frac{d_1^0}{1 + r_1^0} + \frac{d_2^0}{(1 + r_2^0)^2}$$

where x_t for $t = 1, 2$ is a dummy variable that equals 1 if the country chooses to repay at date t, and 0 if it chooses to default (with $x_t = 0$ absorbing and $x_0 = 1$). r_1^0 and r_2^0 are the one-period interest rates paid by the country at date 0 on debt due at dates 1 and 2 respectively. They are endogenously determined in the model as a result of the randomness of incomes, the willingness of the country to repay and the behavior of the Principal.

Dates 1 & 2: *Income realizations.* At the beginning of each period $t = 1, 2$, the country receives its random income. Considering the income process, it *cannot* repay in the bad state of nature because it has no income[2]. Assuming that $\overline{y_t} > \Phi_t$, it can repay in the

[2]The decision to default is always a question of willingness. In this reduced form model, when the country has a zero income and cannot repay any debt, it corresponds to a case where the country does not want to tax its agents to repay its claims and *chooses* to default.

good state of nature, but may not *want* to do so if it is more costly than to default.

Debt structure. At the beginning of period t, the debt structure is characterized by $\{d_\tau^{t-1}\}_{\tau=t}^2$ where d_τ^{t-1} is the remaining claim inherited from date $t-1$ to be paid at a future date τ.

Bailout from the Principal. After the country's income realization at date t, the Principal makes a take-it-or-leave-it offer to the country where he may propose to repurchase some of the debt, either short-term, or long-term, or both if available. He does so if it can prevent a default at the current period at a lower cost than the spillover damage. More precisely, in the case where the country has no income at date t and the Principal wants to prevent a default, he has to repurchase either a part of the claim of date t such that the country is able to rollover all the remaining (when a rollover is available), or the whole claim. In the case where the country is willing to default despite having a positive income and the Principal wants to prevent it, he has to repurchase some debt of various maturities to the point the country is able to rollover some of the claim of date t (when a rollover is available) and willing to repay the rest.

At date 1, knowing the country's income y_1, the Principal can propose to lower the country's debt level from (d_1^0, d_2^0) to (d_1^{1P}, d_2^{1P}) where he repurchases some short-term debt $d_1^0 - d_1^{1P}$ and/or some long-term debt $d_2^0 - d_2^{1P}$. He therefore solves:

$$U_1^P = - \min_{\{d_1^{1P}, d_2^{1P}\}} \left\{ d_1^0 - d_1^{1P} + \delta(d_2^0 - d_2^{1P}) + \delta \mathbb{E}_1 \left\{ x_2(d_2^1 - d_2^{2P}) + (1 - x_2)\phi_2 \right\} \right\}$$

such that:

$$\begin{cases} d_1^0 - d_1^{1P} + \delta(d_2^0 - d_2^{1P}) + \delta \mathbb{E}_1 \left\{ x_2(d_2^1 - d_2^{2P}) + (1 - x_2)\phi_2 \right\} \leq \phi_1 & (IC^P) \\ d_1^1 + \delta \mathbb{E}_1 \left(x_2 d_2^2 + (1 - x_2)\Phi_2 \right) \leq \Phi_1 & (IC^C) \\ d_1^1 \leq y_1 & (PC^C) \\ 0 \leq d_t^{1P} \leq d_t^0 \qquad \text{for } t = 1, 2 & (*) \end{cases}$$

The Principal minimizes the net present value of what he decides at date t to repurchase (short-term and/or long-term debt if available), what he expects to repurchase as well in the future and the collateral damage he expects to incur. Constraint (IC^P) is the

Principal's incentive constraint guaranteeing that he prefers to participate in the bailout at date t rather than let the country default. Constraint (IC^C) is the country's incentive constraint which makes sure that it is willing to repay its debt at date t after the offer of the Principal (and after a potential rollover) rather than default. Constraint (PC^C) guarantees that the country can finally pay back its debt at date t, while constraint $(*)$ ensures that the Principal can only repurchase debt at date t.[3] If there is no solution to the system, then the Principal does not repurchase any debt and proposes $(d_1^{1P}, d_2^{1P}) = (d_1^0, d_2^0)$. There is no bailout at date 1.

Similarly, at date 2, knowing the country's income y_2, the Principal can propose to repurchase some of the country's long-term debt and lower it from d_2^1 to $d_2^{2P} = d_2^2$.[4] He therefore solves:

$$U_2^P = -\min_{\{d_2^2\}} \left\{ d_2^1 - d_2^2 \right\}$$

such that:

$$\begin{cases} d_2^1 - d_2^2 \leq \phi_2 & (IC^P) \\ d_2^2 \leq \Phi_2 & (IC^C) \\ d_2^2 \leq y_2 & (PC^C) \\ 0 \leq d_2^2 \leq d_2^1 & (*) \end{cases}$$

Again, if there is no solution to the system, the Principal does not repurchase any debt and proposes $d_2^2 = d_2^1$. There is no bailout at date 2.

Rollover decision (Date 1 only). At date 1, after the bailout offer from the Principal, the country has (d_1^{1P}, d_2^{1P}) on its portfolio, and can choose to rollover some of its debt to date 2. Defining d_2^{1R} as the additional amount of debt to repay at date 2 after the rollover, the country minimizes the net present value of what it expects to pay (debt and/or default costs):

$$\min_{d_2^{1R}} \left\{ d_1^1 + \delta \mathbb{E}_1 (x_2 d_2^2 + (1 - x_2)\Phi_2) \right\}$$

[3] In case the country has zero income at date t, constraint (PC^C) is binding. In fact, the Principal makes sure that after his bailout, the country is able to rollover all the remaining debt and thus avoid a default. In case the country has a positive income at date t and does not want to rollover and repay without the help of the Principal, constraint (IC^C) is binding. The Principal repurchases some debt to the point the country is willing to rollover and repay. The Principal's bailout does not exceed this point because he is minimizing his own participation.

[4] There is no rollover as date 2 is the last period of the model.

such that:

$$\begin{cases} d_1^1 + \delta \mathbb{E}_1(x_2 d_2^2 + (1-x_2)\Phi_2) \leq \Phi_1 \quad (IC^C) \\ d_1^1 = d_1^{1P} - \frac{d_2^{1R}}{1+r_2^1} \\ d_2^1 = d_2^{1P} + d_2^{1R} \end{cases}$$

where r_2^1 is the one-period interest rate paid by the country at date 1 on debt due at date 2. The marginal utility of borrowing is the same at dates 1 and 2, so the country has an incentive to rollover some of its debt only if it can prevent a default at date 1. If there is no solution to the system, then $d_2^{1R} = 0$ and the country does not rollover any debt[5].

Repayment decision. At date 1, the country expects the following utility:

$$U_1^C(R) = \begin{cases} y_1 - d_1^1 + \delta \mathbb{E}_1(y_2 - x_2 d_2^2 - (1-x_2)\Phi_2) & \text{if it repays} \\ y_1 - \Phi_1 + \delta \mathbb{E}_1(y_2) & \text{if it defaults} \end{cases}$$

If it did not default at date 1, the country expects the following utility at date 2:

$$U_2^C(R) = \begin{cases} y_2 - d_2^2 & \text{if it repays} \\ y_2 - \Phi_2 & \text{if it defaults} \end{cases}$$

Knowing the offer of debt repurchasing made by the Principal, the country decides whether to rollover some of the renegotiated liability d_t^{tP} (when a rollover is available) and repay the rest, or to default. If the country chooses to default at date t, it defaults on all its debt due at dates $t \leq \tau \leq 2$, incurs the default cost Φ_t but still has access to its random incomes in the future. At date t, the country chooses not to default under two conditions : (i) its utility from repaying is higher than its utility from defaulting and (ii) it can repay the remaining liability of date t after the rollover (when it is available).

$$x_t = 1 \Leftrightarrow \begin{cases} U_t^C(R, x_t = 1) \geq U_t^C(R, x_t = 0) \\ d_t^t \leq y_t \end{cases}$$

[5] I assume for simplicity that when the country is indifferent between rollover or not its debt, it chooses not to rollover it, and when it is indifferent between different sizes of rollovers, it chooses the smallest one.

2.2 The role of the ingredients

In the model, there is a conflict of interests between the country and the Principal : the country maximizes the Principal's participation in bailouts in order to extend its borrowing capacity, whereas the Principal wants the country to reduce its borrowing level and therefore his own expected involvement.

To understand the role of each assumption in the model, I characterize the first-best allocation, and study how the optimal allocation evolves as I add one by one the key assumptions of the model. In the following, I denote $(d_1, d_2) = (d_1^0, d_2^0)$ the initial repayment levels, $U^C(R) = U_0^C(R)$ the country's initial utility and $U^P = U_0^P$ the Principal's one for simplicity. The proofs can be found in Appendix 1.

First, I characterize the first-best borrowing strategy (denoted with a star), ie. the one that maximizes the country's utility in the case where the country can commit to repay its debt, the Principal can commit not to bailout the country, and markets are complete.

Lemma 1 (First-best)

$$\left\{ \begin{array}{ll} d_1^{*G} = \bar{y}_1 & d_2^{*GG} = \bar{y}_2 \\ & d_2^{*GB} = 0 \\ d_1^{*B} = 0 & d_2^{*BG} = \bar{y}_2 \\ & d_2^{*BB} = 0 \end{array} \right\}$$

G accounting for the good state at date 1, B for the bad state at date 1, GG for a succession of two good states at dates 1 and 2, BB of two bad states, GB of a good and a bad state, and BG of a bad and a good state. It gives initially the following expected utility to the country and the Principal:

$$U^{C*}(R) = R\delta \left(\alpha\bar{y}_1 + \delta \left(\alpha\bar{\rho} + (1-\alpha)\underline{\rho} \right) \bar{y}_2 \right)$$

$$U^{P*} = 0$$

As the country's marginal utility is the highest at date 0 and there is no distortion, the country optimally chooses to transfer all future incomes to the present, and its maturity structure is balanced. There is no risk of default, and the Principal does not incur any bailout or spillover cost.

Second, I characterize the optimal borrowing strategy when the country cannot commit to repay its debt, but the Principal can commit not to bailout the country and markets are complete.

Lemma 2 (Optimal Allocation with country's lack of commitment)

$$\left\{ \begin{array}{ll} d_1^G = \Phi_1 - \delta\bar{\rho} d_2^{GG} & d_2^{GG} \leq \Phi_2 \\ & d_2^{GB} = 0 \\ d_1^B = \delta\underline{\rho}(\Phi_2 - d_2^{BG}) & d_2^{BG} \leq \Phi_2 \\ & d_2^{BB} = 0 \end{array} \right\}$$

It gives initially the following expected utility to the country and the Principal:

$$U^C(R) = R\delta\left(\alpha\Phi_1 + \delta(1-\alpha)\underline{\rho}\Phi_2\right) + \delta\alpha(\bar{y}_1 - \Phi_1) + \delta^2\left(\alpha\bar{\rho} + (1-\alpha)\underline{\rho}\right)\bar{y}_2 - \delta^2(1-\alpha)\underline{\rho}\Phi_2$$

$$U^P = 0$$

The country's utility is reduced compared to the first-best because of its inability to commit to repay its debt. The maturity structure of its portfolio is indeterminate between short- and long-term debt. The Principal still incurs no bailout or spillover cost.

Third, I characterize the optimal borrowing strategy when the country cannot commit to repay its debt and the Principal cannot commit not to bailout the country, but markets are complete.

Lemma 3 (Optimal Allocation with country and Principal's lack of commitment)

$$\left\{ \begin{array}{ll} d_1^G = \phi_1 + \Phi_1 - \delta\bar{\rho} d_2^{GG} - \delta(1-\bar{\rho})d_2^{GB} & 0 \leq d_2^{GG} \leq \Phi_2 + \phi_2 \\ & 0 \leq d_2^{GB} \leq \phi_2 \\ d_1^B = \phi_1 + \delta\underline{\rho}(\Phi_2 - d_2^{BG}) - \delta(1-\underline{\rho})d_2^{BB} & \Phi_2 \leq d_2^{BG} \leq \Phi_2 + \phi_2 \\ & 0 \leq d_2^{BB} \leq \phi_2 \end{array} \right\}$$

It gives initially the following expected utility to the country and the Principal:

$$U^C(R) = R\delta\left(\phi_1 + \alpha\Phi_1 + \delta(1-\alpha)\underline{\rho}\Phi_2\right) + \delta\alpha\bar{y}_1 - \delta\alpha\Phi_1 + \delta^2\left(\alpha\bar{\rho} + (1-\alpha)\underline{\rho}\right)\bar{y}_2 - \delta^2(1-\alpha)\underline{\rho}\Phi_2$$

$$U^P = -\delta\phi_1$$

The country is able to expand its level of borrowings as a result of the Principal's lack of commitment compared to the previous case. The maturity structure of its portfolio is indeterminate. However, the Principal's utility becomes negative as he expects to incur some bailout and/or spillover costs in the future. In the following, I introduce the key ingredient of incomplete markets and study the impact of the presence of bailouts on the choice of maturity structure of the country.

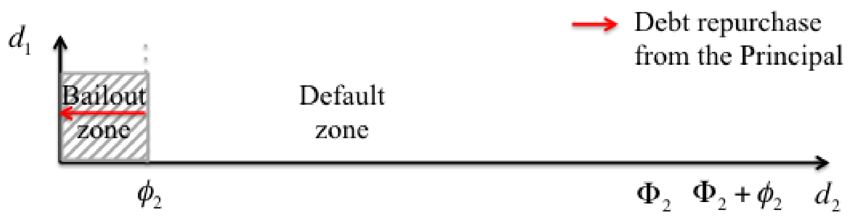
3 The Principal's bailout strategy

In this section, I solve for the optimal bailout strategy of the Principal at dates 1 and 2 by backward induction. At each date, after observing the country's income, the Principal anticipates whether or not it will be willing to default as well as the level of his bailout/spillover costs. Then, he makes a take-it-or-leave-it offer of debt repurchasing to the country if it can reduce his expected participation, i.e. if a bailout costs him less than a default. Eventually, the country decides whether to accept the offer and repay the remaining debt, potentially after a rollover, or to default.

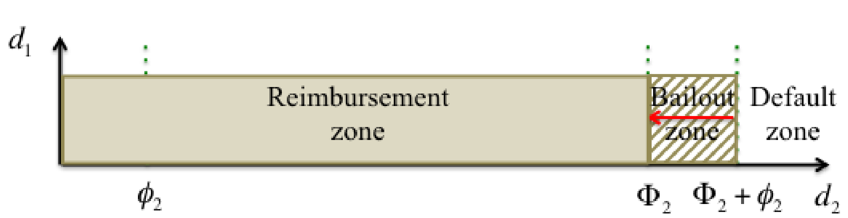
A date 2, the Principal's bailout strategy can involve the repurchasing of long-term debt d_2 only.

Lemma 4 (Date 2, Principal's bailout strategy) *At the beginning of date 2, the country's debt structure is characterized by d_2^1.*

- *In the bad state, the Principal repurchases all the country's long-term debt if $d_2^1 \leq \phi_2$ and lets it default otherwise.*



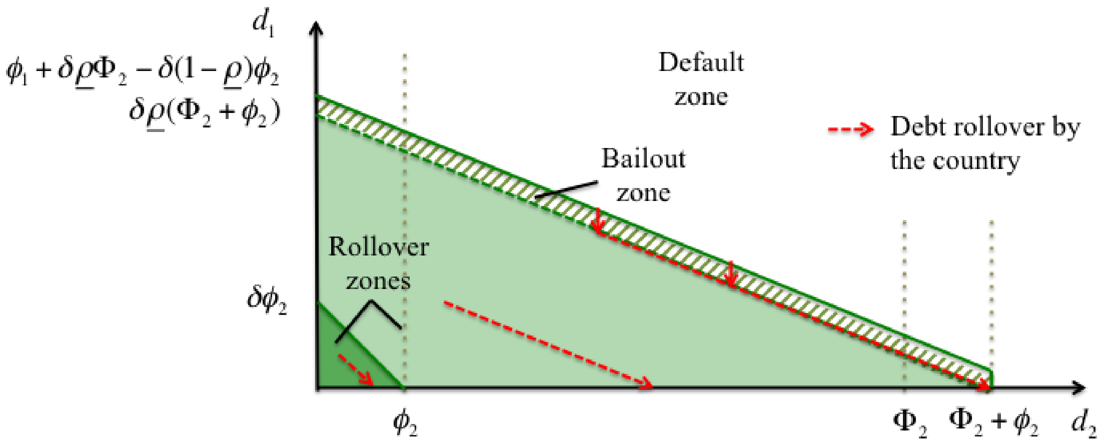
- *In the good state, the country repays its debt without any help if $d_2^1 \leq \Phi_2$. The Principal repurchases some long-term debt if $\Phi_2 < d_2^1 \leq \Phi_2 + \phi_2$ to bring back the debt level to $d_2^{2P} = \Phi_2$ and the country repays. When the debt level exceeds $\Phi_2 + \phi_2$, the Principal does not repurchase any debt and the country defaults.*



At date 1, the Principal can propose to repurchase some of the country's debt (short-term, long-term, or both), taking into account the fact that the country can rollover some of its debt after his offer.

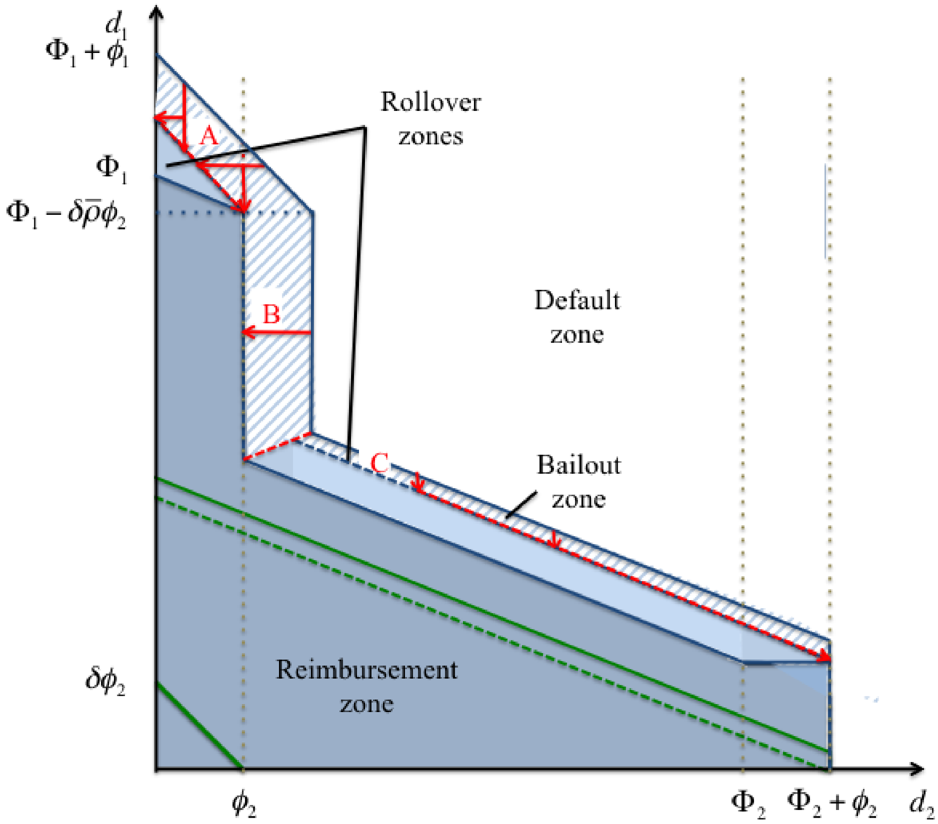
Lemma 5 (Date 1, Principal's bailout strategy) *At the beginning of date 1, the country's debt structure is characterized by* $\{d_1^0, d_2^0\}$.

- *In the bad state, the country rolls over its short-term debt if* $\{d_1^0, d_2^0\}$ *is within one of the Rollover zones. In the Bailout zone, the Principal repurchases some of its short-term debt to bring down the debt level to the frontier of one of the Rollover zones. Otherwise, The Principal does not repurchase any debt and the country defaults.*



- *In the good state, the country repays its debt without any help if* $\{d_1^0, d_2^0\}$ *is within the Reimbursement zone. In the Rollover zones, it rolls over some of its short-term debt and repays the rest. In the Bailout zone, the Principal repurchases some of the country's debt (short-term and/or long-term) to bring down the debt level either to the frontier of one of the Rollover zones, or directly to the frontier of the Reimbursement zone. Beyond the Bailout zone, the Principal does not repurchase any debt and the country defaults.*

Rollover zones At date 1, after the bailout offer of the Principal $\{d_1^{1P}, d_2^{1P}\}$, the country has the possibility to rollover some of its short-term debt to avoid a default. In both the bad and the good state, there are two types of Rollover zones: the *Safe* and the *Risky* ones. In the Safe Rollover zones, the amount of short-term debt to rollover in order to avoid a default is small enough for the level of long-term debt at the end of date 1 to be $d_2^1 \leq \phi_2$. In this case, the rollover makes it possible to avoid a default at date 1, while still guaranteeing that there is no default at date 2. In the Risky Rollover zones, the amount of short-term debt to rollover in order to avoid a default is larger, so that the level of long-term debt at the end of date 1 is $d_2^1 > \phi_2$. In this case, the rollover makes it possible to avoid a default at date 1, but there is a potential default at date 2 if the country is in the bad state.



Bailout zone At date 2, the Principal can repurchase only long-term debt to prevent a default, whereas at date 1, he has the choice between repurchasing short-term, long-term debt, or both. In the two cases, he does so if such a bailout costs him less than a default. In the following, I study the Bailout zone in both the good and the bad states and characterize what maturity of debt it is optimal to repurchase in each case.
On the one hand, repurchasing long-term debt is characterized by a *commitment effect*: the Principal commits to a bailout tomorrow, so that the country accepts to rollover and repay its debt today, knowing that it cannot default tomorrow. In fact, such a bailout alleviates the country's incentive constraints both today and tomorrow, and prevents a default at the two maturities. There are two channels at work for the country: a debt price channel which reduces the cost of a rollover, and a debt level channel which decreases directly the net present value of the repayments. On the other hand, repurchasing short-term debt is characterized by an *option value effect*: the Principal repurchases only short-term debt today such that the country accepts to rollover and repay the rest. Such a bailout prevents a default today through a debt level channel uniquely, without trying to affect the potential outcomes tomorrow.

More precisely, in the bad state, the country cannot repay any debt. In the Bailout zone, the Principal finds it optimal to repurchase short-term debt to prevent a default at date 1 only. In fact, as the Principal minimizes the cost of his bailout, he repurchases debt to reach the frontier of the Risky Rollover zone. It would be more costly to reach the frontier of the Safe Rollover zone and prevent a default at date 2 as well, because the implied cost of repurchasing additional short-term and long-term debt (in some cases) outweighs the benefit from preventing a default at date 2. Furthermore, a bailout involving short-term debt is optimal, because the amount of short-term debt to repurchase in order to reach the frontier of the Risky Rollover zone is lower than the amount of long-term debt that would be required to attain the same objective. Hence, the option value effect dominates.

In the good state, the country's overall debt level is so high that it would rather default today, even though it would have been able to repay the maturing claim. The Bailout zone can be divided into three zones: A, B and C (separated by red dotted lines in the graph). In zone A, there is no risk of default tomorrow, so that the Principal is indifferent between repurchasing short-term or long-term debt to bring back debt to the frontier of the Safe Rollover zone. In zones B and C, a default can happen tomorrow if the country is in the bad state. In zone B, the gain from repurchasing long-term debt to reach the frontier of either the Rollover zone or the Reimbursement zone and prevent a default at both dates outweighs the gain from repurchasing short-term debt and prevent a default at date 1 only. Hence, repurchasing long-term debt is strictly optimal and the commitment effect dominates. In zone C, repurchasing short-term debt to reach the frontier of the Risky Rollover zone and prevent a default at date 1 only is less costly than repurchasing long-term debt to attain the same objective. It is also less costly than repurchasing additional debt to prevent a default at date 2 as well. Hence, repurchasing short-term debt is strictly optimal and the option value effect dominates.

The next Proposition summarizes the optimal bailout strategy for the Principal and the tradeoff between long-term and short-term debt repurchasing.

Proposition 1 (Principal's bailout strategy)

- *Repurchasing long-term debt in the Bailout zone is strictly optimal when a commitment effect dominates: by committing to a bailout tomorrow, the Principal makes the country repay today (backloading result), and prevents a default at the two maturities.*
- *Otherwise, repurchasing short-term debt is optimal and the "option value effect" dominates. It prevents a default today, without trying to affect the potential outcomes tomorrow.*

The proof can be found in Appendix 2. Intuitively, repurchasing long-term debt alleviates the country's incentive constraints through two channels: a reduction in the level of debt and an increase in debt prices that helps the country rollover its debt at date 1. On the contrary, repurchasing short-term debt impacts the country through a reduction in the level of debt only.

4 The country's optimal borrowing strategy

In this section, I solve for the optimal borrowing strategy of the country at date 0, internalizing both the Principal's bailout strategies and the country's rollover and default decisions at dates 1 and 2. I am therefore able to solve simultaneously for the initial borrowing level b_0 and the interest rates paid on debt r_1^0 and r_2^0.

At date 0, the country can choose a debt level $\{d_1^0, d_2^0\}$ lying into 4 possible zones (See Figure 3).

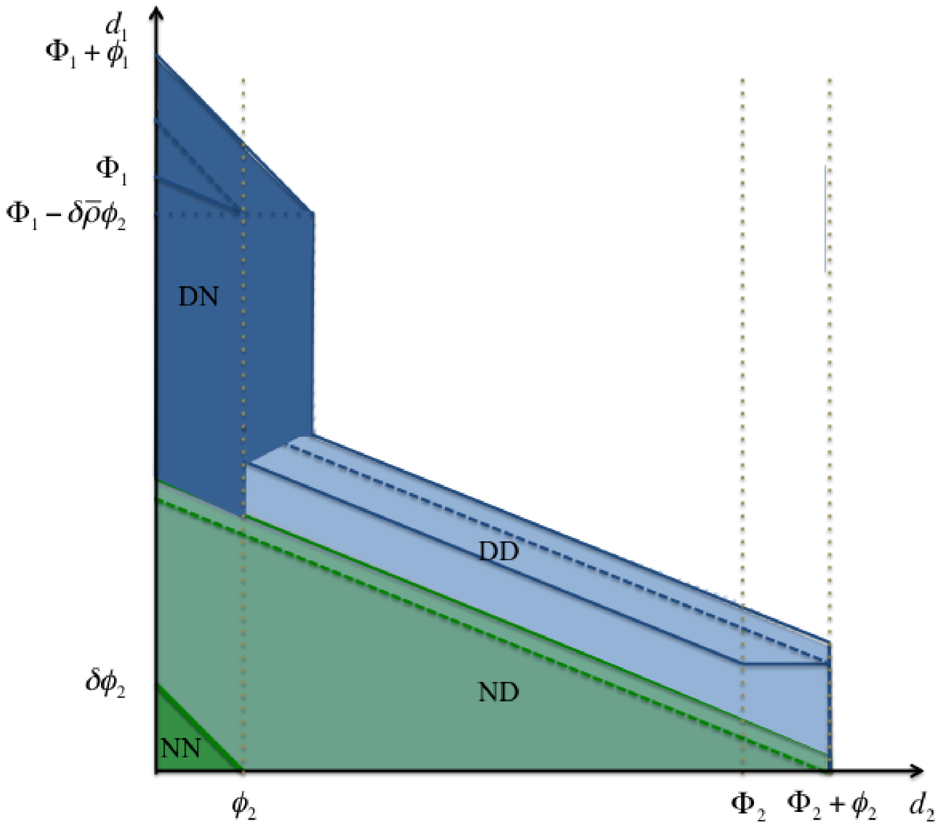


Figure 3: Summary of borrowing strategies

- In the NN zone, the country never defaults: at date 1, it repays its debt in the good state and rolls it over in the bad state. At date 2, it repays in the good state and is bailed out by the Principal in the bad state.

- In the ND zone, there is no default at date 1 but a potential default at date 2: at date 1, the country repays its debt in the good state and rolls it over after a potential bailout from the Principal in the bad state. At date 2, it repays after a potential bailout from the Principal in the good state, but defaults in the bad state.

- In the DN zone, there is a potential default at date 1 but no default at date 2: at date 1, the country repays its debt after a potential rollover and/or bailout from the Principal in the good state, but defaults in the bad state. At date 2, provided that the country did not default at date 1, it repays in the good state and is bailed out by the Principal in the bad state.

- In the DD zone, there is a potential default at both dates: at date 1, the country repays its debt after a potential rollover and/or bailout from the Principal in the good state, but defaults in the bad state. At date 2, provided that the country did not default at date 1, the country repays after a potential bailout from the Principal in the good state, but defaults in the bad state.

Beyond these 4 zones, the country defaults with certainty at one period at least, so that interest rates are infinite. It cannot happen at the equilibrium.

Within each borrowing zone, the interest rates paid on debt at both maturities are determined by default probabilities. Then, the optimal borrowing strategy of the country is a function of its liquidity need R. In the following, I denote $(d_1^i, d_2^i) = (d_1^0, d_2^0)$ the initial repayment levels for a given borrowing strategy i. The next Lemma whose proof can be found in Appendix 2 summarizes the results.

Lemma 6 (Country's borrowing strategy)

- *In the low liquidity need zone ($1 \leq R < R_1$), the country chooses the "Low strategy" within the NN zone by borrowing a small amount of long-term debt:*

$$\begin{cases} d_1^L = 0 \\ d_2^L = \phi_2 \end{cases}$$

- *In the intermediate liquidity need zone ($R_1 \leq R < R_2$), the country chooses the "Intermediate strategy" within the ND zone by additionally borrowing some short-*

term debt:

$$\begin{cases} d_1^I = \phi_1 + \delta \underline{\rho} \Phi_2 - \delta \phi_2 \\ d_2^I = \phi_2 \end{cases}$$

- *In the high liquidity need zone ($R \geq R_2$), the country chooses the "High strategy" within the DN zone by borrowing an even larger amount of short-term debt:*

$$\begin{cases} d_1^H = \phi_1 + \Phi_1 - \delta d_2^{R,ST} \\ d_2^H \leq \frac{\phi_1}{\delta} + \overline{\rho} \phi_2 \end{cases}$$

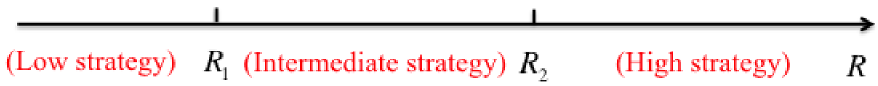


Figure 4: Country's borrowing strategy (1/2)

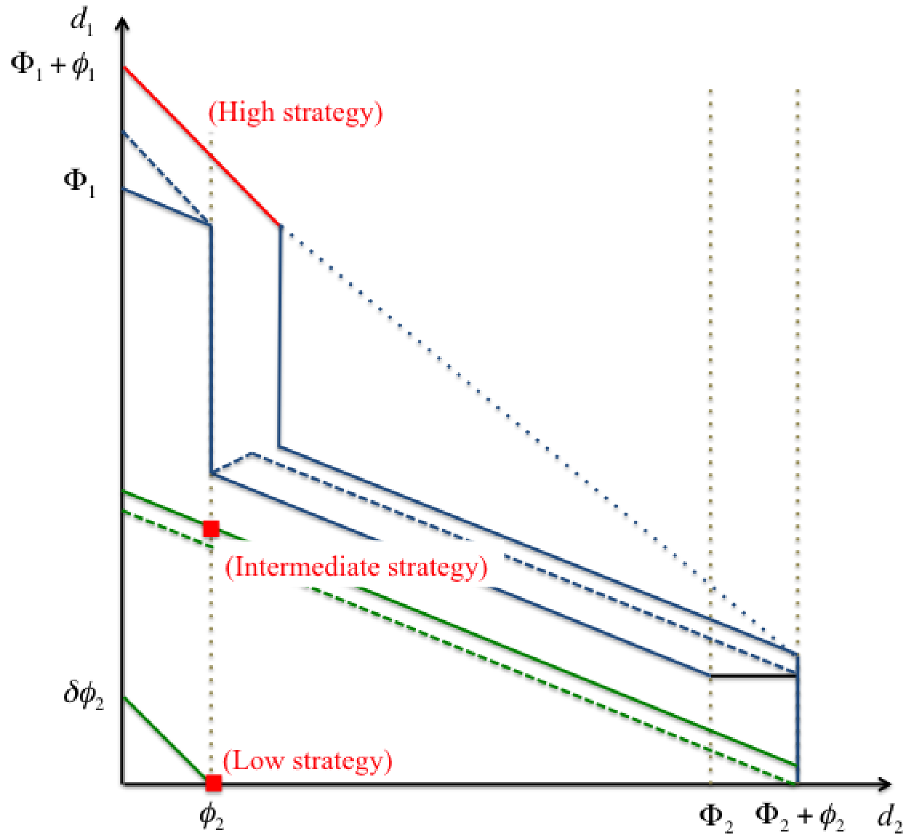


Figure 5: Country's borrowing strategy (2/2)

Figure 4 summarizes the choice of borrowing strategy as a function of the liquidity need R, while Figure 5 represents the borrowing strategies in the space of short-term and long-term debt repayments. In the low liquidity need zone, the country chooses to borrow long-term debt only, such that there is no default at date 2. Indeed, it repays its debt in the good state of date 2 and is bailed out by the Principal in the bad state. In the intermediate liquidity need zone, it additionally borrows some short-term debt and takes a risk of default at date 2. At date 1, it repays its debt in the good state and rolls it over after a bailout from the principal in the bad state. At date 2, it repays after a bailout from the Principal in the good state, but defaults in the bad state. In the high liquidity need zone, it borrows short-term debt further and takes a risk of default at date 1. At date 1, it repays after both a bailout from the Principal and a rollover of some of its debt in the good state, but defaults in the bad state. At date 2, provided that the country did not default at date 1, it repays in the good state and is bailed out by the Principal in the bad state.

Borrowing level evolution Defining b_i as the initial level of borrowing for $i = \{L, I, H\}$, I find that the level of borrowings increases with the liquidity need, as the country switches from the Low to the Intermediate and High strategies. The same can be observed for the level of repayments (see Figure 5).

- In the low liquidity need zone, the country chooses a low level of borrowings:

$$b_L = \delta^2 \phi_2$$

- In the intermediate liquidity need zone, the country increases its level of borrowings and takes a risk of default at date 2:

$$b_I = \delta(\phi_1 + \delta\underline{\rho}\Phi_2 - \delta(1-\alpha)(1-\underline{\rho})\phi_2)$$

- In the high liquidity need zone, the country increases further its level of borrowings and takes a risk of default at date 1:

$$b_H = \delta\alpha(\phi_1 + \Phi_1)$$

Maturity evolution Defining $\beta_i = \frac{d_1^0}{d_1^0 + d_2^0}$ as the ratio of short-term repayments over total repayments for $i = \{L, I, H\}$, I find that the maturity of the country's portfolio shortens when its liquidity need rises, as the country switches from the Low to the Intermediate and High strategies (see Figure 6).

- In the low liquidity need zone, the country optimally chooses to borrow only long-term debt:

$$\beta_L = 0$$

The Principal's help is exclusively concentrated on the long-term maturity.

- In the intermediate liquidity need zone, the country additionally borrows some short-term debt so that the composition of its portfolio is balanced (under realistic assumptions about the parameters[6]):

$$\beta_I = \frac{\phi_1 + \delta \underline{\rho} \Phi_2 - \delta \phi_2}{\phi_1 + \delta \underline{\rho} \Phi_2 + (1-\delta)\phi_2}$$

The Principal's help is shared evenly across the two maturities, so that the country may benefit from repeated bailouts.

- In the high liquidity need zone, the country relies almost exclusively on short-term debt:

$$\frac{\Phi_1 - \delta \overline{\rho} \phi_2}{\Phi_1 + \frac{\phi_1}{\delta} + (1-\delta)\overline{\rho}\phi_2} \leq \beta_H \leq 1$$

Most of the Principal's help is concentrated on the short-term maturity.

Intuitively, the country can credibly commit to repay more debt on the short maturity than on the long one, as there is less uncertainty about future outcomes in the short-term than in the long-term. In a situation of high liquidity need, borrowing mostly short-term debt and concentrating the Principal's help on this maturity is therefore a way for the sovereign to maximize its initial borrowing capacity at the expense of recurrent default risks. On the contrary, when its liquidity need is lower, the country chooses a more balanced portfolio and shares the Principal's help across the two maturities, or even shifts it completely towards long-term debt along with the Principal's help. Then, it is able to moderate or even eliminate default risks.

The respective benefits of short- and long-term debt Performing a counterfactual analysis where the country has access to long-term debt only, I find that, in a situation of high liquidity need, its borrowing capacity as well as its initial expected utility are reduced compared to the benchmark model. On the contrary, when the country has access to short-term debt only, it reaches the same level of borrowing capacity and of initial expected utility (see Appendix 3 for more details).

[6]There are more details in the appendices.

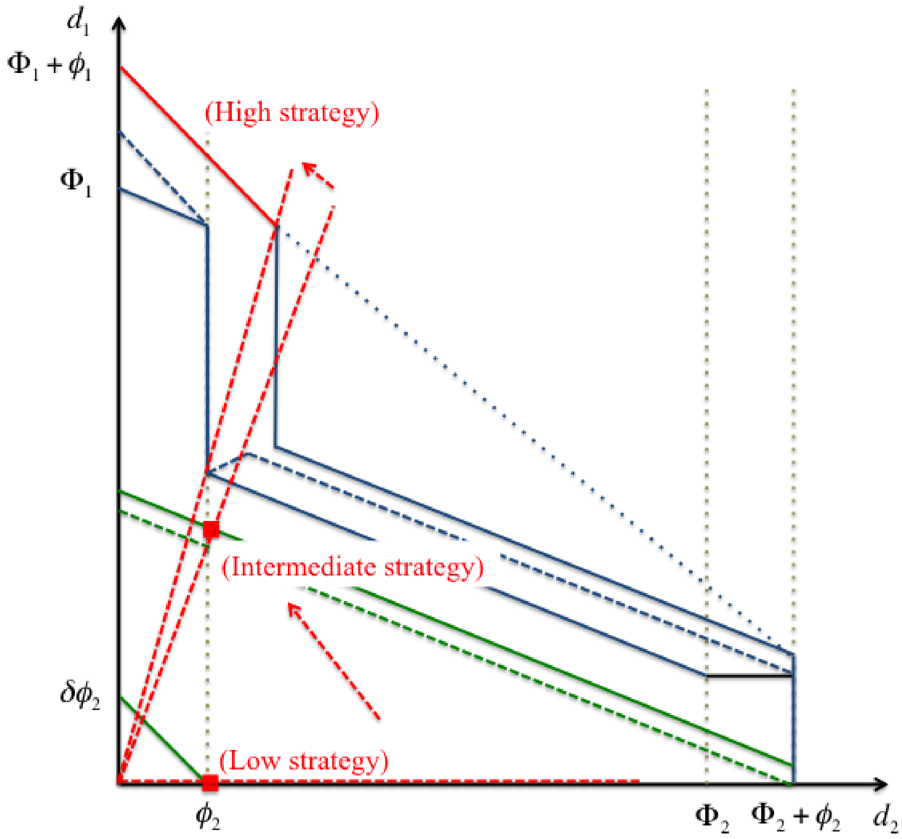


Figure 6: Maturity Evolution

As the country cannot commit to reimburse more than what it will be willing to repay, borrowing at two maturities instead of one can only expand the set of borrowing possibilities. Eventually, it is clear that short-term debt has a borrowing capacity advantage compared to long-term debt when the liquidity need is high, which explains the extensive use of short-term debt in this situation.

5 The impact of the presence of potential bailouts

In this section, I investigate the impact of the presence of potential bailouts on the country's borrowing strategy by comparing the benchmark model with the extreme case where $\phi_1 = \phi_2 = 0$, i.e. there is no bailout from the Principal.

In a setup without potential bailout, the country is able to repay a positive amount of debt at a given maturity in the good state only. In this case, the only strategy with no risk of default at both dates is composed of no debt. All other strategies with a positive amount of debt are strategies with a potential default[7].

[7]The only instrument that the country can use to avoid a default at a given period is to rollover its debt (available only at date 1), but it makes a default possible in the following period to which the debt

Defining R_N as the threshold of liquidity need in this setup (N accounting for No potential bailout), the next Lemma whose proof can be found in Appendix 4 summarizes the country's optimal borrowing strategy:

Lemma 7 (Country's borrowing strategy without potential bailout)

- *When $1 \leq R < R_N$, the country chooses a "Low strategy" with no debt:*

$$\begin{cases} d_1^{N,L} = 0 \\ d_2^{N,L} = 0 \end{cases}$$

  There is no default at both dates.

- *When $R \geq R_N$, the country chooses a "High strategy" with only short-term debt:*

$$\begin{cases} d_1^{N,H} = \Phi_1 \\ d_2^{N,H} = 0 \end{cases}$$

  There is a potential default at date 1 if the country receives no income, but none at date 2.

First, the borrowing level of the country is reduced when there is no potential bailout. In fact, the Low strategy consists of no debt, while the High strategy consists of short-term debt only. When there are potential bailouts, the Low strategy allows the country to take a positive level of debt on the long-term maturity, while the High strategy enables the country to borrow more on the short-term maturity (and on the long-term maturity as well in some cases). As in the benchmark model, as the liquidity need rises, the country issues more short-term debt and takes risks of default.
Second, comparing the liquidity need threshold R_N without potential bailout with the ones in the benchmark model with potential bailouts, they rank as follows:

$$R_1 < R_N < R_2$$

The set of liquidity needs where the country relies on short-term debt and takes risks of default is larger when there are potential bailouts. Hence, the country uses more short-term debt and takes more risks on average.

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is rolled over.

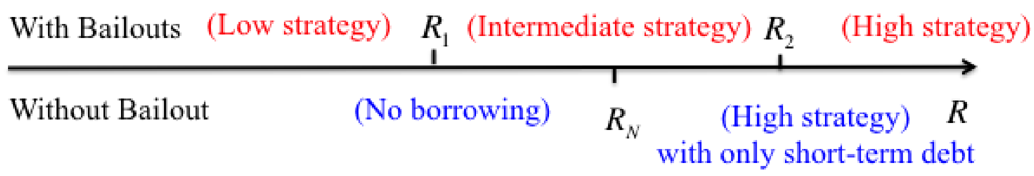

Figure 7: Comparison with/without potential bailouts

Third, the respective nature of long-term and short-term debt can be distinguished. Short-term debt enables the country to raise more money initially, but generates a risk of default at date 1. On the contrary, long-term debt does not allow the country to raise as much money initially, while still generating a risk of default at date 2. Hence, short-term debt always dominates long-term debt when the liquidity need is high, and long-term debt is not used at the equilibrium in this setup.

Note that at the equilibrium, it is never optimal for the country to raise a positive level of debt at *both* maturities. In particular, sharing debt between the two maturities is always dominated by raising only short term debt. In fact, the incentive compatibility constraint that insures repayment at date 1 is tighter when there is some debt at date 2, and the resulting decrease in the ability to borrow at date 1 is not compensated by the increase in borrowings of date 2. Intuitively, the country takes more risk and is able to borrow less when debt is allocated across the two maturities.

Eventually, the lack of commitment of the Principal affects directly his expected utility, as he expects to incur more bailout/spillover costs. This is the case for any level of liquidity needs R, as the country optimally takes advantage of as much help as possible from the Principal. The next Proposition summarizes the results:

Proposition 2 (The impact of the presence of potential bailouts)

The presence of potential bailouts raises the country's borrowing capacity and makes it rely more on short-term debt on average. As short-term debt is associated with debt rollover risk, there are more default risks on average.

6 Welfare Analysis

In this section, I study the ex post welfare of the country and the Principal in the benchmark model with potential bailouts, and study how it could be improved ex ante.

6.1 Ex Post Welfare

First, the Principal's welfare is zero at its maximum and negative when he anticipates bailout and/or spillover costs. Furthermore, it does not depend on the level of the liquidity need R, but only on the country's choice of borrowing strategy. If the High strategy is chosen, the Principal expects the lowest possible utility U_H^P. If the Intermediate strategy is chosen, the Principal expects a higher utility U_I^P. The highest utility he can expect is under the Low strategy, U_L^P.

Second, the country's welfare depends on both its level of liquidity need R and its choice of borrowing strategy. More precisely, the Low strategy brings a higher level of utility to the country when R is below the threshold R_1, the Intermediate strategy is preferable when R is between R_1 and R_2, and the High strategy when R is above R_2.

From the point of view of the Principal, it would be preferable that the country chooses the Low or the Intermediate strategy even in situations of high liquidity needs (see Appendix 5).

6.2 Ex Ante Policies

In this subsection, I am looking for ex ante policies that could improve total welfare in the benchmark model. I follow two different approaches: in the first, I exhibit a regulatory instrument (a maturity limit ratio imposing $\beta \leq \overline{\beta}$, a fixed threshold) and let the country choose its optimal borrowing strategy. In the second, I design a contractual agreement that both the country and the Principal are willing to take out initially. The proofs are in Appendix 6.

Regulatory instrument The combination of a maturity limit ratio with an initial transfer to compensate the country can contain the country's risk-taking behavior, but cannot improve total welfare (see Figure 8). In fact, it reduces the Principal's expected participation in bailout/spillover costs, but the cost of the transfer to compensate the country for a reduced borrowing capacity is larger.

Let us take the example of the following limit $\overline{\beta}$, defined by the maturity ratio of the Intermediate strategy:

$$\beta \leq \overline{\beta} = \frac{\phi_1 + \delta\underline{\rho}\Phi_2 - \delta\phi_2}{\phi_1 + \delta\underline{\rho} + (1-\delta)\phi_2}$$

When the liquidity need is low, the country optimally chooses the Low strategy which is allowed by the maturity limit ratio. In this case, a positive transfer is not necessary because the Principal's utility is already maximized: $U_{ML,L}^C(R) = U_L^C(R)$ (ML accounting

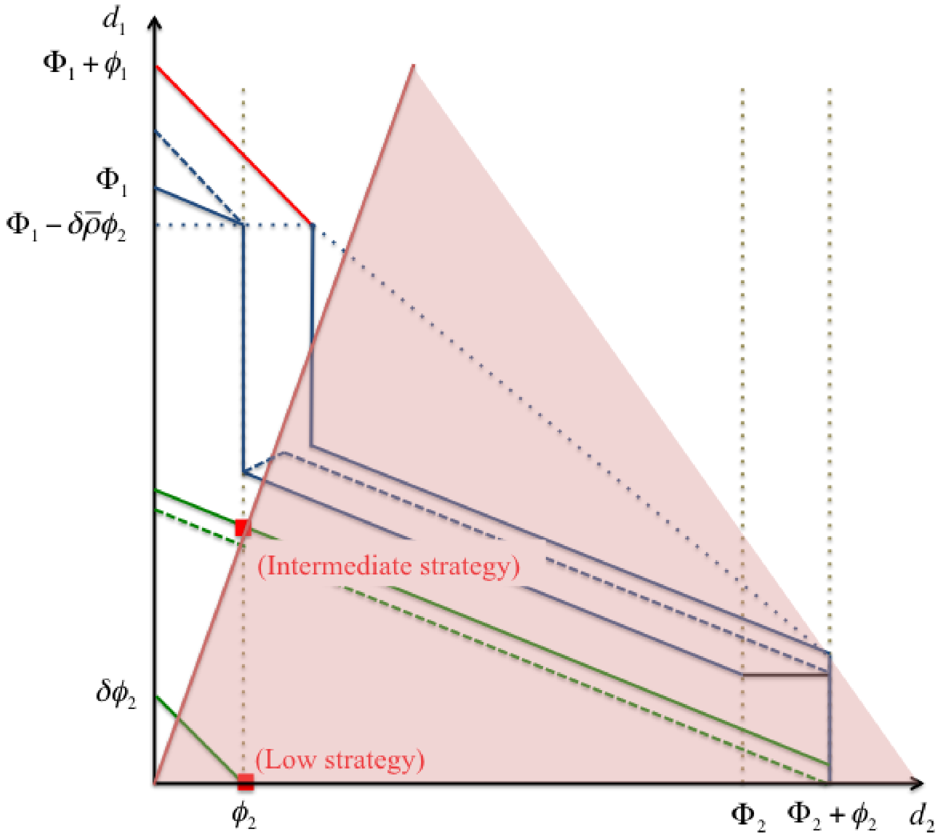

Figure 8: Ex-Ante Policy

for Maturity Limit ratio).

$$T(R) = 0 \quad \forall R < R_1$$

When the liquidity need is intermediate, the country chooses the Intermediate strategy with a larger share of short-term debt which is also allowed by the maturity limit ratio. Again, a positive transfer is not necessary, because the country's utility is already maximized: $U^C_{ML,I}(R) = U^C_I(R)$.

$$T(R) = 0 \quad \forall R_1 \leq R < R_2$$

When the liquidity need is high, the country wants to choose the High strategy with a large share of short-term debt, but it is not allowed by the maturity limit ratio. Below the limit ratio, the country optimally chooses the Intermediate strategy, whose maturity structure is more balanced. On the one hand, the Principal's expected utility is improved compared to the High strategy $U^P_I - U^P_H = \delta\phi_1 - \delta^2(1-\alpha\bar{\rho})\phi_2 > 0$, so that the Principal is strictly better off. On the other hand, the country's utility is reduced to $U^C_{ML,I}(R) = U^C_I(R) < U^C_H(R)$

and a positive transfer is necessary to compensate it:

$$T(R) = \frac{U_H^C(R) - U_I^C(R)}{R} \quad \forall R \geq R_2$$

Yet, the cost of the transfer is strictly larger than the gain in the Principal's expected utility. Hence, this regulatory policy is not welfare-improving.

Intuitively, a limit on the ratio of short-term debt-to-total debt that the country can have on its portfolio contains its risk-taking behavior, because of the strong link between short-term debt and risks of default. Yet, it cannot improve total welfare because it does not address the contradictory incentives of the country and the Principal. It is indeed always optimal for the country to obtain the largest participation possible from the Principal in order to increase its initial borrowing capacity. Hence, a limit on the maturity structure cannot alleviate this incentive, but only delay its impact on default risk to date 2.

Contractual agreement A contractual agreement between the country and the Principal can improve total welfare.

More precisely, suppose that at the beginning of date 0, the Principal anticipates which borrowing strategy the country is willing to choose (Low, Intermediate, or High) and can make a contractual offer to the country. If the country turns down the offer, the outcome is the one studied in the benchmark model.

The contractual offer is composed of borrowings from both the market and the Principal. On the one hand, bonds sold in the financial market have the same characteristics than in the benchmark model. On the other hand, the Principal offers loans on competitive terms, but can make the repayments contingent on the state of nature.

In the contractual offer, if the country were to adopt the Low strategy with the market for any level of liquidity needs, it would reduce the Principal's expected participation in bailout and/or spillover costs. To make this incentive-compatible for the country, the Principal has to lend to the country on top of the market. Furthermore, the Principal designs the contract to guarantee that (i) the country is willing to repay its debt to both the market and the Principal in the good states of nature[8] and that (ii) he is willing to repurchase the country's debt to the market in bad ones. Hence, the country never defaults.

There are two important elements: first, the Principal lends on top of the market in order to prevent the country from borrowing too much from the market and to risk a de-

[8]In this setup, the country incurs a default cost whenever it does not repay one of its claim. Hence, the country always considers either to repay everybody (the market and the Principal) or nobody. There is no issue of seniority.

fault. Second, reimbursements to the Principal are contingent on the realization of good states of nature. It can be understood as guarantors acting as "lenders in last resort" and willing to forgive some of their claims if necessary[9].

Using a mechanism design approach, the contract specifies:

- A borrowing level b and its allocation between the market and the Principal: $b = b^M + b^P$
- Debt repayments to be made to the market and the Principal: $(d_1^M, d_2^M, d_1^P, d_2^P)$ where (d_1^M, d_2^M) are the amounts of non-contingent debt to be reimbursed to the market at dates 1 and 2 (with $b^M = \delta d_1^M + \delta^2 d_2^M$), and (d_1^P, d_2^P) the amounts of contingent debt to be reimbursed to the Principal only if the good state is realized at dates 1 and 2 (with $b^P = \delta \alpha d_1^P + \delta^2(\alpha \bar{\rho} + (1-\alpha)\underline{\rho}) d_2^P$).

Let us study the case where b^M is defined by the level of debt of the Low strategy[10]. If the country is initially willing to choose the Low strategy, the contract includes only borrowings b^M from the market. If the country is initially willing to choose the Intermediate or the High strategy, the contract includes borrowings from the market b^M as well as from the Principal b^P. The Principal designs b^P to make the contract incentive-compatible for the country and to allow it to reach the same level of utility it would have obtained outside of the contract.
Under such a contract, the Principal reduces his participation in bailout/spillover costs. When the liquidity need is low, his expected utility is the one of the Low strategy. When the liquidity need is higher, as he exactly breaks even under his own lending at date 0, his expected utility is again the one of the Low strategy. Furthermore, the country expects the same utility that it would have obtained outside of the contract. Thus, the Principal is better off under the optimal contract, while the country is as well off. It is therefore Pareto-improving.

Proposition 3 (Contractual agreement).

[9] Note that if the country could borrow from the Principal only, it would have access to fully contingent debt and would be able to extend its borrowing capacity compared to the borrowing strategies of the benchmark model.

[10] Note that there is no potential default in this situation, because the country is willing to repay its debt to the market in the good state of date 2 and the Principal repurchases it in the bad state.

- *If the country is to take initially the Low strategy, then the contactual offer is:*

$$\begin{cases} b^M = \delta^2 \phi_2 \\ b^P = 0 \end{cases}$$

- *If the country is to take initially the Intermediate or the High strategy, then a contractual offer is:*

$$\begin{cases} b^M = \delta^2 \phi_2 \\ b^P = \delta \alpha d_1^P + \delta^2 (\alpha \bar{\rho} + (1-\alpha)\underline{\rho}) d_2^P \end{cases}$$

  (d_1^P, d_2^P) *being defined to make the offer incentive-compatible for the country.*

The Pareto-improvement comes from addressing the divergence of interests between the country and the Principal. First, the Intermediate and High strategies enable the country to extend its initial borrowing capacity, but generate at the same time risks of default for the country and collateral damage for the Principal. Second, adopting the Low strategy with no default limits the initial borrowing capacity, whereas the country could commit to reimburse more in the good state of nature. In this situation, contingent borrowing from the Principal on top of the market alleviates these inefficiencies.

To conclude, the Principal prefers the country to take the Low strategy with no default at both dates in order to reduce his participation in bailout and/or spillover costs. Under the assumption that the Principal can lend to the country on top of the market and make his claims contingent on the state of nature, he can design a contract where everybody is better off (at least weakly).

7 Conclusion

The paper highlights the mechanisms behind the choice of maturity structure of a sovereign that internalizes potential bailouts from guarantors. The presence of potential bailouts extends the initial borrowing capacity of the country and makes it rely more on short-term debt on average. Intuitively, when the country is faced with a high liquidity need, it chooses to rely mostly on short-term debt and is able to concentrate the help of guarantors on the short-term, which is the maturity that enables the country to raise initially as much debt as possible. As short-term debt is associated with debt rollover risk, this generates more default risk on average.

When guarantors decide to bailout the country and there is also a potential default in the future that is not too costly to prevent, the optimal bailout involves the repurchasing of long-term debt. Hence, guarantors achieve two goals at the same time, prevent a default both today and tomorrow. Otherwise, the optimal bailout involves the repurchasing of short-term debt only, and prevents a default only today.

Based on the welfare analysis, some policy implications can be derived. First, the intervention of potential guarantors when a country starts having troubles can prevent the adoption of risky borrowing strategies (with potential defaults). More concretely, this article gives a rationale for the intervention of the IMF or the Eurozone. Indeed, guarantors can design a contract where they lend some money to the troubled country at different terms than those proposed by the financial market[11]. Such a contract is welfare-improving and prevents the occurrence of any default at the equilibrium. Second, regulating the maturity of debt issuance in order to guarantee that the country's portfolio remains balanced is able to contain the risk-taking behavior of the country, but is not Pareto-improving. In fact, such a policy does not address the fundamental issue of the setup, which is the divergence of interests between the country and the guarantors.

The paper opens a lot of alleys for further research. In the model, I assume perfect information on the cost of default and on the endowment level. The country, when deciding which borrowing strategy to adopt, anticipates exactly when and by how much guarantors will bail it out. In the Eurozone for example, the country forms expectations about potential bailouts, as there is no formal joint-and-several liability agreement. It is the same issue for guarantors when they decide to bailout a country: they do not know exactly the default cost of the country and have to form expectations about it. It would be interesting to introduce some moral hazard where guarantors form beliefs about the country's health (based on the maturity choice of the country for example) and study how this would affect the behavior of both the country and guarantors.

[11] Note that this supposes some coordination among guarantors.

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Appendix 1: First step Analyses

Lemma 1 (First-best). Given the fact that $R \geq 1$, the marginal utility of borrowing is the highest at date 0, compared to dates 1 and 2. Thus, the optimal borrowing strategy is to transfer all future incomes to the present. The first-best is therefore:

$$\left\{ \begin{array}{ll} d_1^{*G} = \bar{y}_1 & d_2^{*GG} = \bar{y}_2 \\ & d_2^{*GB} = 0 \\ d_1^{*B} = 0 & d_2^{*BG} = \bar{y}_2 \\ & d_2^{*BB} = 0 \end{array} \right\}$$

It gives initially the following expected utility to the country and the Principal:

$$U^{C*}(R) = R\delta \left(\alpha \bar{y}_1 + \delta \left(\alpha \overline{\rho} + (1-\alpha)\underline{\rho} \right) \bar{y}_2 \right)$$

$$U^{P*} = 0$$

Lemma 2 (Optimal allocation with country's lack of commitment). The country solves the following problem, under additional incentive constraints for the country (denoted by IC^C) and rollover constraints (denoted by RO^C)[12]:

$$\max_{d_1^{0G}, d_1^{0B}, d_2^{0GG}, d_2^{0BG}} \left\{ R\delta \left(\alpha d_1^{0G} + (1-\alpha) d_1^{0B} + \delta\alpha\overline{\rho} d_2^{0GG} + \delta(1-\alpha)\underline{\rho} d_2^{0BG} \right) + \delta\alpha(\overline{y_1} - d_1^{1G}) \right.$$
$$\left. + \delta^2(\alpha\overline{\rho} + (1-\alpha)\underline{\rho})\overline{y_2} - \delta^2\alpha\overline{\rho} d_2^{1GG} - \delta^2(1-\alpha)\underline{\rho} d_2^{1BG} \right\}$$

such that:

$$\left\{ \begin{array}{lll} d_1^{1G} + \delta\overline{\rho} d_2^{1GG} \leq \Phi_1 & (IC_G^C) & \\ d_2^{1iG} \leq \Phi_2 & (IC_{iG}^C) & \text{for } i = B, G \\ d_1^{1G} = d_1^{0G} - \delta\overline{\rho} d_2^{1GR} & (RO_G^C) & \\ d_2^{1iG} = d_2^{0iG} + d_2^{1iR} & (RO_{iG}^C) & \text{for } i = B, G \\ d_1^{0B} = \delta\underline{\rho} d_2^{1BR} & (RO_B^C) & \end{array} \right.$$

There is a continuum of solutions:

[12] Notice that the country cannot commit to repay anything in the bad states of date 2, so that $d_2^{0GB} = d_2^{0BB} = 0$

$$\left\{\begin{array}{ll} d_1^G = \Phi_1 - \delta\overline{\rho} d_2^{GG} & d_2^{GG} \leq \Phi_2 \\ & d_2^{GB} = 0 \\ d_1^B = \delta\underline{\rho}(\Phi_2 - d_2^{BG}) & d_2^{BG} \leq \Phi_2 \\ & d_2^{BB} = 0 \end{array}\right\}$$

It gives initially the following expected utility to the country and the Principal:

$$U^C(R) = R\delta\left(\alpha\Phi_1 + \delta(1-\alpha)\underline{\rho}\Phi_2\right) + \delta\alpha(\bar{y}_1 - \Phi_1) + \delta^2\left(\alpha\bar{\rho} + (1-\alpha)\underline{\rho}\right)\bar{y}_2 - \delta^2(1-\alpha)\underline{\rho}\Phi_2$$

$$U^P = 0$$

Lemma 3 (Optimal allocation with country and Principal's lack of commitment).

$$\max_{d_1^{0G}, d_1^{0B}, d_2^{0GG}, d_2^{0GB}, d_2^{0BG}, d_2^{0BB}} \left\{R\delta\left(\alpha d_1^{0G} + (1-\alpha)d_1^{0B} + \delta\alpha\overline{\rho} d_2^{0GG} + \delta\alpha(1-\overline{\rho})d_2^{0GB} + \delta(1-\alpha)\underline{\rho} d_2^{0BG}\right.\right.$$
$$\left.\left. + \delta(1-\alpha)(1-\underline{\rho})d_2^{0BB}\right) + \delta\alpha(\overline{y_1} - d_1^{1G}) + \delta^2(\alpha\overline{\rho} + (1-\alpha)\underline{\rho})\overline{y_2} - \delta^2\alpha\overline{\rho} d_2^{2GG} - \delta^2(1-\alpha)\underline{\rho} d_2^{2BG}\right\}$$

such that:

$$\left\{\begin{array}{ll}
d_1^{1G} = d_1^{1GP} - \delta\overline{\rho} d_2^{1GGR} - \delta(1-\overline{\rho})d_2^{1GBR} & (RO_G^C) \\
d_2^{1Gi} = d_2^{1GiP} + d_2^{1GiR} & (RO_{Gi}^C) \text{ for } i = B, G \\
d_1^{1BP} = \delta\underline{\rho} d_2^{1BR} + \delta(1-\underline{\rho})d_2^{1BBR} & (RO_B^C) \\
d_2^{1Bi} = d_2^{1BiP} + d_2^{1BiR} & (RO_{Bi}^C) \text{ for } i = B, G \\
d_1^{1G} + \delta\overline{\rho} d_2^{2GG} \leq \Phi_1 & (IC_G^C) \\
d_2^{2iG} \leq \Phi_2 & (IC_{iG}^C) \text{ for } i = B, G \\
d_1^{0G} - d_1^{1GP} + \delta\overline{\rho}(d_2^{0GG} - d_2^{1GGP}) + \delta\overline{\rho}(d_2^{1GG} - d_2^{2GG}) & \\
+\delta(1-\overline{\rho})(d_2^{0GB} - d_2^{1GBP}) + \delta(1-\overline{\rho})d_2^{1GB} \leq \phi_1 & (IC_G^P) \\
d_1^{0B} - d_1^{1BP} + \delta\underline{\rho}(d_2^{0BG} - d_2^{1BGP}) + \delta\underline{\rho}(d_2^{1BG} - d_2^{2BG}) & \\
+\delta(1-\underline{\rho})(d_2^{0BB} - d_2^{1BBP}) + \delta(1-\underline{\rho})d_2^{1BB} \leq \phi_1 & (IC_B^P) \\
d_2^{1iG} - d_2^{2iG} \leq \phi_2 & (IC_{iG}^P) \text{ for } i = B, G \\
d_2^{1iB} \leq \phi_2 & (IC_{iB}^P) \text{ for } i = B, G
\end{array}\right.$$

There is a continuum of solutions:

$$\left\{\begin{array}{ll} d_1^G = \phi_1 + \Phi_1 - \delta\bar{\rho} d_2^{GG} - \delta(1-\bar{\rho}) d_2^{GB} & 0 \leq d_2^{GG} \leq \Phi_2 + \phi_2 \\ & 0 \leq d_2^{GB} \leq \phi_2 \\ d_1^B = \phi_1 + \delta\underline{\rho}(\Phi_2 - d_2^{BG}) - \delta(1-\underline{\rho}) d_2^{BB} & \Phi_2 \leq d_2^{BG} \leq \Phi_2 + \phi_2 \\ & 0 \leq d_2^{BB} \leq \phi_2 \end{array}\right\}$$

It gives initially the following expected utility to the country and the Principal:

$$U^C(R) = R\delta\left(\phi_1 + \alpha\Phi_1 + \delta(1-\alpha)\underline{\rho}\Phi_2\right) + \delta\alpha(\bar{y_1} - \delta\alpha\Phi_1 + \delta^2\left(\alpha\bar{\rho} + (1-\alpha)\underline{\rho}\right)\bar{y_2} - \delta^2(1-\alpha)\underline{\rho}\Phi_2$$

$$U^P = -\delta\phi_1$$

Appendix 2: The Model

<u>**Date 2**</u> At the beginning of date 2, the country's debt is characterized by d_2^1.

<u>**First case**</u>**: zero income at date 2**

The Principal's bailout strategy: If the country has a zero income $y_2 = 0$ at date 2, it cannot repay its debt and chooses to default as long as $d_2^2 > 0$. Thus, the Principal has to repurchase its whole debt by offering $d_2^2 = 0$ if he wants to prevent a default. He does so only if a bailout costs less than letting the country default (Bailout zone).

Lemma 4.1 (Date 2, zero income) *The Principal's optimal bailout strategy is:*

- *If $d_2^1 \leq \phi_2$, the Principal converts d_2^1 to $d_2^2 = 0$ and the country does not default.*
- *If $d_2^1 > \phi_2$, the Principal does not repurchase any debt and the country defaults.*

<u>**Second case**</u>**: positive income at date 2**

The Principal's bailout strategy: If the country has a positive income $y_2 = \bar{y_2}$ at date 2, it is willing to pay back its debt when $d_2^2 \leq \Phi_2$. Thus, when $d_2^1 \leq \Phi_2$, the Principal knows that the country will repay and does not intervene (Reimbursement zone). When $d_2^1 > \Phi_2$, the Principal has to repurchase some of the debt if he wants to prevent a default by offering optimally $d_2^2 = \Phi_2$ which is the point where the country is willing to pay back its debt and costs the least to the Principal. The Principal does so only if such a bailout costs less than letting the country default (Bailout zone).

Lemma 4.2 (Date 2, positive income) *The Principal's optimal bailout strategy is:*

- *If* $d_2^1 \leq \Phi_2$*, the Principal needs not repurchase any debt and* $d_2^2 = d_2^1$*. The country repays* d_2^2 *and there is no default.*
- *If* $\Phi_2 < d_2^1 \leq \Phi_2 + \phi_2$*, the Principal offers to repurchase* $d_2^1 - \Phi_2 \leq \phi_2$ *and sets* $d_2^2 = \Phi_2$*. The country repays* $d_2^2 = \Phi_2$ *and there is no default.*
- *If* $d_2^1 > \Phi_2 + \phi_2$*, the Principal does not repurchase any debt and* $d_2^2 = d_2^1$*. The country defaults at the equilibrium*[13].

__Date 1__ At the beginning of date 1, the country's debt is characterized by (d_1^0, d_2^0).

__First case__**: zero income at date 1**

If the country has a zero income $y_1 = 0$ at date 1, it cannot repay its debt, and chooses to default as long as $d_1^1 > 0$. Thus, the Principal can propose to repurchase some of the country's debt $(d_1^0 - d_1^{1P}, d_2^0 - d_2^{1P})$, and the country can decide to rollover some of its debt to the next period d_2^{1R}. After both the bailout from the Principal and the rollover, the remaining debt at date 1 must be $d_1^1 = 0$ if a default is to be prevented.

The Sovereign's rollover strategy: After a bailout from the Principal, the country's remaining debt is (d_1^{1P}, d_2^{1P}). As long as $d_1^{1P} > 0$, the country needs to rollover all the remaining short-term debt if it wants to avoid a default[14]. Let us solve for the optimal rollover strategy by differentiating the cases according to the level of d_2^{1P}:

- When $0 \leq d_2^{1P} \leq \phi_2$ and
  - $0 \leq d_1^{1P} \leq \delta(\phi_2 - d_2^{1P})$, the country is able to rollover debt such that the final level of long-term debt $d_2^1 = d_2^{1P} + d_2^{1R} \leq \phi_2$ with $d_1^{1P} = \delta d_2^{1R}$.
  - $\delta(\phi_2 - d_2^{1P}) \leq d_1^{1P} \leq \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^{1P})$, the country is able to rollover debt such that the final level of long-term debt $d_2^1 = d_2^{1P} + d_2^{1R} \leq \Phi_2 + \phi_2$ with $d_1^{1P} = \delta\underline{\rho} d_2^{1R}$.
  - $d_1^{1P} > \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^{1P})$, the level of short-term debt to rollover in order to avoid a default is too large, and there is no rollover.

[13] Notice that this case cannot happen at the equilibrium, as the country defaults with certainty at date 2. Thus, no lender agrees initially to sell long-term debt above the limit $\Phi_2 + \phi_2$.

[14] The marginal utility of consumption being the same between dates 1 and 2, the country has no incentive to borrow more than what is required to avoid a default at date 1.

- When $\phi_2 \leq d_2^{1P} \leq \Phi_2 + \phi_2$ and
  - $0 \leq d_1^{1P} \leq \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^{1P})$, the country is able to rollover debt such that the final level of long-term debt $d_2^1 = d_2^{1P} + d_2^{1R} \leq \Phi_2 + \phi_2$ with $d_1^{1P} = \delta\underline{\rho} d_2^{1R}$.
  - $d_1^{1P} > \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^{1P})$, the level of short-term debt to rollover in order to avoid a default is too large, and there is no rollover.

The Principal's bailout strategy: The sovereign enters the period with debt levels (d_1^0, d_2^0). Taking into account the fact that the sovereign can rollover some of its debt after his bailout offer, the Principal can propose to repurchase some of the country's debt $(d_1^0 - d_1^{1P}, d_2^0 - d_2^{1P})$ to prevent a default at date 1. He does so only if the sovereign cannot rollover enough debt to avoid a default, and if he can find a bailout offer (d_1^{1P}, d_2^{1P}) that costs him less to than to let the sovereign default. Let us solve for the optimal bailout strategy by differentiating the cases according to the level of d_2^0:

- When $d_2^0 \leq \phi_2$: if $d_1^0 \leq \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0)$, the country can rollover all its short-term debt and avoid a default at date 1. There is no need for the Principal to intervene. Yet, if $d_1^0 > \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0)$, the Principal can propose to repurchase some of the debt of the country if he wants to prevent a default at date 1. In this case, he has to bring back debt levels to (d_1^{1P}, d_2^{1P}) such that $d_1^{1P} \leq \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^{1P})$. He has three possibilities: repurchase debt such that the final level of long-term debt (after the rollover decision of the country) is (1) $\Phi_2 < d_2^1 = d_2^{1P} + \frac{d_1^{1P}}{\delta\underline{\rho}} \leq \Phi_2 + \phi_2$, (2) $\phi_2 < d_2^1 = d_2^{1P} + \frac{d_1^{1P}}{\delta\underline{\rho}} \leq \Phi_2$, or eventually (3) $d_2^1 = d_2^{1P} + \frac{d_1^{1P}}{\delta} \leq \phi_2$. In the three cases, the Principal solves:

$$\min_{d_1^{1P}, d_2^{1P}} \left\{ d_1^0 - d_1^{1P} + \delta(d_2^0 - d_2^{1P}) + \delta\mathbb{E}_1 \left\{ x_2(d_2^1 - d_2^{2P}) + (1 - x_2)\phi_2 \right\} \right\}$$

  such that the overall expected cost is lower than his spillover cost ϕ_1, d_2^1 is in the defined range, and he cannot repurchase more debt than the country has on its portfolio. Eventually, the cost incurred by the Principal is the lowest in case (1), and involves the largest bailout zone. Thus, within the set $\delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0) < d_1^0 \leq \phi_1 - \delta\underline{\rho} d_2^0 + \delta\underline{\rho}\Phi_2 - \delta(1 - \underline{\rho})\phi_2$, the Principal makes a bailout offer $(d_1^{1P} = \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0), d_2^{1P} = d_2^0)$ and the country does not default at date 1[15].

[15] I assume that the Principal makes the minimal bailout offer to the country. More precisely, he is indifferent between all the following bailouts: $(\delta\underline{\rho}(\Phi_2 - d_2^0) < d_1^{1P} \leq \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0), d_2^{1P} \leq d_2^0)$. In this set, it amounts the same to the Principal to repurchase more short-term debt today (implying that the country has less debt to rollover), or to repurchase more debt tomorrow.

- When $\phi_2 < d_2^0 \leq \Phi_2$: if $d_1^0 \leq \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0)$, the country can rollover all its short-term debt and avoid a default at date 1. There is no need for the Principal to intervene. Yet, if $d_1^0 > \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0)$, the Principal can propose to repurchase some of the debt of the country if he wants to prevent a default at date 1. In this case, he has to bring back debt levels to (d_1^{1P}, d_2^{1P}) such that $d_1^{1P} \leq \delta\underline{\rho}(\Phi_2+\phi_2-d_2^{1P})$. He has three possibilities: repurchase debt such that the final level of long-term debt (after the rollover decision of the country) is (1) $\Phi_2 < d_2^1 = d_2^{1P} + \frac{d_1^{1P}}{\delta\underline{\rho}} \leq \Phi_2 + \phi_2$, (2) $\phi_2 < d_2^1 = d_2^{1P} + \frac{d_1^{1P}}{\delta\underline{\rho}} \leq \Phi_2$, or eventually (3) $d_2^1 = d_2^{1P} + \frac{d_1^{1P}}{\delta} \leq \phi_2$. In the three cases, the Principal solves the same problem as before. Eventually, the cost incurred by the Principal is the lowest in case (1), and involves the largest bailout zone. Thus, within the set $\delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0) < d_1^0 \leq \phi_1 - \delta\underline{\rho}d_2^0 + \delta\underline{\rho}\Phi_2 - \delta(1-\underline{\rho})\phi_2$, the Principal makes a bailout offer $(d_1^{1P} = \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0), d_2^{1P} = d_2^0)$ and the country does not default at date 1.

- When $\Phi_2 < d_2^0 \leq \Phi_2 + \phi_2$: if $d_1^0 \leq \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0)$, the country can rollover all its short-term debt and avoid a default at date 1. There is no need for the Principal to intervene. Yet, if $d_1^0 > \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0)$, the Principal can propose to repurchase some of the debt of the country if he wants to prevent a default at date 1. In this case, he has to bring back debt levels to (d_1^{1P}, d_2^{1P}) such that $d_1^{1P} \leq \delta\underline{\rho}(\Phi_2+\phi_2-d_2^{1P})$. He has three possibilities: repurchase debt such that the final level of long-term debt (after the rollover decision of the country) is (1) $\Phi_2 < d_2^1 = d_2^{1P} + \frac{d_1^{1P}}{\delta\underline{\rho}} \leq \Phi_2 + \phi_2$, (2) $\phi_2 < d_2^1 = d_2^{1P} + \frac{d_1^{1P}}{\delta\underline{\rho}} \leq \Phi_2$, or eventually (3) $d_2^1 = d_2^{1P} + \frac{d_1^{1P}}{\delta} \leq \phi_2$. In the three cases, the Principal solves the same problem as before. Eventually, the cost incurred by the Principal is the lowest in case (1), and involves the largest bailout zone. Thus, within the set $\delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0) < d_1^0 \leq \phi_1 - \delta\underline{\rho}d_2^0 + \delta\underline{\rho}\Phi_2 - \delta(1-\underline{\rho})\phi_2$, the Principal makes a bailout offer $(d_1^{1P} = \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0), d_2^{1P} = d_2^0)$ and the country does not default at date 1.

Lemma 4.3 (Date 1, Zero income)
$0 \leq d_2^0 \leq \phi_2$

- *If* $0 \leq d_1^0 \leq \delta(\phi_2 - d_2^0)$*, the country is able to rollover its debt such that the level of long-term debt is eventually* $d_2^1 = d_2^0 + \frac{d_1^0}{\delta} \leq \phi_2$*, and the Principal does not intervene. There is no default at date 1.*

- *If* $\delta(\phi_2 - d_2^0) < d_1^0 \leq \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0)$*, the country is able to rollover its debt such that the level of long-term debt is eventually* $\Phi_2 < d_2^1 = d_2^0 + \frac{d_1^0}{\delta\underline{\rho}} \leq \Phi_2 + \phi_2$*, and the Principal does not intervene. There is no default at date 1.*

- *If* $\delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0) < d_1^0 \leq \phi_1 - \delta\underline{\rho}d_2^0 + \delta\underline{\rho}\Phi_2 - \delta(1-\underline{\rho})\phi_2$, *the Principal proposes to repurchase some of the short-term debt of the country to bring back debt levels to* $(d_1^{1P} = \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0), d_2^{1P} = d_2^0)$. *After the bailout, the country rolls over the remaining debt and* $d_2^1 = \Phi_2 + \phi_2$. *There is no default at date 1.*

- *If* $d_1^0 > \phi_1 - \delta\underline{\rho}d_2^0 + \delta\underline{\rho}\Phi_2 - \delta(1-\underline{\rho})\phi_2$, *the Principal does not intervene, there is too much debt to rollover, and the country defaults at date 1.*

$\phi_2 < d_2^0 \leq \Phi_2 + \phi_2$

- *If* $0 \leq d_1^0 \leq \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0)$, *the country is able to rollover its debt such that the level of long-term debt is eventually* $\Phi_2 < d_2^1 = d_2^0 + \frac{d_1^0}{\delta\underline{\rho}} \leq \Phi_2 + \phi_2$, *and the Principal does not intervene. There is no default at date 1.*

- *If* $\delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0) < d_1^0 \leq \phi_1 - \delta\underline{\rho}d_2^0 + \delta\underline{\rho}\Phi_2 - \delta(1-\underline{\rho})\phi_2$, *the Principal proposes to repurchase some of the short-term debt of the country to bring back debt levels to* $(d_1^{1P} = \delta\underline{\rho}(\Phi_2 + \phi_2 - d_2^0), d_2^{1P} = d_2^0)$. *After the bailout, the country rolls over the remaining debt such that the level of long-term debt is eventually* $d_2^1 = \Phi_2 + \phi_2$. *There is no default at date 1.*

- *If* $d_1^0 > \phi_1 - \delta\underline{\rho}d_2^0 + \delta\underline{\rho}\Phi_2 - \delta(1-\underline{\rho})\phi_2$, *the Principal does not intervene, there is too much debt to rollover, and the country defaults at date 1.*

<u>Second case</u>: positive income at date 1

If the country has a positive income $y_1 = \bar{y}_1$ at date 1, it can repay its debt, but may be willing not to do so if it is more costly for it than to default. In this case, the Principal can propose to repurchase some of the country's debt $(d_1^0 - d_1^{1P}, d_2^0 - d_2^{1P})$, and the country can decide to rollover some of its debt to the next period d_2^{1R}. After both the bailout and the rollover, the remaining debt (d_1^1, d_2^1) must be such that the country is willing to repay d_1^1 if a default is to be prevented. See the details of the resolution in the Appendix online.

Lemma 4.4 (Date 1, Positive income)
$0 \leq d_2^0 \leq \phi_2$

- *If* $0 \leq d_1^0 \leq \Phi_1 - \delta\overline{\rho}d_2^0$, *the country repays its debt without rollover or bailout. There is no default at date 1.*

- *If* $\Phi_1 - \delta\overline{\rho}d_2^0 < d_1^0 \leq \Phi_1 + \delta(1-\overline{\rho})\phi_2 - \delta d_2^0$, *the country rolls over some of its debt such that the level of long-term debt is eventually* $d_2^1 = d_2^0 + \frac{d_1^0 - \Phi_1 + \delta\overline{\rho}d_2^0}{\delta(1-\overline{\rho})} \leq \phi_2$, *and repays the rest. There is no default at date 1.*

- *If* $\Phi_1 + \delta(1-\overline{\rho})\phi_2 - \delta d_2^0 < d_1^0 \leq \phi_1 + \Phi_1 - \delta d_2^0$, *the Principal proposes to repurchase some of the debt of the country to bring back debt levels to* $(d_1^{1P} = \Phi_1 + \delta(1-\overline{\rho})\phi_2 - \delta d_2^{1P}, d_2^{1P} \leq d_2^0)$. *After the bailout, the country rolls over some of the remaining debt such that the level of long-term debt is eventually* $d_2^1 = \phi_2$, *and repays the rest. There is no default at date 1.*

- *If* $d_1^0 > \phi_1 + \Phi_1 - \delta d_2^0$, *the Principal does not intervene, there is too much debt to rollover, and the country defaults at date 1.*

$\phi_2 < d_2^0 \leq \frac{\phi_1}{\delta} + \overline{\rho}\phi_2$

- *If* $0 \leq d_1^0 \leq \Phi_1 - \delta\overline{\rho}d_2^0 - \delta(1-\overline{\rho})\Phi_2$, *the country repays its debt without rollover or bailout. There is no default at date 1.*

- *If* $\Phi_1 - \delta\overline{\rho}d_2^0 - \delta(1-\overline{\rho})\Phi_2 < d_1^0 \leq \delta(1-\overline{\rho})d_2^0 + \Phi_1 - \delta(1-\overline{\rho})\Phi_2 - \delta\phi_2$, *the country rolls over some of its debt such that the level of long-term debt is eventually* $\Phi_2 < d_2^1 = d_2^0 + \frac{d_1^0 - \Phi_1 + \delta\Phi_2}{\delta\overline{\rho}} \leq \Phi_2 + \phi_2$, *and repays the rest. There is no default at date 1.*

- *If* $\delta(1-\overline{\rho})d_2^0 + \Phi_1 - \delta(1-\overline{\rho})\Phi_2 - \delta\phi_2 < d_1^0 \leq \Phi_1 - \delta\overline{\rho}\phi_2$, *the Principal proposes to repurchase some of the long-term debt of the country to bring back debt levels to* $(d_1^{1P} = d_1^0, d_2^{1P} = \phi_2)$. *After the bailout, the country repays the remaining debt. There is no default at dates 1 and 2.*

- *If* $\Phi_1 - \delta\overline{\rho}\phi_2 < d_1^0 \leq \phi_1 + \Phi_1 - \delta d_2^0$, *the Principal proposes to repurchase some of debt of the two maturities to bring back debt levels to* $(d_1^{1P} = \Phi_1 + \delta(1-\overline{\rho})\phi_2 - \delta d_2^{1P}, d_2^{1P} \leq \phi_2)$. *After the bailout, the country rolls over some of the remaining debt such that the level of long-term debt is eventually* $d_2^1 = \phi_2$, *and repays the rest. There is no default at date 1, and no default at date 2.*

- *If* $d_1^0 > \phi_1 + \Phi_1 - \delta d_2^0$, *the Principal does not intervene, there is too much debt to rollover, and the country defaults at date 1.*

$\frac{\phi_1}{\delta} + \bar{\rho}\phi_2 < d_2^0 \leq \Phi_2$

- *If* $0 \leq d_1^0 \leq \Phi_1 - \delta\overline{\rho}d_2^0 - \delta(1-\overline{\rho})\Phi_2$, *the country repays its debt without rollover or bailout. There is no default at date 1.*

- *If* $\Phi_1 - \delta\overline{\rho}d_2^0 - \delta(1-\overline{\rho})\Phi_2 < d_1^0 \leq \Phi_1 - \delta\overline{\rho}d_2^0 - \delta(1-\overline{\rho})\Phi_2 + \delta\overline{\rho}\phi_2$, *the country rolls over some of its debt such that the level of long-term debt is eventually* $\Phi_2 < d_2^1 = d_2^0 + \frac{d_1^0 - \Phi_1 + \delta\Phi_2}{\delta\overline{\rho}} \leq \Phi_2 + \phi_2$, *and repays the rest. There is no default at date 1.*

- *If* $\Phi_1 - \delta(1-\overline{\rho})\Phi_2 + \delta\overline{\rho}\phi_2 - \delta\overline{\rho}d_2^0 < d_1^0 \leq \phi_1 + \Phi_1 - \delta(1-\overline{\rho})(\Phi_2 + \phi_2) - \delta\overline{\rho}d_2^0$, *the Principal proposes to repurchase some of the short-term debt of the country to bring back debt levels to* $(d_1^{1P} = \Phi_1 - \delta(1-\overline{\rho})\Phi_2 + \delta\overline{\rho}\phi_2 - \delta\overline{\rho}d_2^0, d_2^{1P} = d_2^0)$. *After the bailout, the country rolls over some of the remaining debt such that the level of long-term debt is eventually* $d_2^1 = \Phi_2 + \phi_2$, *and repays the rest. There is no default at date 1.*

- *If* $d_1^0 > \phi_1 + \Phi_1 - \delta(1-\overline{\rho})(\Phi_2 + \phi_2) - \delta\overline{\rho}d_2^0$, *the Principal does not intervene, there is too much debt to rollover, and the country defaults.*

$\Phi_2 < d_2^0 \leq \Phi_2 + \phi_2$

- *If* $0 \leq d_1^0 \leq \Phi_1 - \delta\Phi_2$, *the country repays its debt without rollover or bailout. There is no default at date 1.*

- *If* $\Phi_1 - \delta\Phi_2 < d_1^0 \leq \Phi_1 - \delta\overline{\rho}d_2^0 - \delta(1-\overline{\rho})\Phi_2 + \delta\overline{\rho}\phi_2$, *the country rolls over some of its debt such that the level of long-term debt is eventually* $\Phi_2 < d_2^1 = d_2^0 + \frac{d_1^0 - \Phi_1 + \delta\Phi_2}{\delta\overline{\rho}} \leq \Phi_2 + \phi_2$, *and repays the rest. There is no default at date 1.*

- *If* $\Phi_1 - \delta(1-\overline{\rho})\Phi_2 + \delta\overline{\rho}\phi_2 - \delta\overline{\rho}d_2^0 < d_1^0 \leq \phi_1 + \Phi_1 - \delta(1-\overline{\rho})(\Phi_2 + \phi_2) - \delta\overline{\rho}d_2^0$, *the Principal proposes to repurchase some of the short-term debt of the country to bring back debt levels to* $(d_1^{1P} = \Phi_1 - \delta(1-\overline{\rho})\Phi_2 + \delta\overline{\rho}\phi_2 - \delta\overline{\rho}d_2^0, d_2^{1P} = d_2^0)$. *After the bailout, the country rolls over some of the remaining debt such that the level of long-term debt is eventually* $d_2^1 = \Phi_2 + \phi_2$, *and repays the rest. There is no default at date 1.*

- *If* $d_1^0 > \phi_1 + \Phi_1 - \delta(1-\overline{\rho})(\Phi_2 + \phi_2) - \delta\overline{\rho}d_2^0$, *the Principal does not intervene, there is too much debt to rollover, and the country defaults.*

Date 0: Country's optimal borrowing strategy

At date 0, the country chooses its initial debt portfolio (d_1^0, d_2^0) taking into account the bailout strategy of the Principal, and its rollover and default decisions at dates 1 and 2. I solve simultaneously for the initial amount of borrowings b_0 and the interest rates paid on debt.

Country's borrowing strategies

Let us consider the possible borrowing strategies for the country. Strategies that are strictly dominated are excluded from the analysis (see the Appendix online for more details). In every situation, the country solves:

$$\max_{\{d_1^0, d_2^0\}} \left\{ R\left((1+r_1^0)d_1^0 + (1+r_2^0)^2 d_2^0\right) + \mathbb{E}_0\left(\sum_{t=1}^{2} \delta^t (y_t - x_t d_t^t - (1-x_t)x_{t-1}\Phi_t)\right)\right\}$$

- (a) $\begin{cases} d_1^0 \leq \delta(\phi_2 - d_2^0) \\ d_2^0 \leq \phi_2 \end{cases}$
  There is no default at any date so that interest rates are risk-free $1+r_1 = 1+r_2 = \delta$. The country is thus able to borrow initially $b_0 = \delta d_1^0 + \delta^2 d_2^0$. There is a unique solution $(d_1^0 = 0, d_2^0 = \phi_2)$ bringing initial utility to the country:

$$U_a^C(R) = R\delta^2\phi_2 + \delta\alpha\bar{y_1} + \delta^2(\alpha\bar{\rho} + (1-\alpha)\underline{\rho})(\bar{y_2} - \phi_2)$$

- (b) $\begin{cases} \delta\underline{\rho}(\Phi_2 - d_2^0) < d_1^0 \leq \phi_1 + \delta\underline{\rho}\Phi_2 - \delta(1-\underline{\rho})\phi_2 - \delta\underline{\rho}d_2^0 \\ d_2^0 \leq \phi_2 \end{cases}$
  There is no risk of default at date 1, but a risk at date 2 of the country has consecutively no income at both dates. The country is thus able to borrow initially $b_0 = \delta d_1^0 + \delta^2(\alpha + (1-\alpha)\underline{\rho})d_2^0$. There is a unique solution: $(d_1^0 = \phi_1 + \delta\underline{\rho}\Phi_2 - \delta\phi_2, d_2^0 = \phi_2)$ bringing initial utility to the country:

$$U_b^C(R) = R\delta\left(\phi_1 + \delta\underline{\rho}\Phi_2 - \delta(1-\alpha)(1-\underline{\rho})\phi_2\right) + \delta\alpha\bar{y_1} + \delta^2(\alpha\bar{\rho} + (1-\alpha)\underline{\rho})\bar{y_2} - \delta\alpha\phi_1$$
$$-\delta^2(1-\alpha(1-\underline{\rho}))\Phi_2 + \delta^2\alpha(1-\bar{\rho})\phi_2$$

- (c) $\begin{cases} \max(\Phi_1 + \delta(1-\bar{\rho})\phi_2 - \delta d_2^0, \Phi_1 - \delta\bar{\rho}\phi_2) < d_1^0 \leq \phi_1 + \Phi_1 - \delta d_2^0 \\ d_2^0 \leq \frac{\phi_1}{\delta} + \bar{\rho}\phi_2 \end{cases}$
  There is a risk of default at date 1 if the country has no income, but no more risk at date 2 provided the country did not default at date 1. The country is thus able to borrow initially $b_0 = \delta\alpha d_1^0 + \delta^2\alpha d_2^0$. There is a unique solution: $(d_1^0 = \phi_1 + \Phi_1 - \delta d_2^0, d_2^0 \leq \phi_2)$ bringing initial utility to the country:

$$U_c^C(R) = R\delta\alpha\left(\phi_1 + \Phi_1\right) + \delta\alpha\bar{y_1} + \delta^2(\alpha\bar{\rho} + (1-\alpha)\underline{\rho})\bar{y_2} - \delta\Phi_1$$

Analysis of the different borrowing strategies

Let us compare together the different borrowing strategies:

$$U_b^C(R) \geq U_a^C(R) \Leftrightarrow R \geq R_{ba} = \frac{\alpha\phi_1 + \delta(1-\alpha(1-\underline{\rho}))\Phi_2 - \delta(\alpha + (1-\alpha)\underline{\rho})\phi_2}{\phi_1 + \delta\underline{\rho}\Phi_2 + \delta(1 + (1-\alpha)(1-\underline{\rho}))\phi_2}$$
$$U_c^C(R) \geq U_b^C(R) \Leftrightarrow R \geq R_{cb} = \frac{\Phi_1 - \alpha\phi_1 - \delta\Phi_2 + \delta\alpha(1-\underline{\rho})(\Phi_2 + \phi_2)}{\alpha\Phi_1 - (1-\alpha)\phi_1 - \delta\underline{\rho}\Phi_2 + \delta(1-\alpha)(1-\underline{\rho})\phi_2}$$

To simplify the analysis, let us assume that $\Phi_1 = \Phi_2$ and that $\phi_1 = \phi_2 = \gamma\Phi_1 = \gamma\Phi_2$ with $\gamma << 1$. We also assume that income realizations are independent across dates 1 and 2, that is $\alpha = \bar{\rho} = \underline{\rho}$. Under these assumptions, we obtain:

$$R_{ba} = \frac{\alpha\gamma + \delta - \delta\alpha(1-\alpha) - \delta\alpha(2-\alpha)\gamma}{\gamma + \delta\alpha - \delta(\alpha + (1-\alpha)^2)\gamma} =_{\gamma=0} \frac{1-\alpha(1-\alpha)}{\alpha}$$
$$R_{cb} = \frac{1 - \alpha\gamma - \delta(1-\alpha(1-\alpha)(1+\gamma))}{\alpha - (1-\alpha)\gamma - \delta(\alpha - (1-\alpha)^2\gamma)} =_{\gamma=0} \frac{1-\delta(1-\alpha(1-\alpha))}{\alpha(1-\delta)}$$

Notice that for a very small γ, by continuity, both thresholds are above 1. Furthermore, I obtain under the same assumption that:

$$R_{cb} \geq R_{ba}$$

- When $1 \leq R < R_{ba} = R_1$, the country chooses strategy (a)
- When $R_{ba} < R \geq R_{cb} = R_2$, the country chooses strategy (b)
- When $R > R_{cb}$, the country chooses strategy (c)

Remark: the assumption that $R \geq 1$ guarantees that borrowing is always optimal for the country. More precisely, $R \geq \alpha$ would be enough.

Appendix 3: Setup with only Long- or Short-term debt

See the Appendix online.

Appendix 4: Setup without potential bailout

See the Appendix online.

Appendix 5: Welfare Analysis

See the Appendix online.

Appendix 6: ex ante policies

See the Appendix online.