



Centre for Efficiency and Productivity Analysis

**Working Paper Series
No. WP10/2019**

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E. Grifell-Tatjé & C. A. K. Lovell

Date: August 2019

**School of Economics
University of Queensland
St. Lucia, Qld. 4072
Australia**

ISSN No. 1932 - 4398

Dual Productivity Analysis: A Konüs/Shephard Approach

E. Grifell-Tatjé, Universitat Autònoma de Barcelona (emili.grifell@uab.cat)

C. A. K. Lovell, University of Queensland (k.lovell@uq.edu.au)

Abstract

A primal (or direct) productivity index is conventionally defined as the ratio of an output quantity index to an input quantity index. There have been attempts in the literature to define and implement dual and indirect productivity indexes based on price changes rather than quantity changes. Although dual and indirect productivity indexes share a common motivation, the measurement of productivity change when prices are measured more accurately than quantities, they differ analytically, from one another and from primal productivity indexes. We introduce a new dual productivity index, based on contributions of Konüs and Shephard, and we compare our dual productivity index with a primal productivity index based on the work of Malmquist. We also compare these two theoretical productivity indexes with analogous empirical Fisher productivity indexes. We provide an empirical application to US agricultural productivity growth.

Keywords: dual and primal productivity indexes, price distance functions, agricultural productivity

JEL Codes: D24, D33

Acknowledgements

We are grateful to the Generalitat de Catalunya project 2017SGR-1036 and the Spanish Ministerio de Economía, Industria y Competitividad, project ECO2017-86054-C3-1-RX, for their generous financial support of this research.

Dual Productivity Analysis: A Konüs/Shephard Approach

1. Introduction

In his post-war review of US productivity trends, Evans (1947; 221-222) stressed not just the magnitude of productivity changes but also their economic significance, reporting that workers had enjoyed the fruits of productivity growth (which he called technical progress) through wage increases and price declines. Soon thereafter Davis (1955; Chs. 1,5), whose interest centred on the performance of an individual business rather than that of an aggregate economy, argued that business productivity growth benefits customers through lower prices, labour and suppliers of materials through higher remuneration, and owners of the business through higher retained earnings.¹ In his monumental study of productivity trends in the US economy in the first half of the 20th century, Kendrick (1961; 111) echoed the issues raised by Evans and Davis, noting that “[i]f productivity advances, wage rates and capital return necessarily rise in relation to the general product price level, since this is the means whereby the fruits of productivity gains are distributed to workers and investors by the market mechanism.” Kendrick illustrated his claim by calculating productivity growth in the US economy over various time periods using the ratio of total factor price and average product price. Returning to the level of an individual business, Eldor and Sudit (1981) analysed the distribution of productivity gains among consumers, workers and owners at “a major corporation”. This early awareness of the distributional implications of productivity change, at both business and aggregate levels, and the emphasis of Kendrick on necessity, suggest the possibility of using price changes rather than quantity changes to measure productivity change.

Fourastié (1957, 1962) was among the first to infer productivity change from price changes. Noting that both the objective and the result of productivity growth were to “...bring down selling prices and consequently speed up social progress and more particularly raise living standards and the purchasing power of wages and salaries,” (1957; 12) Fourastié developed a crude mathematical model in which productivity was expressed as the ratio of labour’s wage to product price, assuming that the quantities of all other inputs could be expressed in terms of labour hours. He proceeded to argue that in some circumstances prices are easier to measure than quantities, especially at the aggregate level and over long time periods, and he provided a variety of empirical applications in which he used changes in labour’s wage and product prices to estimate productivity change. Fourastié referred to the use of price changes to measure productivity change as the “indirect” approach to productivity measurement.²

Eventually analysis began complementing intuition and evidence, beginning with Siegel (1955; 53), who was the first to state conditions under which a price-based productivity index coincides with a quantity-based productivity index. Arguing that the impact of productivity growth “...is to make output cheap compared to input...” led him

to refine Fourastié's approach by defining an indirect productivity index as the ratio of a cost index to a product price index. Later Siegel (1961; 27), Vincent (1961; 12-13) and Puiseux and Bernard (1965, 1966) were more analytically precise. Siegel showed that (i) if the product test is satisfied for outputs, so that $Y \times P = V$, with Y and P indicating quantity and price indexes for output, and V indicating a "value index" for output, by which he meant revenue change R^{t+1}/R^t ; and (ii) if the product test is satisfied for inputs, so that $X \times W = U$, with X and W indicating quantity and price indexes for input, and U indicating a "value index" for input, by which he meant cost change C^{t+1}/C^t ; then $PY/WX = V/U$, in which PY/WX is profitability change, or Georgescu-Roegen's (1951) "return to the dollar". Additionally if $R^t = C^t$, $t=0,1$, it follows that $V = U$, and consequently a price-based productivity index W/P coincides with a quantity-based productivity index Y/X . Siegel illustrated this result with a Paasche dual productivity index W_P/P_P and a Laspeyres primal productivity index Y_L/X_L . This final assertion is correct but incomplete, however, and we show below that, conditional on satisfaction of the two product tests, a zero profit condition $R^t = C^t$, $t=0,1$, is sufficient but not necessary for $R^{t+1}/R^t = C^{t+1}/C^t$, and consequently for equality of dual and primal productivity indexes. The historical development and decomposition of profitability change into productivity change and price recovery change (the reciprocal of W/P) is detailed in Grifell-Tatjé and Lovell (2015; Chs. 2,3).³

More recently Shapiro (1987), Roeger (1995), Hsieh (2002), Aiyar and Dalgaard (2005) and Fernald and Neiman (2011) have spawned a rapidly growing literature that uses price changes to estimate productivity change, primarily in the context of uncovering the sources of economic growth. Hsieh's motivation arose from anomalies he uncovered in previous research into the growth experience of East Asian countries, Singapore in particular. The methodology underlying this literature is dual productivity accounting, as proposed by Jorgenson and Griliches (1967), in which primal productivity change is estimated as the revenue share-weighted sum of the rates of change of output quantities minus (or divided by) the cost share-weighted sum of the rates of change of input quantities, and dual productivity change is estimated as the cost share-weighted sum of the rates of change of input prices minus (or divided by) the revenue share-weighted sum of the rates of change of output prices. In the Jorgenson and Griliches set-up the accounting identity forces equality between revenue and expenditure in each period, and thus between primal and dual measures of productivity change. However the principal finding of the subsequent empirical literature cited above is that primal and dual estimates of productivity change can and do differ, because the accounting identity can and does break down for a number of reasons. Hsieh mentions four potential sources of divergence: imperfect competition, short-run adjustment costs, omitted or mis-measured inputs (or outputs; Hsieh's model has a single aggregate output), and inconsistencies between the sources of quantity and price data. Fernald and Neiman add another potential source of divergence: heterogeneous tax treatment and capital subsidies arising from government policy interventions.

Our dual approach is similar in spirit, but very different in assumptions, from the dual productivity accounting approach developed by Jorgenson and Griliches. An important assumption underpinning the productivity accounting approach is that producers optimise, with marginal rates of input substitution equalling input price ratios and marginal rates of output transformation equalling output price ratios. This equilibrium assumption justifies replacement of theoretical but unobservable input and output production elasticities with observable input cost shares and output revenue shares. In sharp contrast, although our dual approach also requires quantity data, the quantity data are not linked to an assumption of optimising behaviour, being based on distance functions rather than minimum cost and maximum revenue functions.

Our dual approach also differs from an indirect approach developed by Shephard (1974) and extended to indirect productivity measurement by Färe et al. (1992) and Färe and Grosskopf (1994). Whereas our approach is simultaneously output- and input-oriented, this indirect approach has a single orientation, either a cost-constrained output (or revenue) maximisation orientation in which output quantities are the choice variables, or a revenue-constrained input (or cost) minimisation orientation, in which input quantities are the choice variables. It is based on distance functions, but the distance functions are defined on either cost-deflated input price and output quantity space or revenue-deflated output price and input quantity space, and so it requires quantity information. However as a distance function based approach, it does not require an assumption of optimising behaviour, and perhaps consequently the cost-constrained output maximisation orientation has proved particularly useful for analysing performance in the provision of public services such as education, health care or public safety, in which output prices are missing or heavily subsidized and budgets (rather than resources) are wholly or partly allocated by agencies.

Our dual approach is inspired by the Konüs (1924) cost of living index, the ratio of the minimum cost of producing a given output quantity vector at two different input price vectors. In this set-up input prices are exogenous, and the choice variables are input quantities. We then exploit Shephard's (1970) interpretation of a (minimum) cost function as a distance function in input price space. This insight leads to a re-interpretation of the Konüs cost of living index, in which input quantities are the choice variables, as an input price index, in which input prices are the choice variables. Konüs did not consider the output side of the production relation, but Shephard did, re-interpreting a (maximum) revenue function as a distance function in output price space. This re-interpretation enables us to define the ratio of the maximum revenue obtainable from a given input quantity vector at two different output price vectors as an output price index, in which output prices are the choice variables. Finally, our Konüs-Shephard dual productivity index is defined as the ratio of the input price index to the output price index, with its reciprocal defining a price recovery index. This dual productivity index is consistent with the interpretations of productivity change appearing in the quotes of Kendrick and, especially, Siegel (1955) above.

The rest of our paper is structured as follows. In Section 2 we develop our dual productivity index. In Section 3 we relate our dual productivity index to a primal productivity index based on the work of Malmquist (1953) in a consumer context, as transferred to a producer context by Diewert (1992; 348) and Bjurek (1996; 308). In contrast to Jorgenson and Griliches (1967; 252), who note that their dual and primal productivity indexes "...are dual to each other and are equivalent...", we find no general equivalence between our Konüs-Shephard dual productivity index and a Malmquist primal productivity index. In our analytical framework equivalence requires the input and output price mixes and the input and output quantity mixes to vary in offsetting ways, a razor's edge condition that is unlikely to be satisfied in any empirical setting. In Section 4 we develop a procedure with which to implement our dual productivity index with time series data rather than panel data. In Section 5 we describe our time series data set, which consists of price indexes and implicit quantity indexes for three inputs and three outputs in US agriculture over the 68 year period 1948-2015, and we report our empirical findings. We find estimated productivity growth based on our Konüs-Shephard dual productivity index to differ from an estimate based on a primal Malmquist productivity index, although the two are very close, and they bound estimates of the US Department of Agriculture based on a Fisher productivity index. Section 6 concludes.

2. Analytical Foundations

We follow Shephard (1970) to develop a dual input price index that we use to decompose cost change in Section 2.1, and we develop a dual output price index that we use to decompose revenue change in Section 2.2, following similar procedures. A dual productivity index is defined as the ratio of the two dual price indexes, and is the reciprocal of a price recovery index, which we use to decompose profitability change in Section 2.3. The decompositions of cost change, revenue change and profitability change are essential to uncovering the relationship between dual and primal productivity indexes.

We use the following notation: $y \in \mathfrak{R}_+^M$ and $x \in \mathfrak{R}_+^N$ are quantity vectors of outputs and inputs, the prices of which are $p \in \mathfrak{R}_{++}^M$ and $w \in \mathfrak{R}_{++}^N$. The production set $T = \{(x,y): x \text{ can produce } y\}$ is closed, bounded and satisfies weak disposability of x and y . Input sets are $L(y) = \{x: (y,x) \in T\}$, $y \in \mathfrak{R}_+^M$, and output sets are $P(x) = \{y: (x,y) \in T\}$, $x \in \mathfrak{R}_+^N$. Input distance functions $D_i(y,x) = \max_{\lambda} \{\lambda: (x/\lambda) \in L(y)\}$, $y \in \mathfrak{R}_+^M$, are homogeneous of degree +1 in x , with $D_i(y,x) \geq 1$ if and only if $x \in L(y)$. The cost function $c(y,w) = \min_x \{w^T x: D_i(y,x) \geq 1\}$ is homogeneous of degree +1 in w . Output distance functions $D_o(x,y) = \min_{\theta} \{\theta: (y/\theta) \in P(x)\}$, $x \in \mathfrak{R}_+^N$, are homogeneous of degree +1 in y , with $D_o(x,y) \leq 1$ if and only if $y \in P(x)$. The revenue function $r(x,p) = \max_y \{p^T y: D_o(x,y) \leq 1\}$ is homogeneous of degree +1 in p .

2.1. A Dual Input Price Index and the Decomposition of Cost Change

Shephard (1970; Chapter 5) introduced the cost structure of production technology, defined as the family of subsets of $w \in \mathfrak{R}_{++}^N$ given by

$$\mathfrak{L}(y) = \{w: c(y, w) \geq 1, w > 0\}, \quad (1)$$

in which the cost function $c(y, w)$ is a distance function for the cost structure $\mathfrak{L}(y)$. $\mathfrak{L}(y)$ inherits its properties from those of the production technology, and we follow Shephard by assuming $\mathfrak{L}(y)$ satisfies disposability, closure and convexity properties. Shephard defined an input price vector $w \in \mathfrak{L}(y)$ as efficient if and only if it belongs to the efficient subset of $\mathfrak{L}(y)$ for which $c(y, w') < 1 \forall w' \leq w$.

Since $c(y, w)$ is a distance function in input price space we change notation and define a dual input price distance function on $\mathfrak{L}(y)$ as

$$D_w(y, w) = \max_{\delta} \{\delta: c\left(y, \frac{w}{\delta}\right) \geq 1\}, \quad (2)$$

which seeks the maximum radial reduction in w such that $c(y, w) \geq 1$. Färe (1984) defined a measure of input price efficiency as the radial distance of an input price vector to Shephard's efficient subset of $\mathfrak{L}(y)$ given by $F_w(y, w) = \min_{\gamma} \{\gamma: \gamma w \in \mathfrak{L}(y)\} = 1/D_w(y, w)$. The dual input price distance function $D_w(y, w)$ has the same properties as its primal input quantity distance function counterpart $D_i(y, x)$, with input prices replacing input quantities.

Next we use the dual input price distance function to define a dual input price index, which we then use to decompose cost change. We define a Konüs base period dual input price index as

$$W_K^0(y^0, w^1, w^0) = \frac{D_w^0(y^0, w^1)}{D_w^0(y^0, w^0)}, \quad (3)$$

which is non-negative, non-decreasing and homogeneous of degree +1 in w . $W_K^0(y^0, w^1, w^0)$ is independent of y^0 if, and only if, base period production technology is input homothetic.

We use $W_K^0(y^0, w^1, w^0)$ to decompose cost change, beginning with

$$\frac{C^1}{C^0} = W_K^0(y^0, w^1, w^0) \times XI^0(y^0, x^1, x^0, w^1, w^0), \quad (4)$$

in which $XI^0(y^0, x^1, x^0, w^1, w^0)$ is a base period implicit input quantity index. Transferring a strategy employed by Grifell-Tatjé and Lovell (2015, 2016) from quantity space to price space, we decompose $XI^0(y^0, x^1, x^0, w^1, w^0)$ as follows

$$\begin{aligned}
XI^0(y^0, x^1, x^0, w^1, w^0) &= \frac{C^1/C^0}{W_K^0(y^0, w^1, w^0)} \\
&= \frac{w^{1T} x^1}{w^{1T} x^0} \times \frac{x^{0T} w^1}{x^{0T} w^0} \times \frac{D_w^0(y^0, w^0)}{D_w^0(y^0, w^1)} \\
&= X_P \times WS_L(y^0, x^0, w^1, w^0),
\end{aligned} \tag{5}$$

in which $X_P = w^{1T} x^1 / w^{1T} x^0$ is a Paasche input quantity index and

$$WS_L(y^0, x^0, w^1, w^0) = \frac{x^{0T} [w^1 / D_w^0(y^0, w^1)]}{x^{0T} [w^0 / D_w^0(y^0, w^0)]} \geq 1 \tag{6}$$

is a Shephard-Laspeyres input price mix index with input prices w^1 and w^0 radially expanded or contracted to Shephard's base period efficient subset. This index evaluates the impact on cost of a change in the input price mix from w^0 to w^1 . If base period input quantities are less (more) expensive when evaluated at efficient base period input prices than at efficient comparison period input prices, then $WS_L(y^0, x^0, w^1, w^0) > (<) 1$. Since $D_w^0(y, w)$ is homogeneous of degree +1 in w , the input price mix index has a value of unity, and makes no contribution to cost change, if, and only if, $w^1 = \lambda w^0$, $\lambda > 0$, i.e. if and only if the input price mix is unchanged from base period to comparison period. $WS_L(y^0, x^0, w^1, w^0)$ is independent of y^0 if and only if base period technology is input homothetic.

Inserting (5) into (4) generates a base period decomposition of cost change

$$\frac{C^1}{C^0} = W_K^0(y^0, w^1, w^0) \times X_P \times WS_L(y^0, x^0, w^1, w^0). \tag{7}$$

Thus cost change can be expressed as the product of a Konüs base period dual input price index, a Paasche input quantity index, and a Shephard-Laspeyres input price mix index. $W_K^0(y^0, w^1, w^0)$ and X_P satisfy the product test with C^1/C^0 if, and only if, $WS_L(y^0, x^0, w^1, w^0) = 1$. Otherwise cost change has three drivers.

It is straightforward to repeat the foregoing analysis using a Konüs comparison period dual input price index

$$W_K^1(y^1, w^1, w^0) = \frac{D_w^1(y^1, w^1)}{D_w^1(y^1, w^0)}, \quad (8)$$

which is independent of y^1 if, and only if, comparison period production technology is input homothetic. This generates the cost change decomposition

$$\frac{C^1}{C^0} = W_K^1(y^1, w^1, w^0) \times XI^1(y^1, x^1, x^0, w^1, w^0), \quad (9)$$

in which $XI^1(y^1, x^1, x^0, w^1, w^0)$ is a comparison period implicit input quantity index. This index can be decomposed as

$$\begin{aligned} XI^1(y^1, x^1, x^0, w^1, w^0) &= \frac{C^1/C^0}{W_K^1(y^1, w^1, w^0)} \\ &= \frac{w^{0T} x^1}{w^{0T} x^0} \times \frac{x^{1T} w^1}{x^{1T} w^0} \times \frac{D_w^1(y^1, w^0)}{D_w^1(y^1, w^1)} \\ &= X_L \times WS_P(y^1, x^1, w^1, w^0), \end{aligned} \quad (10)$$

in which $X_L = w^{0T} x^1 / w^{0T} x^0$ is a Laspeyres input quantity index and

$$WS_P(y^1, x^1, w^1, w^0) = \frac{x^{1T} [w^1 / D_w^1(y^1, w^1)]}{x^{1T} [w^0 / D_w^1(y^1, w^0)]} \lesseqgtr 1 \quad (11)$$

is a Shephard-Paasche input price mix index with input prices w^1 and w^0 radially expanded or contracted to Shephard's comparison period efficient subset. This index also evaluates the impact on cost of a change in the input price mix from w^0 to w^1 . If comparison period inputs are less (more) expensive when evaluated at efficient comparison period input prices than at efficient base period input prices, then $WS_P(y^1, x^1, w^1, w^0) < (>) 1$. $WS_P(y^1, x^1, w^1, w^0)$ is independent of y^1 if, and only if, comparison period technology is input homothetic.

Inserting (10) into (9) yields a comparison period cost change decomposition

$$\frac{C^1}{C^0} = W_K^1(y^1, w^1, w^0) \times X_L \times WS_P(y^1, x^1, w^1, w^0), \quad (12)$$

which states that cost change also can be expressed as the product of a Konüs comparison period dual input price index, a Laspeyres input quantity index, and a Shephard-Paasche input price mix index. Since $D_w^1(y, w)$ is homogeneous of degree +1 in w , the input price mix index has a value of unity if, and only if, the input price mix remains constant, in which case $W_K^1(y^1, w^1, w^0)$ and X_L satisfy the product test with C^1/C^0 .

Taking the geometric mean of (7) and (12) generates the cost change decomposition

$$\begin{aligned} \frac{C^1}{C^0} &= [W_K^0(y^0, w^1, w^0) \times W_K^1(y^1, w^1, w^0)]^{1/2} \times [X_P \times X_L]^{1/2} \\ &\quad \times [WS_L(y^0, x^0, w^1, w^0) \times WS_P(y^1, x^1, w^1, w^0)]^{1/2} \\ &= W_K(y^1, y^0, w^1, w^0) \times X_F \times WS_F(y^1, y^0, x^1, x^0, w^1, w^0), \quad (13) \end{aligned}$$

in which X_F is a Fisher input quantity index. Thus cost change is expressed as the product of a Konüs dual input price index, a Fisher input quantity index, and a Shephard-Fisher input price mix index. The Konüs dual input price index is independent of output quantities if both base period and comparison period technologies are input homothetic. It satisfies the product test with the Fisher input quantity index if, and only if, the input price mix index is unity, which does not require an unchanged input price mix since it does not require either of its base period and comparison period components to be unity. Thus $WS_L(y^0, x^0, w^1, w^0) = WS_P(y^1, x^1, w^1, w^0) = 1$ is sufficient, but not necessary, for satisfaction of the product test. A necessary and sufficient condition is $WS_L(y^0, x^0, w^1, w^0) = 1/WS_P(y^1, x^1, w^1, w^0)$.⁴

2.2. A Dual Output Price Index and the Decomposition of Revenue Change

Again following Shephard (1970; Chapter 10), we define the revenue structure of production technology as the family of subsets of $p \in \mathfrak{R}_{++}^M$ given by

$$\mathcal{P}(x) = \{p: r(x, p) \leq 1, p > 0\}. \quad (14)$$

$\mathcal{P}(x)$ inherits its properties (disposability, closure and convexity) from those of the primal production technology, and the primal revenue function $r(x,p)$ is a distance function for the revenue structure $\mathcal{P}(x)$.

An output price distance function is defined on $\mathcal{P}(x)$ as

$$D_p(x, p) = \min_v \left\{ v : r \left(x, \frac{p}{v} \right) \leq 1 \right\}, \quad (15)$$

and a radial measure of output price efficiency is defined as $F_p(x, p) = 1/D_p(x, p)$. The dual output price distance function $D_p(x, p)$ has the same properties as its primal output quantity counterpart $D_o(x, y)$, with output prices replacing output quantities.

A Konüs base period dual output price index is defined as

$$P_K^0(x^0, p^1, p^0) = \frac{D_p^0(x^0, p^1)}{D_p^0(x^0, p^0)}, \quad (16)$$

which is non-negative, non-decreasing and homogeneous of degree +1. $P_K^0(x^0, p^1, p^0)$ is independent of x^0 if and only if base period production technology is output homothetic.

We decompose revenue change as

$$\frac{R^1}{R^0} = P_K^0(x^0, p^1, p^0) \times YI^0(x^0, y^1, y^0, p^1, p^0), \quad (17)$$

in which $YI^0(x^0, y^1, y^0, p^1, p^0)$ is a base period implicit output quantity index that can be decomposed as

$$\begin{aligned} YI^0(x^0, y^1, y^0, p^1, p^0) &= \frac{R^1/R^0}{P_K^0(x^0, p^1, p^0)} \\ &= \frac{p^{1T}y^1}{p^{1T}y^0} \times \frac{y^{0T}p^1}{y^{0T}p^0} \times \frac{D_p^0(x^0, p^0)}{D_p^0(x^0, p^1)} \\ &= Y_p \times PS_L(x^0, y^0, p^1, p^0), \end{aligned} \quad (18)$$

in which $Y_p = p^{1T}y^1/p^{1T}y^0$ is a Paasche output quantity index and

$$PS_L(x^0, y^0, p^1, p^0) = \frac{y^{0T} [p^1/D_p^0(x^0, p^1)]}{y^{0T} [p^0/D_p^0(x^0, p^0)]} \leq 1 \quad (19)$$

is a Shephard-Laspeyres output price mix index that evaluates the impact on revenue of a change in the output price mix. Inserting (18) into (17) generates a base period decomposition of revenue change

$$\frac{R^1}{R^0} = P_K^0(x^0, p^1, p^0) \times Y_p \times PS_L(x^0, y^0, p^1, p^0). \quad (20)$$

Since $D_p^0(x, p)$ is homogeneous of degree +1 in p , $PS_L(x^0, y^0, p^1, p^0) = 1$ if, and only if, $p^1 = \lambda p^0$, with $\lambda > 0$, and then $R^1/R^0 = P_K^0(x^0, p^1, p^0) \times Y_p$. Thus $P_K^0(x^0, p^1, p^0)$ satisfies the product test with Y_p if, and only if, the output price mix is unchanged between base and comparison periods. Otherwise revenue change has three drivers.

Moving from base period to comparison period, a Konüs comparison period dual output price index is defined as

$$P_K^1(x^1, p^1, p^0) = \frac{D_p^1(x^1, p^1)}{D_p^1(x^1, p^0)}, \quad (21)$$

which is independent of x^1 if and only if comparison period production technology is output homothetic. We decompose revenue change as

$$\frac{R^1}{R^0} = P_K^1(x^1, p^1, p^0) \times YI^1(x^1, y^1, y^0, p^1, p^0), \quad (22)$$

in which $YI^1(x^1, y^1, y^0, p^1, p^0)$ is a comparison period implicit output quantity index. We express this index as

$$\begin{aligned} YI^1(x^1, y^1, y^0, p^1, p^0) &= \frac{R^1/R^0}{P_K^1(x^1, p^1, p^0)} \\ &= \frac{p^{0T} y^1}{p^{0T} y^0} \times \frac{y^{1T} p^1}{y^{1T} p^0} \times \frac{D_p^1(x^1, p^0)}{D_p^1(x^1, p^1)} \\ &= Y_L \times PS_p(x^1, y^1, p^1, p^0), \end{aligned} \quad (23)$$

in which $Y_L = p^{0T} y^1 / p^{0T} y^0$ is a Laspeyres output quantity index and

$$PS_P(x^1, y^1, p^1, p^0) = \frac{y^{1T} [p^1 / D_p^1(x^1, p^1)]}{y^{1T} [p^0 / D_p^1(x^1, p^0)]} \geq 1 \quad (24)$$

is a Shephard-Paasche output price mix index. Substituting (23) into (22) generates a second decomposition of revenue change

$$\frac{R^1}{R^0} = P_K^1(x^1, p^1, p^0) \times Y_L \times PS_P(x^1, y^1, p^1, p^0), \quad (25)$$

the product of a Konüs comparison period dual output price index, a Laspeyres output quantity index, and a Shephard-Paasche output price mix index that provides a second measure of the impact on revenue of a change in the output price mix. A Konüs comparison period dual output price index satisfies the product test with a Laspeyres output quantity index if, and only if, the output price mix is unchanged between base and comparison periods.

Taking the geometric mean of (20) and (25) generates the final decomposition of revenue change

$$\begin{aligned} \frac{C^1}{C^0} &= [P_K^0(x^0, p^1, p^0) \times P_K^1(x^1, p^1, p^0)]^{1/2} \times [Y_P \times Y_L]^{1/2} \\ &\quad \times [PS_L(x^0, y^0, p^1, p^0) \times PS_P(x^1, y^1, p^1, p^0)]^{1/2} \\ &= P_K(x^1, x^0, p^1, p^0) \times Y_F \times PS_F(x^1, x^0, y^1, y^0, p^1, p^0). \quad (26) \end{aligned}$$

Revenue change is expressed as the product of a Konüs dual output price index, a Fisher output quantity index, and a Shephard-Fisher output price mix index. The Konüs dual output price index is independent of input quantities if both base period and comparison period technologies are output homothetic. It satisfies the product test with the Fisher output quantity index if, and only if, $PS_F(x^1, x^0, y^1, y^0, p^1, p^0) = 1$. Thus $PS_L(x^0, y^0, p^1, p^0) = PS_P(x^1, y^1, p^1, p^0) = 1$ is sufficient, but not necessary, for satisfaction of the product test, a necessary and sufficient condition being $PS_L(x^0, y^0, p^1, p^0) = 1/PS_P(x^1, y^1, p^1, p^0)$.

2.3. A Price Recovery Index and the Decomposition of Profitability Change

Combining the geometric mean decomposition of cost change in Section 2.1 with the geometric mean decomposition of revenue change in Section 2.2 generates the following decomposition of profitability change

$$\frac{R^1/C^1}{R^0/C^0} = \frac{P_K(x^1, x^0, p^1, p^0)}{W_K(y^1, y^0, w^1, w^0)} \times \frac{Y_F}{X_F} \times \frac{PS_F(x^1, x^0, y^1, y^0, p^1, p^0)}{WS_F(y^1, y^0, x^1, x^0, w^1, w^0)}. \quad (27)$$

Thus profitability change is the product of a Konüs dual price recovery index, a Fisher productivity index, and a Shephard-Fisher price mix index. A Konüs dual price recovery index is independent of all quantities if, and only if, both base period and comparison period technologies are inversely homothetic. It satisfies the product test with the Fisher productivity index if, and only if the price mix index is unity.

Solving (27) for the Konüs dual productivity index, which is the reciprocal of the Konüs dual price recovery index, yields

$$\frac{W_K(y^1, y^0, w^1, w^0)}{P_K(x^1, x^0, p^1, p^0)} = \rho(y^1, y^0, x^1, x^0, p^1, p^0, w^1, w^0) \times \frac{Y_F}{X_F}, \quad (28)$$

in which

$$\rho(y^1, y^0, x^1, x^0, p^1, p^0, w^1, w^0) = \frac{PS_F(x^1, x^0, y^1, y^0, p^1, p^0)/WS_F(y^1, y^0, x^1, x^0, w^1, w^0)}{(R^1/C^1)/(R^0/C^0)}.$$

Expression (28) provides the relationship between the Konüs dual productivity index and a Fisher productivity index. The numerator of ρ takes a value of one if the Fisher price mix index is unity, a requirement that imposes restrictions on the behaviour of output and input prices. And since $R^h = (1 + g^h)C^h$, $h = 1,0$ where g expresses the margin of the production unit, the denominator takes a value of one when the margin is unchanged which, as we noted in our discussion of Siegel (1961) in Section 1, does not require $R^h = C^h$, $h = 1,0$. This requirement does, however, preclude any trend in market power. Both conditions are most likely to occur when the analysis is conducted over short time periods or in sectors not subject to frequent disruptive changes to the two price mixes. It is possible to test the hypothesis that $\rho(y^1, y^0, x^1, x^0, p^1, p^0, w^1, w^0) = 1$, and therefore that $W_K(y^1, y^0, w^1, w^0)/P_K(x^1, x^0, p^1, p^0) = Y_F/X_F$.

Also since $\frac{Y_F}{X_F} = \frac{R^1/C^1}{R^0/C^0} \times \frac{W_F}{P_F}$ it follows that

$$\frac{W_K(y^1, y^0, w^1, w^0)}{P_K(x^1, x^0, p^1, p^0)} = \frac{PS_F(x^1, x^0, y^1, y^0, p^1, p^0)}{WS_F(y^1, y^0, x^1, x^0, w^1, w^0)} \times \frac{W_F}{P_F}, \quad (29)$$

which provides the relationship between the Konüs dual productivity index and the Fisher dual productivity index, which is the reciprocal of the Fisher price recovery index. Equality between the two productivity indexes is also a testable hypothesis.

As we show in the next section, relationship (28) is essential in deriving the desired relationship between the Konüs dual productivity index and a Malmquist productivity index, as well as between each of these theoretical productivity indexes and their empirical Fisher counterparts.

3. The Relationship between Dual and Primal Productivity Indexes

The input distance function provides the foundation for a Malmquist (1953) consumer standard of living index, which in our context is a producer input quantity index, given in geometric mean form by

$$\begin{aligned} X_M(y^1, y^0, x^1, x^0) &= [X_M^0(y^0, x^1, x^0) \times X_M^1(y^1, x^1, x^0)]^{1/2} \\ &= \left[\frac{D_i^0(y^0, x^1)}{D_i^0(y^0, x^0)} \times \frac{D_i^1(y^1, x^1)}{D_i^1(y^1, x^0)} \right]^{1/2}. \end{aligned} \quad (30)$$

The output distance function provides the foundation for an analogous producer output quantity index, given in geometric mean form by

$$\begin{aligned} Y_M(x^1, x^0, y^1, y^0) &= [Y_M^0(x^0, y^1, y^0) \times Y_M^1(x^1, y^1, y^0)]^{1/2} \\ &= \left[\frac{D_o^0(x^0, y^1)}{D_o^0(x^0, y^0)} \times \frac{D_o^1(x^1, y^1)}{D_o^1(x^1, y^0)} \right]^{1/2}. \end{aligned} \quad (31)$$

Diewert (1992) and, more explicitly, Bjurek (1996) defined a Malmquist productivity index as

$$\frac{Y_M(x^1, x^0, y^1, y^0)}{X_M(y^1, y^0, x^1, x^0)} = \left[\frac{D_o^0(x^0, y^1)/D_o^0(x^0, y^0)}{D_i^0(y^0, x^1)/D_i^0(y^0, x^0)} \times \frac{D_o^1(x^1, y^1)/D_o^1(x^1, y^0)}{D_i^1(y^1, x^1)/D_i^1(y^1, x^0)} \right]^{1/2}. \quad (32)$$

We want to derive the relationship between the Konüs dual productivity index $W_K(y^1, y^0, w^1, w^0)/P_K(x^1, x^0, p^1, p^0)$ and the Malmquist productivity index $Y_M(x^1, x^0, y^1, y^0)/X_M(y^1, y^0, x^1, x^0)$. We have related the Konüs dual productivity index to profitability change in Section 2.3. We next relate the Malmquist productivity index to profitability change, and from these two relationships it is possible to relate Konüs dual and Malmquist productivity indexes.⁵

Grifell-Tatjé and Lovell (2015, 2016) have expressed profitability change as

$$\frac{R^1/C^1}{R^0/C^0} = \frac{Y_M(x^1, x^0, y^1, y^0)}{X_M(y^1, y^0, x^1, x^0)} \times \frac{P_F}{W_F} \times \frac{YM_F(x^1, x^0, p^1, p^0, y^1, y^0)}{XM_F(y^1, y^0, w^1, w^0, x^1, x^0)}, \quad (33)$$

and solving (33) for the Malmquist productivity index yields

$$\frac{Y_M(x^1, x^0, y^1, y^0)}{X_M(y^1, y^0, x^1, x^0)} = \delta(y^1, y^0, x^1, x^0, p^1, p^0, w^1, w^0) \times \frac{W_F}{P_F}, \quad (34)$$

in which

$$\delta(y^1, y^0, x^1, x^0, p^1, p^0, w^1, w^0) = \frac{(R^1/C^1)/(R^0/C^0)}{YM_F(x^1, x^0, p^1, p^0, y^1, y^0)/XM_F(y^1, y^0, w^1, w^0, x^1, x^0)},$$

and $YM_F(x^1, x^0, p^1, p^0, y^1, y^0)/XM_F(y^1, y^0, w^1, w^0, x^1, x^0)$ is a Malmquist-Fisher quantity mix index analogous to the Shephard-Fisher price mix index in (27). Expression (34) provides the relationship between a Malmquist productivity index and a Fisher dual productivity index. And since $\frac{W_F}{P_F} = \frac{Y_F}{X_F} / \left[\frac{R^1/C^1}{R^0/C^0} \right]$ it follows that

$$\frac{Y_M(x^1, x^0, y^1, y^0)}{X_M(y^1, y^0, x^1, x^0)} = \frac{Y_F/X_F}{YM_F(x^1, x^0, p^1, p^0, y^1, y^0)/XM_F(y^1, y^0, w^1, w^0, x^1, x^0)}, \quad (35)$$

which provides the relationship between a Malmquist productivity index and a Fisher productivity index. The two productivity indexes are equal if and only if $YM_F = XM_F$. However if variation in the quantity mixes is approximately offsetting, in the sense that any change in the output quantity mix has approximately the same effect on revenue

as any change in the input quantity mix has on cost, then theoretical Malmquist and empirical Fisher productivity indexes are approximately equal.

Expressions (28) and (29) relate a theoretical Konüs dual productivity index to empirical primal and dual Fisher productivity indexes, and expressions (34) and (35) relate a theoretical Malmquist productivity index to empirical dual and primal Fisher productivity indexes. Conditions for equality between theoretical and empirical productivity indexes involve the *behaviour of the data*, expressed in terms of the trend in the margin of revenue over cost and trends in quantity and price mixes. No assumption of optimizing behaviour or about the structure of technology is required. In contrast, Färe and Grosskopf (1992) and Balk (1993) relate a theoretical input-oriented CCD Malmquist productivity index to a Fisher productivity index. They impose no restrictions on the behaviour of the data. Their conditions for equality involve *optimizing behaviour*, requiring allocative efficiency, both within and between periods. Finally, to complete the linkages among productivity indexes, Färe and Grosskopf (1996) relate the theoretical Malmquist and CCD Malmquist productivity indexes, with equality requiring *restrictions on the structure of technology and the way it changes*, namely constant returns to scale and inverse homotheticity. O'Donnell (2012) found constant returns to scale and the absence of technical change to suffice for equality, a requirement Mizobuchi (2016) relaxed to constant returns to scale and Hicks-neutral technical change.

Finally, equating expressions (35) and (28) yields the relationship between the Konüs dual and Malmquist productivity indexes

$$\frac{W_K(y^1, y^0, w^1, w^0)}{P_K(x^1, x^0, p^1, p^0)} = \gamma(y^1, y^0, x^1, x^0, p^1, p^0, w^1, w^0) \times \frac{Y_M(x^1, x^0, y^1, y^0)}{X_M(y^1, y^0, x^1, x^0)} \quad (36)$$

in which

$$\begin{aligned} \gamma(y^1, y^0, x^1, x^0, p^1, p^0, w^1, w^0) \\ = \frac{\frac{PS_F(x^1, x^0, y^1, y^0, p^1, p^0)}{WS_F(y^1, y^0, x^1, x^0, w^1, w^0)} \times \frac{YM_F(x^1, x^0, p^1, p^0, y^1, y^0)}{XM_F(y^1, y^0, w^1, w^0, x^1, x^0)}}{(R^1/C^1)/(R^0/C^0)}. \end{aligned}$$

Thus equality of the Konüs dual and Malmquist productivity indexes requires the product of the Shephard-Fisher price mix index and the Malmquist-Fisher quantity mix index to equal profitability change. Even if the accounting identity holds, a condition satisfied by the Jorgenson and Griliches data, equality between Konüs dual and Malmquist productivity indexes is not guaranteed. Equality also would require any departure from price mix constancy to be offset by a reciprocal departure from quantity

mix constancy. However as above the hypothesis that $\gamma(y^1, y^0, x^1, x^0, p^1, p^0, w^1, w^0) = 1$, and thus $\frac{W_K(y^1, y^0, w^1, w^0)}{P_K(x^1, x^0, p^1, p^0)} = \frac{Y_M(x^1, x^0, y^1, y^0)}{X_M(y^1, y^0, x^1, x^0)}$, is testable.

The relationships linking the two theoretical productivity indexes with the two empirical productivity indexes are collected in Table 1, in which each cell refers to an equation in this and the previous section.

Insert Table 1 about here

4. Implementation

The data set we use comprises a single time series, and so we resort to a global productivity index proposed by Pastor and Lovell (2005) defined on an intertemporal production set $T^G = \text{conv}\{T^1 \cup \dots \cup T^T\}$ introduced by Tulkens and Vanden Eeckaut (1995). A global productivity index has the virtue of being a fixed base index, and therefore is circular, whereas a contemporaneous Malmquist productivity index is not, unless severe restrictions are imposed on the structure of technology and the way it changes (Färe and Grosskopf, 1996) or on the data and the way they change (Pastor and Lovell, 2019b).

With a global technology defined on a single time series of data, the dual productivity index is expressed as

$$\frac{W_K^G(y^{t+1}, y^t, w^{t+1}, w^t)}{P_K^G(x^{t+1}, x^t, p^{t+1}, p^t)} = \frac{[W_K^G(y^t, w^{t+1}, w^t) \times W_K^G(y^{t+1}, w^{t+1}, w^t)]^{1/2}}{[P_K^G(x^t, p^{t+1}, p^t) \times P_K^G(x^{t+1}, p^{t+1}, p^t)]^{1/2}}, \quad (37)$$

and the primal productivity index is expressed as

$$\frac{Y_M^G(x^{t+1}, x^t, y^{t+1}, y^t)}{X_M^G(y^{t+1}, y^t, x^{t+1}, x^t)} = \frac{[Y_M^G(x^t, y^{t+1}, y^t) \times Y_M^G(x^{t+1}, y^{t+1}, y^t)]^{1/2}}{[X_M^G(y^t, x^{t+1}, x^t) \times X_M^G(y^{t+1}, x^{t+1}, x^t)]^{1/2}}, \quad (38)$$

in which the superscript “G” indicates the global technology consisting of data from all time periods, and $t = 0, 1, \dots, T-1$.

Both productivity indexes are based on distance functions, and we use data envelopment analysis (DEA) (Charnes, Cooper and Rhodes, 1978; Banker, Charnes and Cooper, 1984) to estimate these distance functions. The envelopment program for the two distance functions comprising the dual Konüs input price index $W_K^G(y^{t+1}, y^t, w^{t+1}, w^t)$ is

Envelopment Programs for $W_K^G(y^{t+1}, y^t, w^{t+1}, w^t) = D_w^G(y^t, w^{t+1})/D_w^G(y^t, w^t)$

$$[D_w^G(y^t, w^s)]^{-1} = \min \mu$$

Subject to

$$\sum_{t=1}^{68} \lambda^t w_n^t \leq \mu w_n^s \quad n = 1, \dots, 3$$

$$y_m^t \leq \sum_{t=1}^{68} \lambda^t y_m^t \quad m = 1, \dots, 3$$

$$\sum_{t=1}^{68} \lambda^t = 1, \lambda^t \geq 0 \quad t = 1, \dots, 68$$

$s=t+1$ in numerator, $s=t$ in denominator

and the envelopment program for the two distance functions comprising the dual Konüs output price index $P_K^G(x^{t+1}, x^t, p^{t+1}, p^t)$ is

Envelopment Programs for $P_K^G(x^{t+1}, x^t, p^{t+1}, p^t) = D_p^G(x^t, p^{t+1})/D_p^G(x^t, p^t)$

$$[D_p^G(x^t, p^s)]^{-1} = \max v$$

Subject to

$$v p_m^s \leq \sum_{t=1}^{68} \lambda^t p_m^t \quad m = 1, \dots, 3$$

$$\sum_{t=1}^{68} \lambda^t x_n^t \leq x_n^t \quad n = 1, \dots, 3$$

$$\sum_{t=1}^{68} \lambda^t = 1, \lambda^t \geq 0 \quad t = 1, \dots, 68$$

$s=t+1$ in numerator, $s=t$ in denominator

The envelopment programs for the distance functions comprising the primal Malmquist quantity indexes have the same structure, with input quantities replacing input prices in the first pair of programs, and output quantities replacing output prices in the second pair of programs.

5. The Data and the Results

Wang, Nehring and Mosheim (2018) provide background information generated by the US Department of Agriculture's Economic Research Service (ERS) on the productivity performance of US agriculture over the 68 year period 1948-2015. Using ERS

aggregate quantity indexes, normalized to unity in 2005, of total output and total farm input, Wang, Nehring and Mosheim report output growth of 170% and input growth of 7%, from which they infer total factor productivity growth of 152%, or 1.38% per annum. Using ERS aggregate price indexes of total output and total farm input, also normalized to unity in 2005, we calculate input price growth of 788% and output price growth of 253%, implying growth in the ratio of input prices to output prices of 152%, or 1.38% per annum, exactly the same as the ERS estimate of total factor productivity growth.⁶

This data source also contains price and implicit quantity indexes, normalized to unity in 2005, of three outputs (livestock and products, crops, and other farm-related outputs) and three farm inputs (capital, labor, and intermediate goods). We use these indexes to estimate the two theoretical productivity indexes and to calculate the two empirical productivity indexes appearing in Table 1. The two decompositions of profitability change (which is one by construction) in expressions (27) and (33) are collected in Table 2 for the entire 1948-2015 period, and for peak-to-peak sub-periods used by the ERS. The decomposition of profitability change in expression (27), normalized to unity in 1949 and cumulated to 2015, is depicted in Figure 1.

Insert Table 2 about here

Insert Figure 1 about here

Over the entire period our Konüs-Shephard dual and the Malmquist primal productivity estimates bound the Fisher estimates very closely, with average annual growth rates of $W_K^G/P_K^G = 1.42\% > Y_F/X_F = 1.38\% > Y_M^G/X_M^G = 1.13\%$. The closeness of the two bounding estimates is due to the very small contributions of the two mix indexes, with average values of the price mix index $PS_F^G/WS_F^G = 1.0007$ and average values of the quantity mix index $YM_F^G/XM_F^G = 1.0025$. Inserting the relevant mix indexes into expression (28) yields $W_K^G/P_K^G = 1.0007 \times Y_F/X_F = 1.0007 \times W_F/P_F$ and into expressions (34) and (35) yields $Y_M^G/X_M^G = 1.0025 \times W_F/P_F = 1.0025 \times Y_F/X_F$. Finally, from expression (36) the relation between dual Konüs and primal Malmquist productivity indexes is $W_K^G/P_K^G = 1.0032 \times Y_M^G/X_M^G$. The small differences among the four productivity indexes occurs despite modest changes in the output mix, with crop production growing faster than livestock production, and “significant” changes in the input mix, with intermediate inputs and capital growing and land and labor declining, documented by the ERS.⁷

Insert Figure 2 about here

Figure 2 portrays three cumulative productivity indexes: two theoretical indexes, the dual Konüs index and the primal Malmquist index, and one empirical index, the Fisher index (primal and dual Fisher indexes are equal since the data satisfy

zero annual profit by construction). The three indexes grow together until approximately 1982-83, when the Malmquist index begins to lag the other two indexes. This pattern suggests that most of the impact of the quantity mix index in equation (34), and the combined quantity mix index and price mix index in equation (36) has occurred since then. This has indeed occurred, with YM_F/XM_F (1984-2016) > YM_F/XM_F (1949-1983) consistent with $Y_M/X_M < W_F/P_F$ during (1984-2016), and YM_F/XM_F (1984-2016) > YM_F/XM_F (1949-1983) and PS_F/WS_F (1984-2016) > PS_F/WS_F (1949-1983) consistent with $Y_M/X_M < W_K/P_K$ during (1984-2016).

The period averages in Figure 2 conceal sub-period variability and dual/primal differences. Thus, for example, the Konüs dual productivity index grew at a rate of 4.4% during 1981-1990 and declined at a rate of 1.6% during 1969-1973, while the Malmquist primal productivity index grew at a rate of 4.5% during 1979-1981 and declined at a rate of 0.2% during 1973-1979. These period averages also conceal substantial annual productivity declines that occurred in 1978, 1983, 2009, 2014 and 2015. These findings are roughly in agreement with those of the ERS, which finds substantial sub-period productivity variation, with rapid productivity growth in 1979-1981 and productivity declines in 1973-1979 and 2007-2015. It also finds substantial annual productivity declines in 1978, 1983, 1995 and 2014 associated with transitory events such as energy shocks or supply shortages due to adverse weather conditions.

The cumulated results depicted in Figures 1 and 2 offer a compelling illustration of the distributive impacts of productivity growth emphasized by Siegel (1955; 53) and Kendrick (1961; 111). In US agriculture sustained productivity growth meant that by 2015 input prices were over 2.5 times higher relative to output prices than they were in 1949.

Insert Tables 3 and 4 about here

Tables 3 and 4 decompose the sources of profitability change results in Table 2 into the sources of revenue change and the sources of cost change. In Table 3 theoretical and empirical indexes agree that output price growth has dominated output quantity growth as the prime source of revenue growth, with $P_K^G > Y_F$ and $P_F > Y_M^G$. Table 3 tells a similar story: input price growth has been the primary driver of cost growth, with $W_K^G > X_F$ and $W_F > X_M^G$.

6. Conclusions

In this paper we propose a new dual productivity index based on the work of Konüs and Shephard. The index exploits the fact, noted by Shephard, that the cost functions Konüs used to define a cost of living index in quantity space, with input quantities as the choice variables, can be re-interpreted as input distance functions in price space, with input prices as the choice variables. These input distance functions can then be

used to construct an input price index. Similarly revenue functions can be re-interpreted as output distance functions in output price space and used to construct an output price index. This leads to the definition of a dual productivity index as the ratio of an input price index to an output price index.

We have derived the relationship between our Konüs-Shephard dual productivity index and a Malmquist primal productivity index, and we have demonstrated that the two are not generally equal. The relationship between the two depends on two features of the underlying data: the trend in the margin of revenue over cost, and the trends in the output quantity mix and the input quantity mix. We have also shown that the dual and primal theoretical indexes are not generally equal to their Fisher empirical counterparts, with the relationships depending on the same features of the underlying data.

Our new dual productivity index is a theoretical index, in contrast to the empirical Fisher index, and so must be estimated rather than calculated. We have used a time series of US agricultural data to estimate our dual and primal productivity indexes, and to calculate Fisher primal and dual productivity indexes. Although no two productivity indexes are equal, all four have approximately the same magnitude. This finding is attributable to two features of the data mentioned above: the margin is unity by construction (i.e., the accounting identity is satisfied in all years), and departures of the output price mix from constancy have been approximately offset by departures of the input price mix from constancy, and similarly for output and input quantity mixes.

We conclude by recalling that Fourastié motivated a dual approach to productivity measurement by arguing that in some circumstances prices are easier to measure, or measured more accurately, than quantities. Later Hsieh estimated primal and dual productivity indexes for four East Asian countries, and found the two to diverge dramatically in Singapore, with estimated dual productivity growth and primal productivity decline. He attributed this divergence to a significant overstatement of investment spending in the Singapore national accounts that artificially depressed estimated primal productivity change.

However all previous dual and indirect productivity indexes cited in Section 1 rely on quantity data, typically in the form of output revenue shares used to weight output price changes and input cost shares used to weight input price changes. Indeed our dual productivity index uses output quantities in the construction of a dual input price index and input quantities in the construction of a dual output price index. This universal reliance on quantity data requires either a belief in the accuracy of national (or sectoral, or business) account quantity data, or a conviction in the inaccuracy of such data combined with a willingness to develop and defend alternative quantity data, as Hsieh has done. An alternative approach to generating and defending alternative quantity data would exploit the multiplier duals to the DEA envelopment programs in Section 4. These multiplier programs provide shadow output quantities and shadow input quantities, which may be compared to observed quantity data as a credibility check.

This dilemma suggests that a logical extension of the dual productivity index introduced in this paper would be the development of an alternative theory-based dual productivity index that is entirely independent of quantity data. Such an index would be analogous to a Malmquist productivity index, which is entirely independent of price data. Our dual productivity index is independent of quantity data if and only if production technology is inversely homothetic. It would be desirable to develop a theory-based dual productivity index independent of quantity data that does not rely on such an unrealistic assumption about the structure of production technology.

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| Productivity Indexes | | Theoretical | | Empirical | |
|----------------------|-----------|-------------|-----------|-----------|-----------|
| | | W_K/P_K | Y_M/X_M | W_F/P_F | Y_F/X_F |
| Theoretical | W_K/P_K | | (36) | (29) | (28) |
| | Y_M/X_M | (36) | | (34) | (35) |
| Empirical | W_F/P_F | (29) | (34) | | (29) |
| | Y_F/X_F | (28) | (35) | (29) | |

Table 1 Relationships Linking Four Productivity Indexes

Table 2 Sources of Profitability Change

| | $\frac{P_K^G(x^1, x^0, p^1, p^0)}{W_K^G(y^1, y^0, w^1, w^0)}$ | $\frac{P_S^G(x^1, x^0, y^1, y^0, p^1, p^0)}{W_S^G(y^1, y^0, x^1, x^0, w^1, w^0)}$ | $\frac{Y_F}{X_F}$ | $\frac{R^1/R^0}{C^1/C^0}$ | $\frac{Y_M^G(x^1, x^0, y^1, y^0)}{X_M^G(y^1, y^0, x^1, x^0)}$ | $\frac{YM_F^G(x^1, x^0, p^1, p^0, y^1, y^0)}{XM_F^G(y^1, y^0, w^1, w^0, x^1, x^0)}$ | $\frac{P_F}{W_F}$ |
|------------------|---------------------------------------------------------------|-----------------------------------------------------------------------------------|-------------------|---------------------------|---------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------|
| 1948-53 | 0.997 | 1.000 | 1.003 | 1.000 | 1.015 | 0.988 | 0.997 |
| 1953-57 | 0.994 | 1.001 | 1.005 | 1.000 | 1.003 | 1.002 | 0.995 |
| 1957-60 | 0.965 | 1.006 | 1.030 | 1.000 | 1.009 | 1.021 | 0.971 |
| 1960-66 | 0.980 | 1.008 | 1.012 | 1.000 | 1.011 | 1.001 | 0.988 |
| 1966-69 | 1.002 | 0.976 | 1.023 | 1.000 | 1.006 | 1.017 | 0.978 |
| 1969-73 | 1.016 | 0.963 | 1.022 | 1.000 | 1.010 | 1.011 | 0.979 |
| 1973-79 | 0.983 | 1.010 | 1.008 | 1.000 | 0.998 | 1.010 | 0.992 |
| 1979-81 | 0.967 | 0.996 | 1.039 | 1.000 | 1.045 | 0.994 | 0.963 |
| 1981-90 | 0.958 | 1.022 | 1.021 | 1.000 | 1.018 | 1.004 | 0.979 |
| 1990-00 | 0.980 | 1.005 | 1.016 | 1.000 | 1.012 | 1.003 | 0.985 |
| 2000-07 | 0.987 | 1.004 | 1.009 | 1.000 | 1.019 | 0.991 | 0.991 |
| 2007-15 | 1.010 | 0.985 | 1.005 | 1.000 | 1.004 | 1.001 | 0.995 |
| 1948-2015 | 0.986 | 1.001 | 1.014 | 1.000 | 1.011 | 1.003 | 0.986 |
| Max | 1.208 | 1.112 | 1.164 | 1.000 | 1.100 | 1.118 | 1.154 |
| Min | 0.815 | 0.832 | 0.867 | 1.000 | 0.922 | 0.892 | 0.859 |

Table 3 Sources of Revenue Change

| | $P_K^G(x^1, x^0, p^1, p^0)$ | $PS_F^G(x^1, x^0, y^1, y^0, p^1, p^0)$ | Y_F | R^1/R^0 | $Y_M^G(x^1, x^0, y^1, y^0)$ | $YM_F^G(x^1, x^0, p^1, p^0, y^1, y^0)$ | P_F |
|------------------|-----------------------------|----------------------------------------|--------------|--------------|-----------------------------|----------------------------------------|--------------|
| 1948-53 | 0.997 | 0.975 | 1.010 | 0.982 | 1.019 | 0.991 | 0.972 |
| 1953-57 | 1.003 | 0.973 | 1.005 | 0.981 | 1.001 | 1.004 | 0.977 |
| 1957-60 | 1.008 | 0.993 | 1.038 | 1.039 | 1.013 | 1.024 | 1.001 |
| 1960-66 | 1.024 | 1.008 | 1.011 | 1.044 | 1.010 | 1.001 | 1.032 |
| 1966-69 | 1.028 | 0.974 | 1.023 | 1.024 | 1.007 | 1.016 | 1.001 |
| 1969-73 | 1.106 | 1.014 | 1.025 | 1.150 | 1.017 | 1.009 | 1.122 |
| 1973-79 | 1.067 | 0.979 | 1.025 | 1.069 | 1.017 | 1.007 | 1.044 |
| 1979-81 | 0.995 | 1.042 | 1.026 | 1.064 | 1.027 | 0.999 | 1.037 |
| 1981-90 | 1.011 | 0.997 | 1.008 | 1.016 | 1.009 | 0.999 | 1.008 |
| 1990-00 | 0.998 | 0.996 | 1.018 | 1.011 | 1.017 | 1.001 | 0.993 |
| 2000-07 | 1.036 | 1.008 | 1.010 | 1.056 | 1.014 | 0.996 | 1.045 |
| 2007-15 | 1.027 | 0.999 | 1.007 | 1.033 | 1.013 | 0.994 | 1.025 |
| 1948-2015 | 1.024 | 0.995 | 1.015 | 1.034 | 1.013 | 1.002 | 1.019 |
| Max | 1.304 | 1.097 | 1.140 | 1.429 | 1.077 | 1.118 | 1.364 |
| Min | 0.867 | 0.899 | 0.860 | 0.867 | 0.965 | 0.891 | 0.880 |

Table 4 Sources of Cost Change

| | $W_K^G(y^1, y^0, w^1, w^0)$ | $WS_F^G(y^1, y^0, x^1, x^0, w^1, w^0)$ | X_F | C^1/C^0 | $X_M^G(y^1, y^0, x^1, x^0)$ | $XM_F^G(y^1, y^0, w^1, w^0, x^1, x^0)$ | W_F |
|------------------|-----------------------------|----------------------------------------|--------------|--------------|-----------------------------|----------------------------------------|--------------|
| 1948-53 | 1.000 | 0.975 | 1.007 | 0.982 | 1.004 | 1.002 | 0.975 |
| 1953-57 | 1.010 | 0.972 | 1.000 | 0.981 | 0.998 | 1.002 | 0.982 |
| 1957-60 | 1.045 | 0.986 | 1.008 | 1.039 | 1.005 | 1.003 | 1.031 |
| 1960-66 | 1.045 | 1.000 | 0.999 | 1.044 | 0.999 | 1.000 | 1.045 |
| 1966-69 | 1.026 | 0.998 | 1.000 | 1.024 | 1.001 | 0.999 | 1.024 |
| 1969-73 | 1.089 | 1.053 | 1.004 | 1.150 | 1.006 | 0.997 | 1.146 |
| 1973-79 | 1.085 | 0.969 | 1.017 | 1.069 | 1.020 | 0.997 | 1.052 |
| 1979-81 | 1.029 | 1.046 | 0.988 | 1.064 | 0.983 | 1.005 | 1.077 |
| 1981-90 | 1.054 | 0.976 | 0.987 | 1.016 | 0.991 | 0.996 | 1.029 |
| 1990-00 | 1.018 | 0.991 | 1.002 | 1.011 | 1.005 | 0.998 | 1.009 |
| 2000-07 | 1.050 | 1.004 | 1.001 | 1.056 | 0.995 | 1.006 | 1.055 |
| 2007-15 | 1.017 | 1.014 | 1.002 | 1.033 | 1.009 | 0.993 | 1.031 |
| 1948-2015 | 1.039 | 0.995 | 1.001 | 1.034 | 1.002 | 0.999 | 1.033 |
| Max | 1.188 | 1.196 | 1.067 | 1.429 | 1.085 | 1.055 | 1.400 |
| Min | 0.836 | 0.884 | 0.932 | 0.867 | 0.957 | 0.963 | 0.856 |

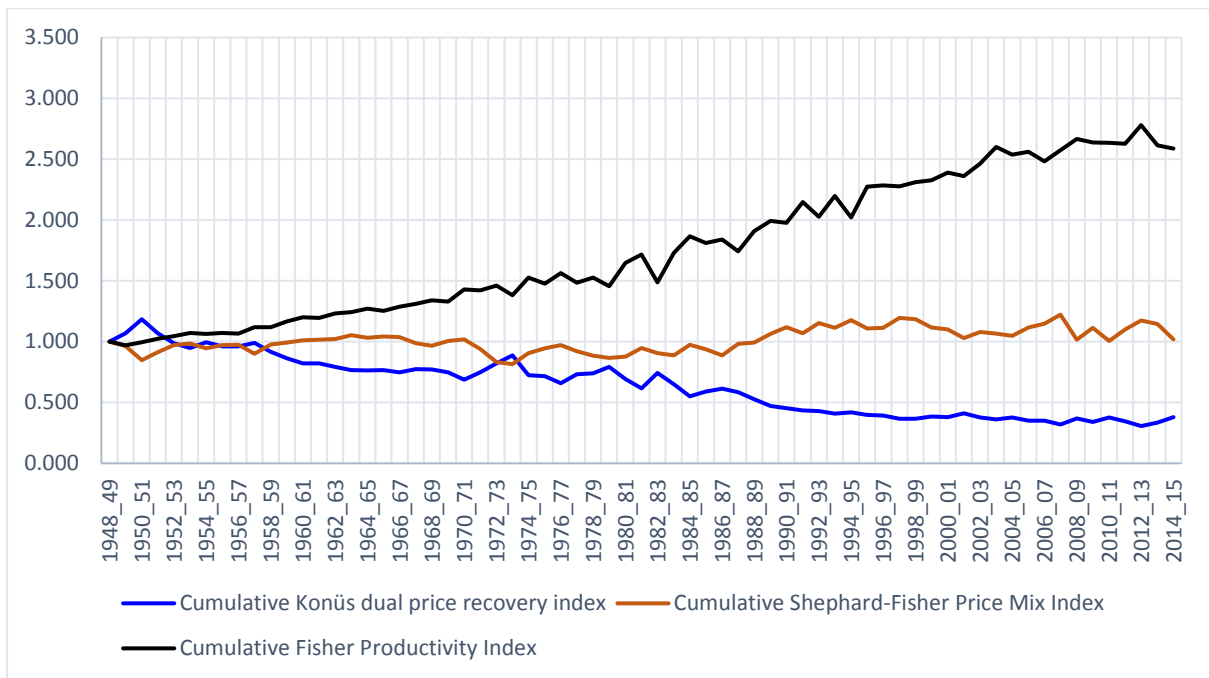


Figure 1 Sources of Profitability Change

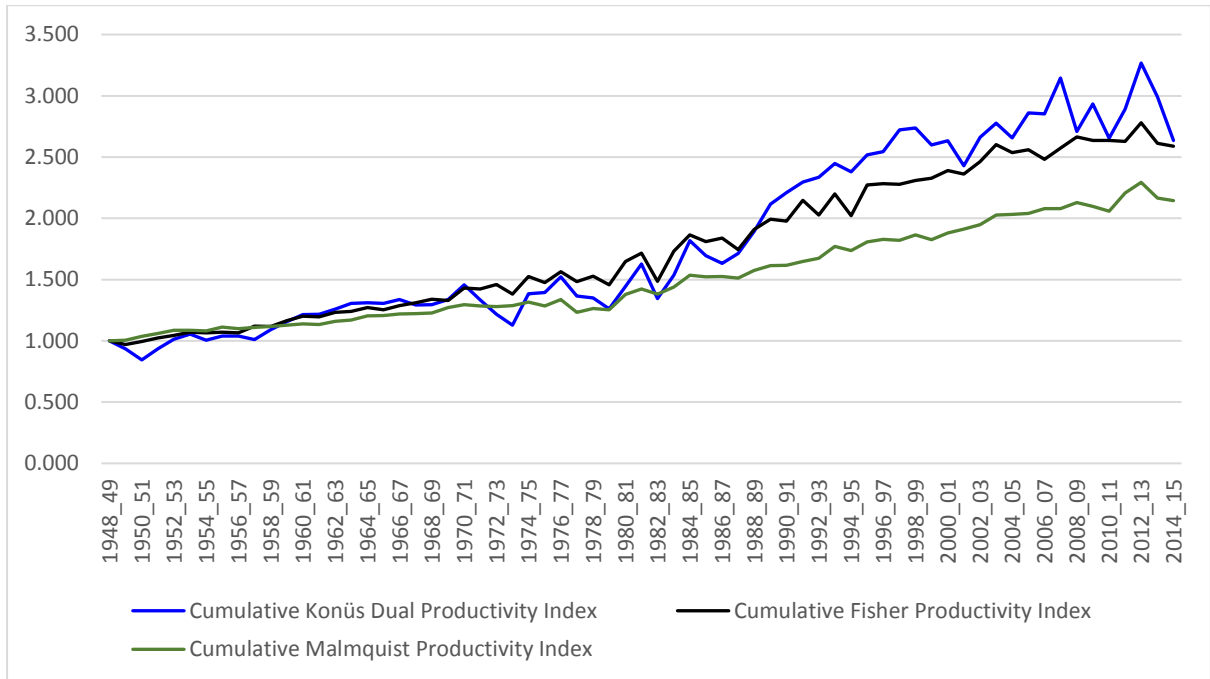


Figure 2 Three Cumulative Productivity Indexes

Endnotes

¹ Davis also emphasized the generation and distribution of non-pecuniary benefits such as improved product quality and improved working conditions.

² Fourastié acknowledged Rostas (1948), Dayre (1951) and Roy (1953) as having inspired his indirect productivity index.

³ A special case of a price recovery index is a terms of trade index, the ratio of an export price index to an import price index.

⁴ A similar sufficiency condition holds for all geometric mean functions. For example, Pastor and Lovell (2019a) have shown that a Fisher price index can be circular even if neither of its Laspeyres and Paasche components is circular.

⁵ The Malmquist productivity index in (32) is simultaneously output- and input-oriented. This desirable feature distinguishes it from an alternative Malmquist productivity index introduced by Caves, Christensen and Diewert (1982), which is either output-oriented or input-oriented, but not both.

⁶ Equality of the ERS implicit quantity-based and price-based productivity indexes is consistent with the data satisfying zero annual profit by construction and the use of Fisher indexes. The ERS agricultural productivity data are publicly available, and can be accessed at <https://www.ers.usda.gov/data-products/agricultural-productivity-in-the-us/>.

⁷ See https://www.ers.usda.gov/webdocs/publications/45387/53417_err189.pdf?v=0.