# Deconstructing the macroeconomics' IS–LM model: Its Assumptions and Simplifications

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The IS-LM model emerges from the academic impact of Keynes' *General Theory of Employment, Interest and Money* (GT). It was developed afterwards, in the late 1930s, starting to be called the Hicks-Hansen model; though other academics such as Harrod and Mead contributed to it. In this regard, the model consisted of a way of mathematising the core of Keynes' GT (though Keynes did not recognise it as such); it was fully developed and consolidated as the academic new paradigm across the 1950s and 1960s. However, it was not (and is not) properly considered as a formalisation of the GT but as an independent development inspired by it. A compromise distinction in this regard has become generally accepted: the label 'economics of Keynes' for referring to his GT and derived developments, and 'Keynesian economics' for the IS-LM construct and its derivatives[1].

With its double equilibrium (Investment=Savings; Money Supply=Demand for Liquidity), the model sort of plays in standard Macroeconomics a parallel role to that of the 'General-Equilibrium-of-competitive-markets' in Economics/Microeconomics. This IS-LM model remains either as fundamental or is simply present in most popular mainstream macroeconomic manuals, as well as those of economic policy –especially monetary policy– texts.

## 1. The previous "45 degrees"' *Aggregate Demand* model

It develops around the idea of equilibrium between *Production* (or *Aggregate Supply*) and *Aggregate Demand*. In this standard paradigm, the meaning of 'equilibrium' is just that of a stable situation where Production's *value* = Aggregate Demand's *value*. Which entails that *Savings* and *Investment* will coincide; and that there is not unwanted increases in firms' stocks, (which in national accounting terms would be [unintended] Investment). These flow variables refer to a given time unit; usually one year, but may also be one quarter, etc.

This preliminary '45$^{o}$ paradigm' builds upon a series of macro-variables definitions ($^{d}$), accounting identities ($^{ai}$), identities or definitions assumed in the model ($^{mi}$), and equilibrium conditions in the model ($^{e}$): (Terminology and simplifications as from Blanchard (2016)'s Macroeconomics book):

[1] It may be seen in this regard: "The Ascent of Keynesianism: The IS-LM Hegemony", at Arnon, 2022:133-149.

- National accounts' identities:

***Production*** of goods (products & services), in market's monetary terms $=^{d}$ $\mathbf{Y}^{*}$.
Composed of: *Consumer* goods + *Investment goods:* $Y^* = Y^*_C + Y^*_I$

**Income** distributed to citizens (salaries, profits, rents, ..) $=^{d}$ **Y**.

*Production* $=^{ai}$ *Income* , $\mathbf{Y^*} =^{ai}$ $\mathbf{Y}$

*Government expenses* ('public *consumption*) $=^{d}$ **G**

- *Net taxes collected*: $\mathbf{T} =^{mi}$ $\mathbf{t} \cdot Y$ – Transfers to private sector
- ***Consumption*** $=^{m}$ Households Consumption + Public consumption; $\mathbf{C} = C_h + G$
- Households Consumption, $C_h =^{mi}$ $C_0 + c_1\,(Y–T)$ ;
  Where $c_1$, (<1), is the *propensity to consume,* regarding Disposable Income, (Y-T).
- Firms' ***Investment*** (value), $I =^{d}$ $Y^*_I$ + (eventually) unintended increases in firms' stocks.
- Firms' *decided/planned Investment* (demand of investment goods) $=^{d}$ $\bar{\mathbf{I}}$
- *Aggregate* ***Demand*** #, $AD =^{mi}$ $(C_h + G) + \bar{I} \equiv [C_0 + c_1\,(Y–T) + G] + \bar{I}$
- *Autonomous Demand*, $\hat{A} =^{mi}$ $C_0 – c_1 \cdot T + G + \bar{I}$
- *Dependent Demand* [2] $DD =^{mi}$ $c_1 \cdot Y$
- Equilibrium condition: $AD =^{e}$ $Y^{*}$ ; which entails, $I = \bar{I}$

(#) in some texts, it is denoted by *Z*

The above identities entail, as can be seen, the following simplifications/ancillary assumptions: 1) There are no indirect taxes (as, f.e., VAT); 2) Income taxes, Profits taxes, and Social Security contributions, are subsumed in $t{\cdot}Y$; and 3) the equilibrium condition entails that no increases in firms' stocks have been produced; i.e., that $Y^*_C$ is equal to *Consumption*, and $Y^*_I$ coincides with the *Investment planed* by firms, $\bar{I}$.

As for the core equilibrium condition, $\boldsymbol{AD} =^{e}$ $Y^*$ it is usually deployed as:

$$\mathbf{AD} =^{mi} \hat{A} + DD \equiv [C_0 - c_1 \cdot T)] + G + \bar{I}\,] + c_1 \cdot Y =^{e} Y^*_C + Y^*_I =^{d} \mathbf{Y^*}$$

In this standard paradigm, the above concept of 'autonomous demand' (Â) plays a core role. Regarding it, an ancillary assumption is made in the model: that its components are (parametrically) stable, in the sense that they remain constant unless a public decision (ΔT, ΔG) or a business undertakings' strategic decision ($\Delta\bar{I}$) is taken. Therefore, its component $C_0$ can be understood as the amount of private consumption associated with a supposed social vital minimum per person. That is, as a *social parameter*, which, as such, may vary in the long long-run and substantively differ among countries (because of significant differences in per-capita income).

[2] These two definitions imply, therefore, the simplification/assumption that *T* is a constant –since Â is assumed to have a constant value (see next standard diagram)–. This, in turn, implies that, when *Y* varies, income tax's rate, *t*, is conveniently modified, in reverse sense, by the government –so to maintain *T* constant–; or *Transfers to private sector* is conveniently modified, to keep *T* unchanged (or a mix of both kinds of governments decisions).

### *1.1 The* **multiplier***, a key analytical piece*

The following step in the model consists of applying it to ask: –starting from an equilibrium situation, $Y^*=AD =^m \hat{A} + c_1 \cdot Y$,– what the consequences would be from a given change (increase) in the *autonomous demand*, $\Delta\hat{A} = \boldsymbol{a}$ € (and, specifically, in its component: Firms' planned investment, $\bar{I}$)? The answer to this is: an induced increase in *Production*, and therefore in *Income*, something higher than the initial $\Delta\hat{A}$; this, because of a *multiplicative* effect occurring over a given time stretch. The value of the *multiplier* depends on parameter $c_1$ (households' propensity to consume). A description of such a multiplicative effect follows:

Starting from an equilibrium situation, $(AD=Y^*)_0$, the immediate consequence of an increase in the aggregate demand ($=\Delta\bar{I},=\Delta\hat{A},=AD,=\boldsymbol{a}$ euros) would be, following the model, an equal increase in firms' *Production* ($\Delta Y^*_1 = a$), within a given 'delivery time'. That, in turn, will translate into an equal increase in *Incomes* ($a$), within a given 'remuneration time', in the way of new Salaries and Profits ($\Delta Y=a$). And this will, in turn, translate into a something lower ($c_1 \cdot a$) increase in household demand for consumer goods & services; i.e., a new, induced, increase in the aggregate demand. This sequence will have taken a given time: the sum of the two above-mentioned time intervals; say, for example, one and a half months. But, because of this new increase in AD (in this case, from household demand) a second cycle starts, where this induced increase in AD, $c_1 \cdot a$, leads to a subsequent (something lower: $c_1^2\ a$) increase in AD. And so on, cycle after cycle –till those successive dwindling increases in AD become insignificant, so reaching a new equilibrium $(AD=Y^*)_1$, at an upper value. (A formal development of the above may be seen in Appendix).

As can be seen, these successive declining increases in AD, ($a + c_1 \cdot a + c_1^2\ a + \ldots$), over $\boldsymbol{n}$ cycles define a geometrical series with ratio $\mathbf{c_1}$. The sum of whose terms is, applying the standard statistical formula in this respect (where the term between brackets is the *multiplicator*, $\boldsymbol{m}$):

$$\text{Sum of increases in AD } = a \cdot \left( \frac{1}{1-c_1} - \frac{c_1^{\,n}}{1-c_1} \right) \equiv a \cdot m$$

Thus, if, for example, $c_1=0.4$, a cycle spans 1.5 months, and the number of cycles is n=6, (9 months in total), the cumulated increases in AD would be equal to $\boldsymbol{a}\cdot(1.656)$. That is, the sum of the increases in AD over 9 months would be ≈1.656 times the initial, punctual, increase in the Autonomous Demand ($\boldsymbol{a}$ €); (the last, 6th, increase in AD would be just around 1% of the initial one). Strictly speaking, this means that after those 9 months an equilibrium, AD=Y*, would have not exactly been reached but it does in practical terms; (for example, extending our scope over 12 cycles (one and a half years), the corresponding figure for the *multiplier,* ***m****,* is 1.66). Obviously, if we simply consider a 'quite large' number of cycles (formally, $n \to \infty$), the second term of the above expression between brackets will be 0; thereby, the *multiplier* would be (this is the usual expression): $\boldsymbol{m} = \mathbf{1/(1\text{-}c_1)} =$ 1.6666.., and the 'excess of Demand over Production' virtually zero.

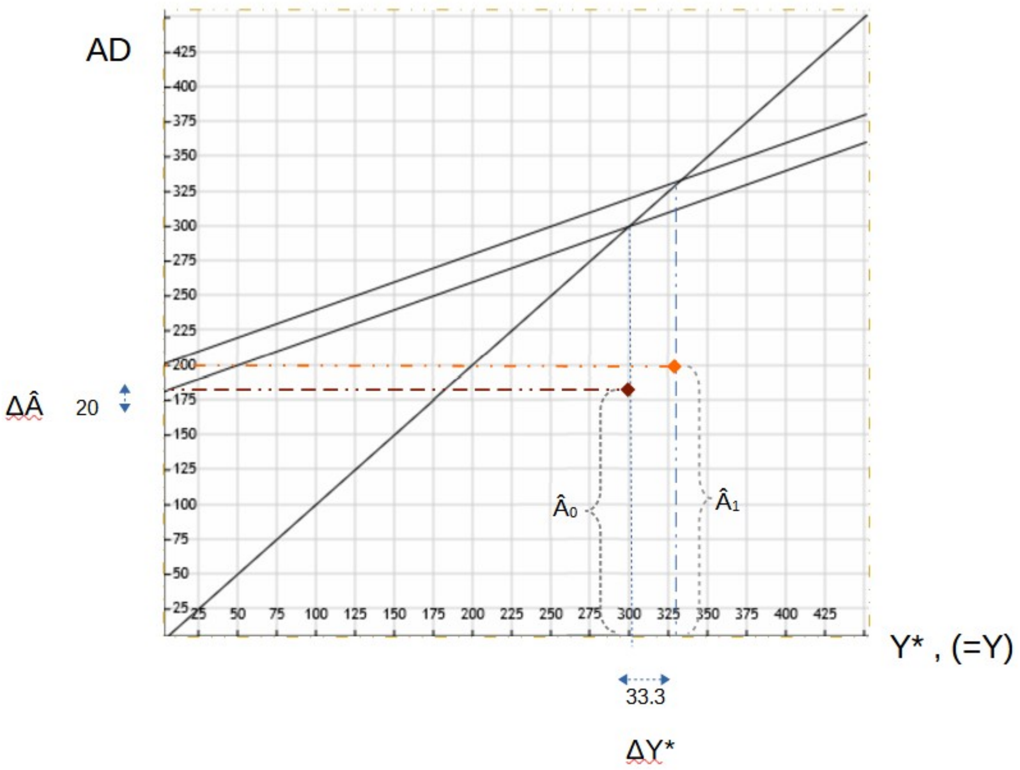
This numerical excursion allows us to show the important fact that the theoretical cumulated repercussion of a given $\Delta\hat{A}$ over AD would not be something immediate. Reaching the new equilibrium will require a given number of cycles, i.e., an *adjustment time*. Therefore, arguing about the multiplier for $n\to\infty$ –which is the usual practice in standard textbooks– entails indeed referring to the middle/long-run; say, to several months.

Nevertheless, the reasoning about the multiplier so calculated tends to be rather developed arguing as if the adjustment process ($\Delta\hat{A} \to \Delta\ AD$) were associated with the short-run; even more, and contradictory, like something rather immediate[3].

[3] This misleading practice comes induced by the fact that the same quantitative result as above, for $n\to\infty$, can simply be mathematically deduced by assuming that the new macroeconomic equilibrium has simply been already achieved, instantly, i.e., just within one cycle (!) (see Appendix 1, 'b'); as well as through expressing the final equilibrium in terms of the chain of conditions: $S=I=\bar{I}$, (see Appendix 1, 'c').

The above paragraphs illustrate the arguments behind the classical “45º” graphical representation of the transition from an initial equilibrium in the Economy, AD$=^{c}$ Y*, to another one, as a consequence of a change in the Autonomous Demand, $\hat{A}$. Let us note that this transition is supposed to be internal to the considered accounting period (say, a year); that is, the model is in fact static.

The example below has been built using the following specific values: $c_1$=0.4 (thus, multiplier, *m*=1,66), an initial equilibrium situation AD=$Y^*$=300 billion €, and $\Delta\hat{A}$= ***a*** = 20 billion €.




The key elements of this explanatory model are, thus, 1) a ‘basic, parameter-like’ *Autonomous Demand* ($C_0 - c_1 \cdot T + G + \bar{I}$) and that of a *Dependent Demand* in terms of a ‘complementary’ consumption, $c_1 \cdot Y$, and 2) the assumption of a given jump in the former ($\Delta\hat{A}$).

## 2. The ‘IS’ model: the *interest rate* as a core macroeconomic variable

The ‘IS’ function, a concept introduced by John Hicks, is an explanatory drawing built on the above “45º” Aggregate Demand’s model/paradigm/scheme. The starting point consists of introducing in it the assumption that the ‘decided/planned’ Investment, $\bar{I}$, is not a constant but depends on the interest rate, in these terms:

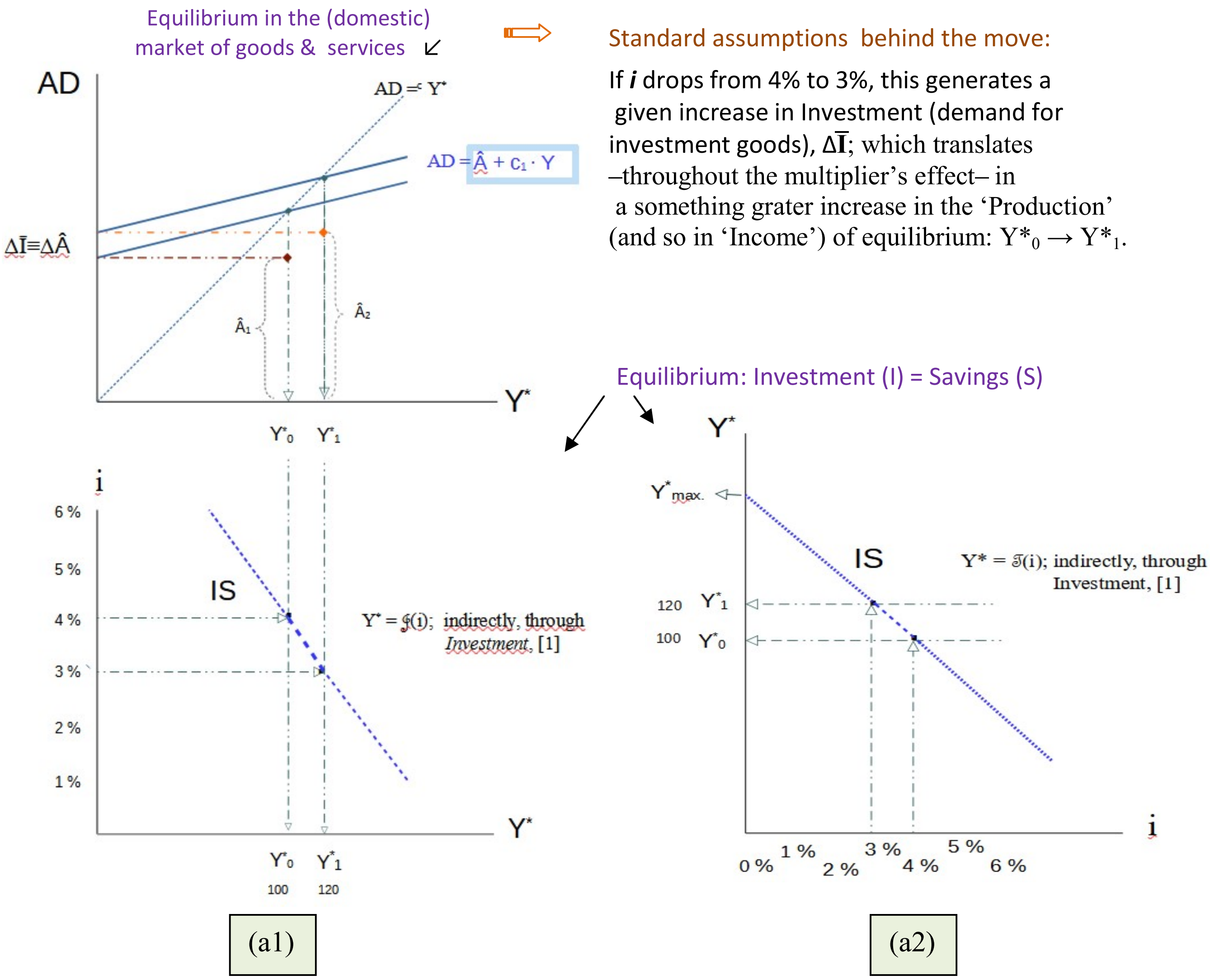
$$\bar{I} =^{mi} I_0 - b \cdot i \qquad [1]$$

where ***i***, is the average interest rate banks apply to lending and $\boldsymbol{I_0}$ is the investment that business undertakings and households would decide/plan if the interest rate was zero.

Additionally, the subsequent conceptual use of the above model implicitly suggests that $\bar{\mathbf{I}}$ depends substantially on the interest rate; i.e., that coefficient *b* has a significant value. This is made explicit in graphical language, when the traditional IS diagram (below) –a postulated functional relationship between *interest rate* → *Production (= Income)*– is built by drawing a proportionally significant increase in Aggregate Demand (AD), and consequently in Production (Y*) as a consequence of a modest reduction in the interest rate.

### *2.1 The IS model: the* **causal relationship** *between GDP (Y*) and Interest rate*

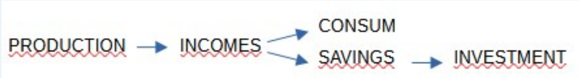
Let us summarise such a graphical construct. Back to the "45º" scheme, the above assumed function [1] means that a change in the interest rate will induce an inverse-sense change in $\bar{I}$, i.e., in $\hat{A}$, and thereby, amplified by the multiplier effect, in AD. Thus, if the interest rate drops, for example, from an initial 4% to 3%, a given increase in $\bar{I}$ will result; and thus the same increase in $\hat{A}$, and amplified increase in AD. This entails that the economy will move from the initial equilibrium, $Y^*_1$, to a new equilibrium, $Y^*_2$, through such a cumulative process (multiplier), commented in the previous section. When we represent these two points (3%, $Y_1$; 4%, $Y_2$) in a second panel in the space ***i*** ∟ ***Y***, (beneath in the left: Fig. a1) we obtain a decreasing segment. If the relationship between $\bar{I}$ and *i* is assumed to be linear (as in [1]), such a segment may be extended left and right, so depicting a declining straight line, a linear relationship *i*→*Y**, which is then termed 'the *IS* function'.



We could ask, why to include ‘S’ in the name, when is just Investment, not Savings, that what is behind this traditional diagram? This is for, in the upper panel AD∟Y*, the equality S=I is pre-supposed; and by terming the resulting line in the panel beneath as ‘IS’ this assumed equality is so underlined in the theorised relationship i ∟Y. In other words, this step within the whole IS-LM model implies the assumption that Investment increases the same amount that Savings.

But, where this increase in Savings has come from since we are supposed to have started from an equilibrium situation? In it, Savings are assumes to be fully employed financing Investment; how then, in the model, the assumed $\Delta\,\overline{I}$ has been possible (its financing)?

The chain of causal relationships in the model is, as has been seen:




Thus, how, in the former explanatory diagram, is the assumed increase in Investment, $\Delta\overline{I}$, financed when no previous additional Production exists? (We will retake this further on).

The meaning of this ‘derived’ IS line is, thus: the pairs $i$, $Y^*$ associated with an equilibrium: AD$=^c$ $Y^*$. This, under the previous assumptions of: 1) planned Investment, $\overline{I}$, equal to actual Investment, $I$, –i.e., no (unwanted) increases in business’ stocks of final goods–; 2) ‘Investment = Savings’; and 3) the relationship ‘Investment’ ← interest rate’ stated in [1], where parameter $b$ has (as for the usual IS diagrams) a significant value.

The above two equalities make up the ‘strong’ premise. It equals talking about a macroeconomic equilibrium’s condition and how a new equilibrium is automatically achieved, but starting by assuming that two fundamental conditions for such an equilibrium situation are already met: $I=\overline{I}$ and $I=S$. Thus, the second one means that the sum of the savings by households (employees, shareholders, self-employees, etc.) and firms (non-distributed profits) during, for example, one year coincides with the sum of the actual investments by firms and households, over such a year.

Note that these assumptions behind the IS diagram/model are not inductive –i.e., not drawn from exploratory observations regarding this or that country and period– but the result of deductive reasoning (hypotheses). Hypotheses that are specifically focused on the effects of changes in the interest rate; a variable that is taken as associated with either the financial market or economic policy’s decisions by public powers (usually, the corresponding central bank).

### 2.2. Misleading problems

The causal sequence embedded in the standard IS model is, as we have seen, $\nabla \boldsymbol{i} \rightarrow \Delta\overline{I} \Rightarrow \Delta AD = \Delta \boldsymbol{Y^*} \equiv \Delta \boldsymbol{Y}$, (or the reverse); which means that a given causal relationship between $i$ and $Y$ –indirectly, through $I$–: $Y=\Im(i)$. However, the usual in textbooks is to graphically represent the causal relationship the other way round, as in the previous panel a2 (down on the left): *Production//Income* in the abscissas axis. Whose formal reading is: ‘*Income* is the independent variable and the *interest rate* the dependent one, $i=f(Y)$; i.e., as if the cause-effect relationship was $\pm(\Delta Y \rightarrow \nabla i)$ .

In an academic paper, this might be taken as a purely instrumental question, the cause-effect relationship $\nabla i \rightarrow \Delta Y$ being understood as implied. But in textbooks, this graphical language’s licence, which formally inverts the cause-effect relationship between the two variables (the diagram’s direct reading is: the greater *Income*, the lower the interest rate), has easily misleading effects on readers, regarding the interpretations of the whole IS-LM model itself to explain this parcel of an economy’s workings.

Graphical abstract-mathematic language in the way of diagrams does play a fundamental role in mainstream economics to formalise cause-effect relationships. Therefore, what a diagram 'tells' formally to readers cannot be ignored. And, what this formal 'reading' of the usual IS diagram/function is that «the interest rate applied by banks decreases $\nabla i$ because the economy grows ($\Delta Y^*$)». A 'drawn' statement that certainly is the reverse of the narrative argument in the corresponding manual; besides being rather at odds with the most elemental observational evidence from statistical series of $Y$ (a periodical macroeconomic monetary amount) and $i$ (average rate of interest along the corresponding period).

The point here is that the *sense* of the causal relationship between some given variables matters for explanatory purposes. The statement resulting from a direct reading of the standard IS diagram/function in the left –an increase in ***Y*** leads to a decrease in ***i***– does not mean the same that 'a reduction in ***i*** induces a given increase in ***Y***, all the rest affecting ***Y*** the same' (the reading of diagram in the right), which is indeed the postulated cause-effect relationship within the standard narrative reasoning itself.

Therefore, we may well ask, why such an inverted graphical representation in the standard model (***Y*** as if independent variable, while ***i*** as the dependent one), which induces in readers a false, contradictory explanation? (Excuse me the reiteration). Let us leave the answer for the end of the following step in the model.

Summing up, the relationship the IS model refers to is

$$Y=\Im(i) \Rightarrow \quad \boldsymbol{Y} = Y^*_0 - b\cdot\boldsymbol{i}\cdot m$$

Where :$\mathbf{Y^*_0}$ is the level of Production//Income when the interest rate is 0; ***b*** is (as in [1]) the parameter measuring the sensibility of firms' Investment to the *interest rate*; and ***m*** is the multiplier of a change in the Autonomous Demand, which, in turn –as seen before– depends on the propensity to consumption in the economy: $m= 1/(1-\boldsymbol{c_1})$.

A different point would be to question the likelihood of the hypothesis, implicit in the above: that the causal relationship $\boldsymbol{i} \rightarrow \boldsymbol{Y}$ is significant. I. e., that the above parameter ***b*** has actually a significant value, as it is assumed –through diagrams' language– in the standard IS model here summarised. Thus, for example, in the prior familiar IS diagram the assumed value for $Y_1$ is 100 (we may think, 'billions' of monetary units), for $Y^*_0$ is 180, and parameter ***b*** is around 13.6 (for the rate of interest expressed in percentage points). And the implicit value of the *multiplier*, 1.4. This entails assuming a quite high sensibility of private Investment (and therefore, of the Production//Income) to the interest rate. Thus, a decrease of the latter from 4% to 3% is supposed to cause an increase of 13.6 % in firms' Investment, and, as a consequence, of 19 % in Production//Income.

This so-high sensibility of *Investment* to *interest rate*, conveyed by the usual diagrams' language of standard texts, appears highly unrealistic. Overall macroeconomic data regarding many countries show us the scenario of a certain trend of economic growth ($\Delta Y^* \equiv^{ai} \Delta Y$) over several years as something rather normal; a trend that logically entails that a certain annual increase in *Investment* –as well as in *Consumption*– occurs. And this is observed to happen while simultaneously the average interest rate applied by banks, over those years, appears as notably unrelated (see Appendix 2).

Widening the perspective, in a private-enterprises market economy the amount of firms' *Investment* depends on more than one variable: business expectations, trending in economic growth, technological change, international state-of-the-affairs, etc., as well as also, though not mainly, on the money market's *interest rate*.

Furthermore, to assume that *Investment* depends basically or mainly on the *interest rate*, and thereby that –as in the standard presentation of the IS model– *Income* also depends to a great extent on the *interest rate*, is plainly unrealistic. This makes IS a confusion-leading model to explain the workings of our economies. See, in this regard, the statistical data on ***Y*, I,*** and ***i*** in Appendix 2, referred to the EU countries over 22 years. Though a simple comparative reading of the two charts in the appendix does not properly constitute an empirical research's proof on the sensibility of Investment to the interest rate, it does give a first rough evidence of the type of exploratory work that is supposed to precede or lead to formulate reasonable hypotheses or deductive assumptions on the topic. In this respect, such a comparative reading of the plotted data illustrates to which extent the IS model's assumptions about the causal relationships between the above three variables are, unnecessarily, far from the realm of our economies' reality.

## 3. The 'LM' model, a theory on *Money Supply* and the *Interest rate*

The model is intended as a theoretical explanation of the causal relationships between the interest rate applied by banks, the people's demand for available money (Liquidity), and Income//Production. Its standard terminology and preliminary definitions are:

- Stock of Money
  - M1 = Currency in circulation + bank current accounts (sight)
  - M2 = M1 + Investment funds + banks term deposits (short-time availability)
  - M3 = M2 + non-banks deposits + banks long-term deposits
  - M = *Money supply/stock*' (Its scope is understood in a broad sense, not always making precise if it is used as referring to M1, M2, etc. Though, in the model development, M is usually supposed to refer just to M1)
- 'Real assets': Producers' equipment, land, buildings, premises' facilities, etc., plus bonds and firms' shares. Consequently, fixed assets of stock firms (their equipment, premises, etc. in their balance sheets) should not be taken into account (to avoid double accounting).
- Model definitions:
  - M = Monetary mass or *Money Supply*, in 'nominal' terms (as stamped in bills)
  - P = Consumer prices' index (base=1)
  - L = *Money Demand* (or Preference for *Liquidity*) –i.e., purchasing power kept available by households, firms, and governments: M1–, *in real terms*; that is, stamped nominal values deflated by P.
  - i = *Interest rate* (in % points) applied by banks, in their lending as well as borrowing[4]. The latter referred to receive and keep middle-long term deposits from particulars.
  - co = Cost (in % points) for banks' clients of converting non-sight deposits into available money (M1).

### *3.1 The explanatory model for the relationship between L and i*

The 'LM' model starts by stating a condition of *monetary equilibrium* for the economy: *Money Demand* (or *Preference for Liquidity)* equal to *Money Supply* in real terms (deflate),

$$L =^{e} M/P \qquad [2]$$

[4] Note that this assumption entails that banks would operate at losses, since their operating differential rate [(% charged to money lent) – (% paid to money deposited by clients)] would be zero. Thus, banks would regularly present losses equal to, at least, their operating expenses.

together with formulating a generic causal-relationships to explain the changes in $L$:

$$\underset{+}{L} \underset{\Leftarrow}{=^{mi}} f(\underset{+,}{Y}, \underset{-,}{i}, \underset{+}{co}), \quad [3]$$

Where the signs for the respective causal relationships the model postulates are those below each variable.

Thus, the model assumes that (§1) «the lower the interest rate, ***i***, the higher ***L***, the volume of Liquidity citizens want/prefer to keep, all the rest the same». As well as that the responsiveness of ***L*** to changes in ***i*** is significant; this comes usually implicit, basically through graphic language: by drawing a steeped declining line in the space ***i***∟***L*** to represent the relationship $L \leftrightarrow i$; more precisely, an abstract linear decreasing function $i=\delta(L)$, like the green line in diagram 'b1' below.

The underlying standard reasoning in the above assumption is: if banks increase the interest rate, ***i***, then citizens decide to keep less of their monetary wealth available, by moving a given part of it from 'available' (say, cash and current accounts) to middle-long term deposits at banks. And the reverse for reductions of the interest rate.

> Insofar the quantitatively dominant part of such 'preference for liquidity', *L*, would refer to households (employees, self-employed, shareholders), as a citizen you might object to the above that the assumption that we people are ordinarily attentive to the evolution of banks' rate of interest, in order to modify the structure of our monetary wealth sounds rather unrealistic. But let us leave this kind of questions for the next entry and continue deploying the standard model.

The above schematic causal-relationship $L \leftarrow i$ is then applied to develop (using the popular diagram 'b1' below) what happens when ..

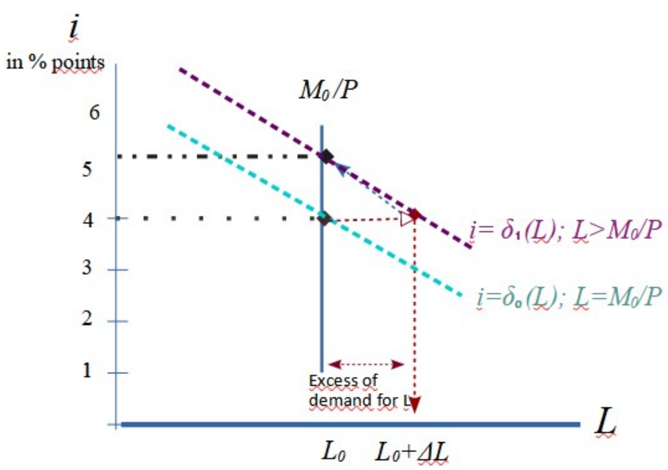
..the Money Supply ($M_0/P$) is a constant, and starting from an equilibrium position $L_0=M_0/P$, this equilibrium is broken because of an increase in the demand for *Liquidity* (of households, firms, shareholders, etc.); this increase generated, in turn, by an autonomous rise in domestic *Production* and therefore in *Income*: $\Delta Y^* \rightarrow \Delta Y \rightarrow \Delta L$. In this regard, it is also assumed that the sensibility of $L$ to $Y$, ($\Delta L/\Delta Y$), remains stable; and therefore that such $\Delta L$ will be proportional to $\Delta Y$. More determinant, however: it is assumed that the said ΔL is not a move over the $i=\delta(L)$ green line in the diagram but a parallel shift of the line upward; that is, the demand for Liquidity is now higher for any value of the interest rate (line in purple in b1).

In this scenario, it is then assumed in the model that (§2): since *money supply* remains unchanged, $M_0$, there is no way for banks to attend such $\Delta L$. And, as a consequence, (§3) «this 'excess of Demand for Money' leads banks to increase the interest rate, to the point that such an excess of demand for Liquidity simply disappears». Thereby, «demand for Liquidity' goes back to the previous level, but now the rate of interest is higher» (sic); (backward arrow in diagram 'b1' below). In short, the volume of *Liquidity* in the hands of households (and firms, etc.) remains unchanged, but the *interest rate* is now higher.

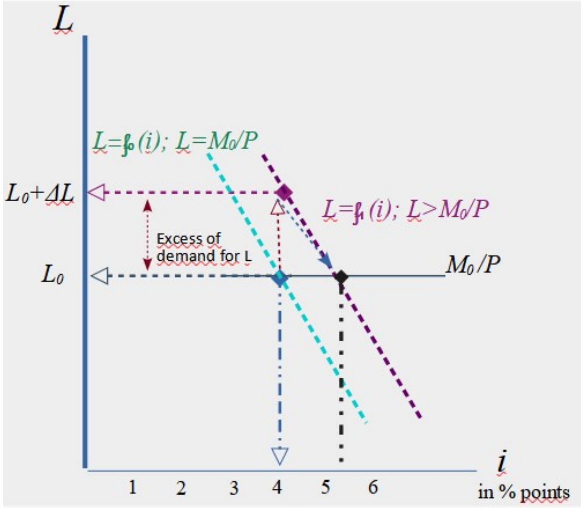
This would be because, «since households (and firms, etc.) want now to have something more money available, they will request loans to banks. But since banks have the same lending capacity as before (M0) –and it is fully used (starting equilibrium)– they put the interest rate up».

This standard argument suggests a sort of market dynamics in the banks sector rather puzzling: before the new demand, banks raise the price ($i$), but since they have not new money to lend, not any new lending is contracted. Thus, we are forced to conclude that the new higher price ($i$) would rather be virtual.

(b1)




(b2)




Whatever the case, the above standard curious argumentative sequence –«there is an increase in the-preference-for-*L*, but at the end there is any increase in *L* because such an increase in demand of Liquidity has made the interest rate to rise»– is what is compactly represented by the traditional diagram 'b1' (see arrows); which, again, has its axis exchanged with regard to the direction of the cause-effect relationship postulated. Diagram 'b2' would be the equivalent, according to such a causal direction.

The former counterintuitive statement regarding *L* comes from an adding implicit into the former: the assumption that: (§4) «As a consequence of the increase in *Income* −that has, in turn, generated the increase in the demand for Liquidity–, *demanders* (households, etc.) have changed their pattern of behaviour to this respect, in the sense of they *preferring more liquidity than before, for any given interest rate*» (because they having become richer?) [5]. Moreover, Note that this argument means to change the sense of the causal relationship represented in the standard diagram b1, [$i \leftarrow L$], to that of diagram b2: [$L \leftarrow i$], which is in fact the sense of the causal relationship in the initial standard postulate [3]. As well as that the functional relation in such a postulate has quantitatively become modified: $L=f_0(i)$ ···→ $L=f_1(i)$ in diagram b2.

*Model's inconsistencies*

Diagrams in Economics play as a language; also for implicitly introducing assumptions as those enumerated so far –and others further on–. Let us, thus, 'read' the traditional diagram 'b1', from this outlook:

1) As in the previous case of the IS model– it has its axes exchanged: the *rate of interest*, *i*, appears to the readers as if the dependent variable, and the demand for *Liquidity*, *L*, as if the independent one; when the causal relationship argued about in standard texts is the reverse [3]. That is not innocuous. Besides inducing confusion, it hinders the 'reading' of the embedded reasoning regarding the cause-effect relationships assumed. In any case, what we could expect as a graph scheme condensing the standard model's reasoning and assumptions (§2, §3, §4) about the relationship $L \leftarrow i$ would simply be the diagram on the right, 'b2'.

[5] In order to search for the logics of the reasoning, the assumption could respond to the premise that «the Income's increase has gone to the same prior households, self-employed, shareholders, etc., which are thus something richer; richer enough as to pay more for having the same available money from banks». A premise, however, that becomes arbitrarily unreal.

2) The fact of using in macroeconomics textbooks the diagram 'b1' –instead of the *direct* one, 'b2'– as a basic explanatory device for the dynamics between *demand for Liquidity* and *Interest rate*, gives way to inconsistencies within the texts themselves: it is talked about a rise of ***L*** at the time that of ***i***, contrarily to what the model formally postulates [3]. Should we understand that it is being supposed in the model to be different causal relationships between *L* and *i*: one for autonomous △▽ of *L* and another one for autonomous △▽of *I* ? An implicit answer to these questions by the devoted authors seems to be: «Yes; whenever there is a rise in *Y*, the demand for liquidity increases *for every value of i*», (because demanders are now richer?). Then, we could well re-ask, what does the initial decreasing line, in blue, (the higher ***i***, the lower ***L***) stand for? A possible answer, from involved authors, to this could be that «'Liquidity' and 'Demand-for-Liquidity' are different concepts, though both denoted by '*L*'». From this outlook, the standard underlying reasoning would be: «The Δ *Demand-for-L* does not materialise, because banks –expecting it– raise the interest rate».

3) An additional implicit assumption conveyed by the above diagram/s is that the sensitivity of people's preference for *Liquidity*, regarding banks' average interest rate, is relevant: the usual high slope depicted for the line representing the relationship $L \leftarrow i$ is telling that to readers.

*On assumptions' likelihood*

In any case, note that the financial behaviour that the standard reasoning is presupposing for households is that of wise and dedicated investors dealing with banks: they would marginally reduce their otherwise demand for available money when are offered a higher interest rate for their term deposits, (or they avoid to increase their available money via borrowing from banks because these have raised loans' interest rates). Thus, to theorise on the causal relationship L=f̌(i), , the model relies, on the paradigm of the homo economicus' behaviour.

It merits to underline that the assumptions or hypotheses the above explanatory model relies on (as §1 to §4) are normally not justified or argued in macroeconomics textbooks –and rarely in advanced works–. Neither are they illustrated with an argument based upon some anecdotic observation regarding this or that country. Simply, such assumptions are purely deductive. They are enunciated in textbooks simply under the register of "monetary dynamics are like that". In any case, they are, more than oversimplified, rather unrealistic indeed.

Thus, even using the same standard macroeconomics scheme, logic tells us that an autonomous *ΔY* necessarily means a rise in households' (and firms', etc.) available income 'at hand' (wages, dividends, etc.); and, in this respect, we could accept that a given marginal part of such a new income may not go neither to consumption nor to savings deposited in banks (term deposits, investment funds, etc.) but to be kept in cash or current accounts. Consequently, no resort (of households, etc.) to banks would be necessary for people to marginally increase their Liquidity. Therefore, no reason for banks to raise the interest rate would exist. In other words, a given increase in *ΔY* entails that producers (business firms) will have had to pay such additional amount to employees and shareholders¸ which implies that the money supply in the system will necessarily have somehow increased. Therefore, the assumption, reflected in the above traditional diagram, that the money supply remains constant ($M_0$) results implausible.

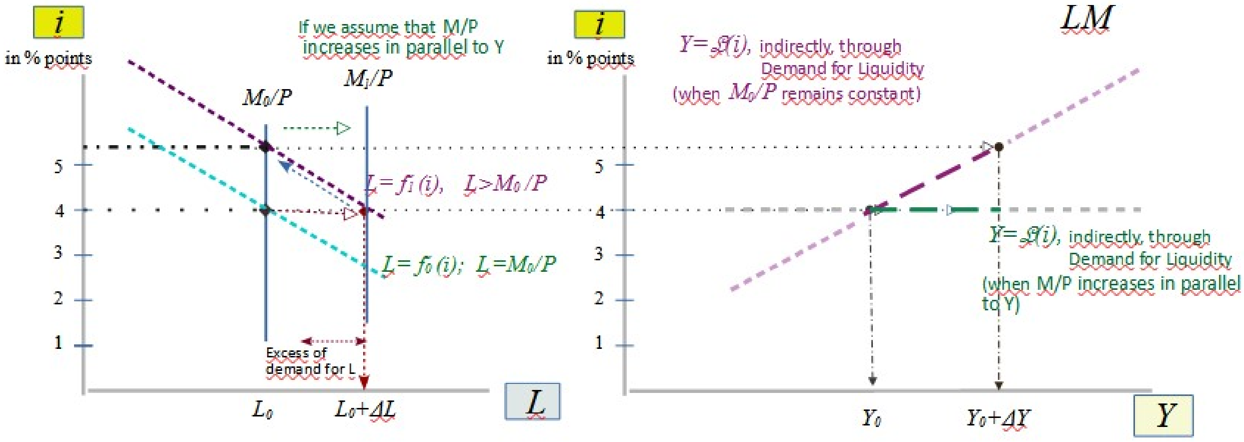
Looking at this the other way around: if Liquidity remains in fact unchanged, this entails that no new loan, neither a drawing from term deposits, for increasing Liquidity has been required by 'demanders' (households, firms, etc.) to banks. Thus, what is the logic for postulating that banks will have raised the interest rate when no new money banking transaction has been carried out?

In short, the core deductive assumption behind the above traditional theorem/diagram on currency in the system, people's behaviour regarding Liquidity –embedded in diagram b1 (or b2)– appears as surprisingly and arbitrarily unreal.

### *3.2 The derived theorem/diagram on* **Income** *and* **Interest rate***: The monetary equilibrium's line 'LM'*

Nevertheless, the reasoning in the standard overall IS-LM model focuses on keeping the prior referred assumption, and using the former diagram (b1) to deduce/define the below traditional 'LM' diagram/function (c1), which represents the relationship between *Income* and *Interest rate*, (through variable *Demand of Liquidity*), $Y=\mathcal{L}(i)$:

(b1) (c1)




In the above diagram on the right (c1), an alternative possibility (rightward flat line in green) has been added to the traditional upward LM line, as some authors defend (f.e., Blanchard, 2017:72, 91-97). Such a flat line would correspond with the reasonable deduction argued before: that currency's amount necessarily increases in an economy, in parallel to an increase in *Income*//Production. Thereby, the *interest rate* would remain unchanged (4% in the above example) since no people's requirements to increase Liquidity will have been made to banks. And consequently the resulting 'LM function' would be just a flat line.

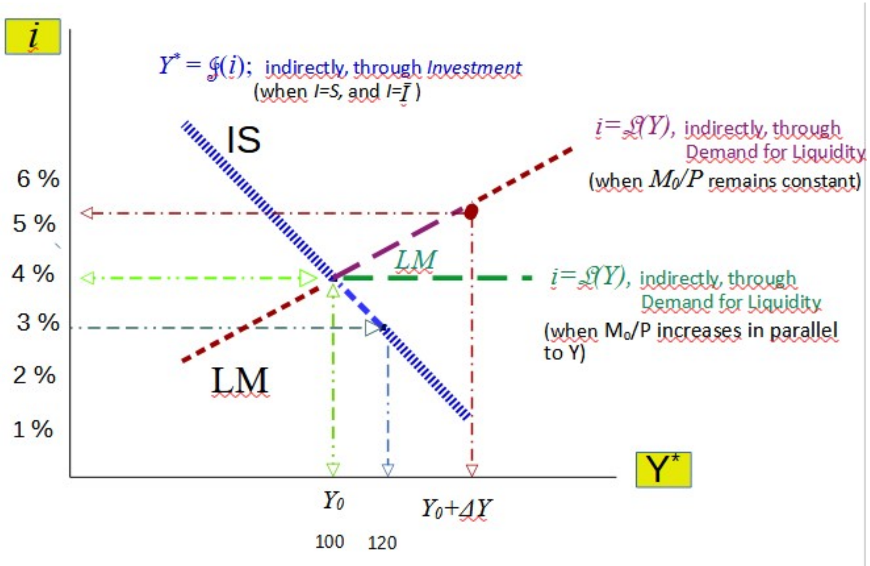
As traditionally in standard textbooks, in these diagrams the axes appear exchanged as for the dependent/independent variable, i.e., regarding the sense of the causal relationship considered –as before regarding the IS function. The observations there in this respect are thus applicable here. However, for the sake of simplicity, the corresponding b2 and c2 graphs have been avoided here. Nevertheless, the sense of the assumed causal relationships has been highlighted by writing the corresponding functions within the respective diagram.

## 4. The 'IS-LM' explanatory model: the macroeconomic 'double equilibrium'

### *4.1 Simultaneous Economic & Financial equilibria*

Finally, in the following step of the standard macroeconomics model, the reasoning develops basically just in graphical language. This consists of a new diagram where the above upward (or flat) 'LM' line, $i=\mathfrak{L}(Y)$ in diagram *b1*, is overlapped to the 'IS' downward line, $i=\mathfrak{J}(Y)$, of former diagram *a1*:

(d)




In this diagram, the intersection point of these IS and LM lines is then termed 'general (or double) equilibrium' since it defines a given pair of 'rate-of-interest ($i$) and Income ($Y$)' associated with both an economic equilibrium ($I=S$, and $I=\bar{I}$) and a financial equilibrium ($L=M/P$). That is, a point [$i$, $Y$] for which the *star* variable in the model, the *rate of interest*, is that of a 'stable equilibrium' in both the goods & services market (= Production) and the financial (=Demand for Liquidity) market. All this, provided that all the assumptions and premises that have been underlined so far would be met in the economy and that the rest of the determining variables (of $i$, $\bar{I}$, $L$, and $Y$) –that do not appear in the model– remain constant.

The epistemological parallelism of the above macroeconomics' Equilibrium (crossing point in diagram 'd') with the model/paradigm of the 'General Equilibrium of (perfectly) Competitive Markets', that vertebrates mainstream Economics-Microeconomics, results indeed remarkable.

In some texts, the IS line/function is drawn in the corresponding diagram (former figs. a1, a2) as a curve significantly concave regarding the coordinates' origin –instead of linear (as assumed in [1])– [6]. This entails having assumed that a reduction of, say, one point in the *interest rate* causes a higher increase in *Decided Investment* –and thus in Production//Income– the lower the starting interest rate is; or, the other way round: successive increases of one point of the interest rate generate lower and lower reductions in *Decided Investment*, and thereby in $Y^*$. In the referred texts, however, this assumption does not use to appear justified or argued.

In parallel, usually in the same texts[7], the LM growing function is represented not as linear but as a curve with a slightly increasing slope in the space $i$∟$Y$ (decreasing slope in the space $Y$∟$i$). This, in turn, means to assume that successive increases in the *interest rate* –as a consequence of decreasing the *Demand for Liquidity* ($L$)– generate lower and lower effects

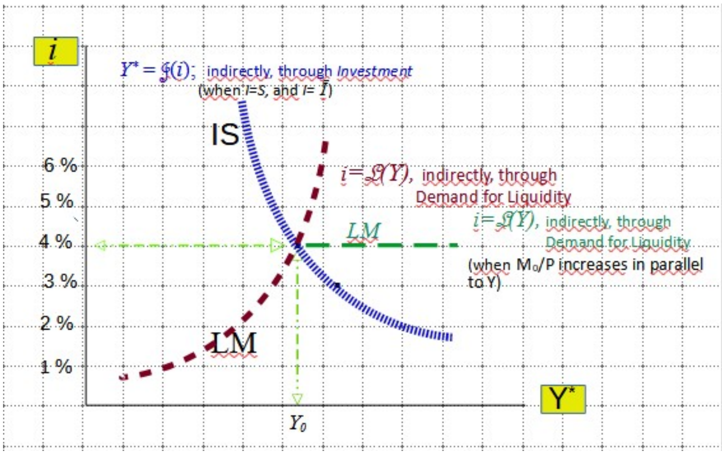
[6] For example: Mankiw (2016: 325); Blanchard (2017:109-12); and Williamson (2018:311-26). Also in Boianovsky's 2004; though in this case the IS function is assumed both as concave (p. 110-11) and convex (p. 107-8).

[7] For example: Boianovsky (2004:109-110); Mankiw (2016:331-3); and Williamson (2018:327-28). A different option is that in the cited Blanchard's book: different flat (parallel) LM functions, for every value of the interest rate (Blanchard, 2017:116); i.e., like the LM option drawn in green in Fig. (d).

(decreases) on the monetary *Income (Y) of equilibrium*. Again, the absence of arguments about this alternative assumption in the referred texts must be underlined

Thus, in the cited texts, the final graph/postulate of the IS-LM theory of macroeconomic equilibrium is:

(d2)




In short, the different steps or pieces summarised in the present summary & analysis of the standard macroeconomics' IS-LM model highlight the fact that its explanatory scheme on the workings of our market economies is built upon a set of hypotheses and premises, dominated by the financial variable *interest rate* (*i*), that draws a picture that is difficult to relate to the whole real workings in a market economy, and thus scarcely useful (if not misleading) to explain it. This, besides the model being excessively simplified, even as a scheme, insofar it leaves aside fundamental determining variables (e.g., the main factors behind firms' Investment decisions) and does not include in it the key variable 'time'.

*'Reading' the IS-LM model/diagram*

The above IS-LM diagrams (both, d and d2) entail, however, a notorious inconsistency:

[In the above Fig. *d* –as in its precedents, Fig. *b1* and *c1*– the starting situation (assumed here, by way of example, as *i*=4%, *Y*=100, for both the IS and the LM) and the movements assumed in the standard model to build/define the IS and LM functions, have been plotted by scale representations]. So underlined, let us observe that the finally resulting double equilibrium point in the IS-LM standard model ($Y_0$, 4% in our example) means that the *Production//Income* of equilibrium comes finally out to be rightly the starting one, $Y_0$, $i_0$ (sic); which plainly sounds as an absurd result. It would mean that no increase in *Income* is stable in the economy because, in the end, it induces a rise in the *interest rate*, which in turn leads somehow to making such *Income* increase not viable/stable.

Certainly, the economic-financial equilibrium shown by the above figures *d, d2* ($Y_0$, 4%), comes from arbitrary initial example-values –for the interest rate and Income, and their moves– taken for illustrative purposes. Nevertheless, the IS-LM model conclusions highlighted in the above paragraph would be the same if other numerical values had been used to deploy/formulate the standard model. Let us develop the above assertions, going schematically back to the chain of cause-effect relationships formerly identified for each of the two standard functions:

> The reasoning regarding the IS function (diagrams *b1* and *d*) is, schematically: There is a $\nabla i$, which generates $\rightarrow \Delta\bar{I}$, (and thus, $\Delta\hat{A}$), what, in turn, causes a something higher ($m\cdot\Delta\hat{A}$) $\rightarrow \Delta AD$, which thereby causes an equal $\Rightarrow \Delta Y^{*}=\Delta Y$; and the resulting quantitative causal relationship, $\Delta Y/\nabla i$, is finally extrapolated left and right to so draw the IS function.

And, referring to the same economy and timing (we must assume so), the reasoning the LM function responds to is: An $\Delta Y$ has occurred (its origin is not made precise), which has generated → an $\Delta L$, while the monetary supply, $M/P$, remains constant; and, as a consequence, such an $\Delta L$ causes → $\Delta i$;[8] (which, we may add, will lead, in turn, to $\nabla \bar{I}$, → … etc.); and, finally, the resulting quantitative causal relationship $\Delta i/\Delta Y$ is extrapolated to so build the LM function.

Therefore, when we consider both functions simultaneously –graphically, overlapping them– the simultaneous dynamic of the LM ($\Delta Y \Rightarrow \Delta i$) neutralises that of the IS ($\nabla i \Rightarrow \Delta Y$); or the reverse. The graphical result, the neutralisation, becomes explicit in Fig. d/d2: the so-called IS-LM double-equilibrium point/economic situation) is simply (equal to) the previously assumed as the starting one ($i$=4%, $Y$=100).

This absurd result is simply a consequence of the IS and the LM models being static, in the sense that time is not actually a variable in them (Arnon, 2022:147-8), and of the formerly commented (sections 2 and 3) arbitrarily unrealistic assumptions the traditional IS and LM 'functions' rely upon; or, in other words, of the model being concerned with a static economy.

◊

Nevertheless, the IS-LM model –either with linear or non-linear assumptions about the two functions– remains as a core topic in almost all Macroeconomics manuals. To cite just two of the rare exceptions: 1) Hubari and O'Brien (2009) only introduce the 45º-degrees diagram (here, section 1) but just to deal with the *Aggregate Demand* and the *multiplier*, and to deploying a model that relates *Prices' index* and *Production//Income*'s dynamics (their Chapters 11 and 12). And 2) Acemoglu, Laibson and List (2016), which deal with the financial market dynamics using an $i \llcorner L$ model/diagram, but as an ordinary market's Price↔Demand dwindling function, like that in the former Figure *b1*.

## 5. Comparative Statics and inconsistencies: shifts of IS and LM functions

Related to the absence of the variable 'time', a final step (or development) of the IS-LM model consists of talking about 'shifts' of the IS function and/or of the LM function; and thereby about movements of the IS-LM double-equilibrium point; something that involves exercises of a sort of comparative static.

The main case usually considered in this line is an *autonomous* increase in the Aggregate Demand, in the sense of it being not caused by a change in the interest rate. Normally what is considered in this regard is 'a rise in Government Expenses (ΔG)'. And the conclusion in this regard is that such a ΔG will shift the IS curve rightward. This, for, in the initial double diagram *a1*, an increase in AD is produced, and thereby the corresponding multiplied effect in Y* results –while the interest rate remains constant (at the level of 4% in our diagram a1)–. And this specific movement to a point to the right in the space $i \llcorner Y$ is then graphically extrapolated left and right, so drawing an entire new IS line rightward in parallel to the initial one.

The same reasoning is usually made or extended regarding an increase in autonomous AD coming from an rise in the 'constant/autonomous' Investment, $I_0$ (see expression [1] on page 4).

[8] If we pass to make the alternative assumption some authors make for LM (f.e. Blanchard, 2016:91-97), in the sense that the Money Supply does not remain constant but automatically adapts or is adapted (increases, in the above example), then the LM would be a simple flat line (the broken in-green one, in Fig. c1 and d), though the interest rate, $i$, would also remain as in the initial situation (=4% in the example).

Also frequent in this regard is to consider a change in Government Taxes (Δ,∇T). Thus, following the former reasoning, a reduction in taxes (∇T) would make the IS line also shift in parallel rightward. In this case, the increase in Aggregate Demand would be $c_1 \cdot \nabla T$; and of $(1-c_1) \cdot \nabla T$ in Savings. That is, again an increase in the *autonomous* Aggregate Demand without having changed the interest rate; and thereby a multiplied increase in the Production//Income of equilibrium. And, conversely, a given increase in Taxes, ΔT, would generate a reduction in Aggregate Demand of c1.ΔT, etc.; which would make the IS line shift in parallel leftward. The same formal consequences as an equivalent ∇G.

As for the LM line, the shifts usually considered in textbooks and literature would come from decisions on monetary policy. Firstly, when the Government/Central Bank modify Money Supply (Δ,∇M); or the Central Bank fixes the interest rate (i). In the case of an increase in Money Supply (Mo/P in diagrams b1, b2) it would shift the LM curve outward (to the right in the space *i*∟*Y*). In fact, this assertion may be illustrated by our former Figure *c1*, where the real Money Supply is assumed to rise to just cover the increase experienced by Demand for Liquidity (L). This defines a point *i*∟*Y* on the righy, which, by extrapolation, gives way to a new LM line/function: thus, the LM line shifts rightward; so leading, for the same interest rate, to a new equilibrium point Yo+ΔY. In the cited texts –as in the case of IS' shifts– any specific movement rightward is generalised by extrapolation, giving way to an in-parallel shift of the LM line.

As for considering the event that the interest rate is fixed by the Central Bank, the LM curve will then shift simply to a flat line. This is particularly the case considered by, f.e., Blanchard in his cited 2017 text: he uses/draws specific flat LM lines for each value of the interest rate (Blanchard, 2017: 116-17).

But, let us consider the above shifts from another perspective; f.e.: 1) how the above referred ΔG would be financed? A given equilibrium is supposed to exist before, defined by specific values I=S, L=M, and the corresponding IS-LM crossing point. Would such a ΔG be financed by a simultaneous money issuing –i.e, an increase in Money Supply–? This would mean that also the LM will have changed ... . And 2): If an ΔG has been carried out (financed, f.e., by a simultaneous money issuing), which leads to a something higher increase in Production//Income (Y*=Y), that means we are implicitly assuming that initially there were idle production factors: Labour, and firms' physical Capital endowments. Then, what kind of *virtue* would the starting equilibrium point –in the IS-LM diagram– have? A macroeconomic *equilibrium* with idle resources, without full employment?

More in general, *autonomous* changes like those considered in mainstream texts (ΔG, ∇T, $\Delta I_0$, ΔM, as well as the respective opposed moves) would not just cause, in the model, a simple in-parallel shift of IS or of LM rightward (or leftward). This type of observation is what led some authors to coin the expression "the pseudo-dynamics" of the IS-LM model (Backhouse and Laidler, 2004:32).

These remarks show once again that the IS-LM is a model intended to formalise the conditions of the *economic and financial stability* of a market economy; i.e., about a *static equilibrium*. In other words, that *time* is not indeed a variable in the model. Therefore, when we pass to consider that the economy is something dynamic, subject to changes, but try nevertheless to use the said model to explain such moves (through side shifts in IS or LM) then we get into contradictions or incoherencies as those discussed before; which leads to question the usefulness of the whole model as an explanatory device. These types of critiques/statements have given rise, of course, to new developments in academic literature, on the line of dynamic models; but this goes beyond the intended scope of this paper; an analysis in this regard may be seen in Backhouse and Laidler (2004:34-52).

Las but not least, the model conveys graphically, as its core idea, an implausible overestimation of the effect of the *interest rate* over the economic macro-magnitudes. This especially regarding both *investment* decisions (that mainly depend on business/firms' plans and expectations) and people's behaviour (mainly on households) regarding keeping money available (*preference for liquidity*). In short, the interest rate is indeed the centre of gravity of the IS-LM model.

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Simultaneously to the above underlined persistence of the IS-LM –as 'the Macro model' in current manuals–, some variants of it 'with curves' shifts' passed to also include *foreign sector flows* so evolving somehow towards dynamic models. Notably, in this respect, the Mundell-Fleming model –which incorporated the *exchange rate* and *capital's foreign flows* as variables–; as well as the Prescot's model, which introduced –and focused on– the effects of *productivity* changes on the usual macro-variables. 'Real Business Cycle' (RBC) and 'New Keynesian model' have been in this regard academic labels related to such evolution. And, as a later development of this trend, we have the 'Dynamic Stochastic General Equilibrium' models (DSGE), which have become a new academic paradigm for years; these are the kind of models currently used by, among others, most central banks to follow up and forecast the economy's trends, and define their monetary policy; the interest rate in the foremost.

All of that, however, entails in itself an entire new macroeconomics topic, that overcomes the purpose of this paper.

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## Appendix 1

(a)

The *multiplicator* of an increase in the Autonomous Demand, ΔÂ:

| Cycle | ΔÂ, ΔAD | ←time→ | Δ Prod. ($\Delta Y^*$) | ←time→ | Δ Incomes (ΔY) | Δ Aggregate Demand (ΔAD) |
|---|---|---|---|---|---|---|
| 1 | a | | a | | a | $c_1 \cdot a$ |
| 2 | $c_1 \cdot a$ | | $c_1 \cdot a$ | | $c_1 \cdot a$ | $c_1^2 \cdot a$ |
| 3 | $c_1^2 a$ | | $c_1^2 \cdot a$ | | $c_1^2 \cdot a$ | $c_1^3 \cdot a$ |
| : | | | | | | |
| : | | | | | | |
| n | $c_1^{n-1} \cdot a$ | | $c_1^{n-1}$ a | | $c_1^{n-1} \cdot a$ | $c_1^n \cdot a$ |

Sum of ($\Delta AD$) = $a \cdot \left( \frac{1}{1-c_1} - \frac{c_1^{\ n}}{1-c_1} \right) \equiv a \cdot m$; Sum $_{\text{when } n\to\infty}$ = $a \cdot \left( \frac{1}{1-c_1} \right)$, → m = $\frac{1}{1-c_1}$

§

(b)

The same quantitative result as above, for n→∞, can simply be mathematically deduced by directly assuming that the new macroeconomic equilibrium has been already achieved, instantly, just within the time unit (year, quarter, ..):

$$\left.\begin{array}{l} AD =^{mi} \hat{A} + c_1 \cdot Y \\ AD =^{e} Y \end{array}\right\} \begin{array}{l} \to Y(1-c_1) =^{m} \hat{A}, \\ \to \Delta AD =^{c} \Delta Y; \end{array} \Bigg\} \to \mathbf{\Delta Y}\ (1-c_1) = \mathbf{\Delta\hat{A}} \equiv a;\ \ \Delta Y =^{mi} a/(1-c_1) \quad \Delta AD = a\frac{1}{1-c_1}$$

§

(c)

And also through expressing the final equilibrium in terms of S=I= $\bar{I}$:

Since the standard macroeconomic equilibrium condition also requires, or entails, Investment = Savings, I=S, and Demand of investment goods, $\bar{I}$, equal to Investment goods produced I ≡ $Y^*_I$, the same above conclusion can also be reached via the I=S condition. Let us summarise the algebra for this alternative path to deduce the *multiplier* (for an increase in the Autonomous Demand) in some Macroeconomics or Principles' textbooks:

- Production of goods and services: $Y^* =^{d} Y^*_C + Y^*_I$ ; $Y^*_I =^{d} I$; $Y^* =^{ai} Y$
- Income expending: $Y =^{m} (C + S) + T$ ; where: $C =^{m} C_h + G \equiv C_0 + c_1 \cdot (Y-T) + G$
- Equilibrium condition: $S =^{c} I =^{c} \bar{I}$; ;

$\to \ Y = [C_0 + c_1 \cdot (Y-T)] + G + S \equiv [(C_0 - c_1 \cdot T) + G + \bar{I}] + c_1 \cdot Y =^{m} AD$

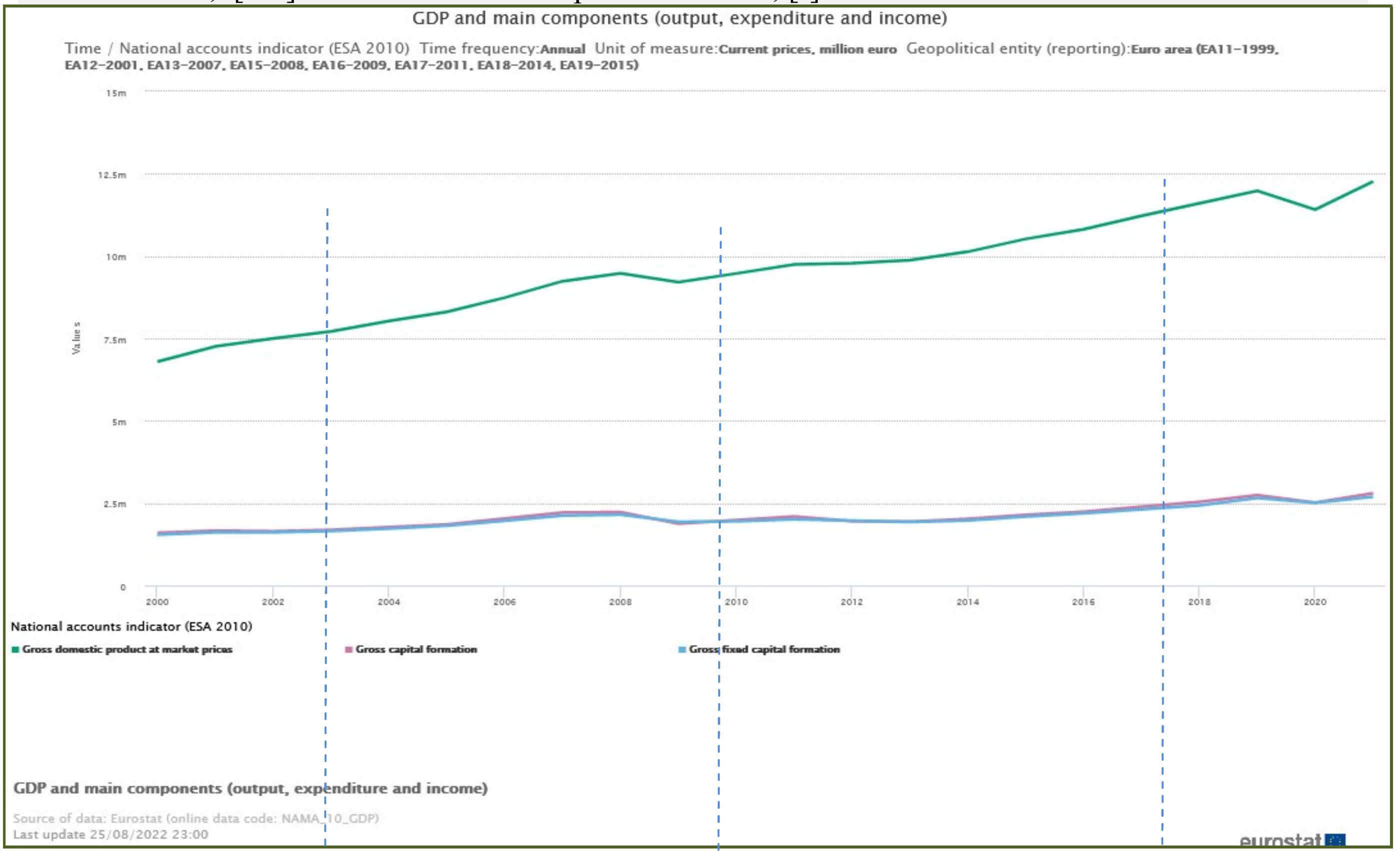
$\equiv Y(1- c_1) = (C_o - c_1 \cdot T) + G + \bar{I} =^{m} \hat{A}$ ; $\to Y =^{c} AD = \hat{A} \cdot \frac{1}{1-c_1}, \to \Delta AD = \Delta\hat{A} \cdot \frac{1}{1-c_1}$

Thereby $\Delta AD \equiv \Delta Y^*$ = $a\frac{1}{1-c_1}$

## Appendix 2

On the sensitivity of *Investment* and *Production//Income* to the *interest rate*

EU : GDP, [ *Y** ] and Gross Fixed Capital Formation, [*I*] 2000-2022




EU: Monetary Markets' *Interest rates*, [i] 2000-2022