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The Great Gatsby Curve and the Carnegie Effect*

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Abstract

We show that US commuting zones with higher income inequality exhibit less upward social mobility at the bottom of the income distribution, more downward social mobility at the top, and lower average income. We explain this empirical evidence through a life-cycle model in which investment in education and effort increase labor income, and individuals are altruistic, suffer disutility from exerting effort, and face a credit constraint. We propose two mechanisms driving those findings. First, due to the credit constraint, investment in education and income of individuals born into low-income families is constrained by parental wealth, which explains that upward social mobility at the bottom is lower in commuting zones with higher inequality where low-income families have less wealth. Second, individuals born into affluent families exert less effort and earn lower labor income when they inherit a larger wealth, which explains that downward social mobility at the top is larger in the most unequal commuting zones where affluent families are wealthier.

JEL classification: D64, E21, E71, I24.

Keywords: Inequality, social mobility, education, effort.

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1 Introduction

The measurement of intergenerational social mobility attempts to quantify the relationship between the income of parents and that of their children. Corak (2013), using cross-country data, and Chetty et al. (2014a, 2014b), using within-country data, show that social mobility is lower when cross-section income inequality is larger.[1] The literature on social mobility calls this negative relationship between inequality and social mobility the Great Gatsby curve (Durlauf, 2022). This literature has shown that this relationship is a robust empirical finding that holds under different measures of social mobility.

The literature studying the Great Gatsby curve has focussed on poverty and its persistence across generations (see, for instance, Jarrim and Macmillan, 2015; Halter, 2015; and Caucutt and Lochner, 2020). This is obviously important for equal opportunity rights, and it is also very relevant for efficiency, since the persistence of poverty is generally explained by market imperfections that lead to a misallocation of talent and, thus, to a loss of efficiency. In contrast, in this paper we study the relationship between income inequality and social mobility of individuals born into families whose income is not only at the bottom of the income distribution but also at the top. We thus contribute to the literature by studying the relationship between inequality and social mobility of individuals born into families whose income falls at different parts of the income distribution. We argue that studying the decisions of individuals born into affluent families is relevant because these decisions explain a significant part of the relationship between inequality, social mobility, and average income.

We first use data from Chetty et al. (2014a) on social mobility for each commuting zone (CZ) in the US to study the correlation between income inequality and social mobility. In the CZs with more income inequality, we find less upward social mobility for individuals born in families whose income falls in the lower part of the income distribution (first two quintiles), more downward social mobility for individuals born in families that fall in the upper part (last two quintiles), and a significantly lower average income. Note that these results imply that the Great Gatsby curve is only explained by individuals born in low-income families, since the correlation between income inequality and social mobility becomes positive for individuals born in high-income families.

We rationalize these findings using a life-cycle model in which labor income increases with

[1] Other empirical papers that study social mobility are Blanden (2013), Björklund and Jantti (2019), Cervini-Pla (2015), Corak (2006), d'Addio (2007), Isaacs (2007), Jantti, et al. (2006), and Solon (2002).

education and effort, and individuals receive an inheritance motivated by joy-of-giving altruism. The two mechanisms driving the relationship between income inequality and social mobility are a credit constraint and the disutility from exerting effort. Credit constraints were introduced in the seminal papers on parental investment in education by Becker and Tomes (1979, 1986). In these papers, credit constraints impose a limit on the investment in the education of children born into low-income families. As a result, those children earn low income as adults so that credit constraints generate persistence of low-income across generations. In addition, since the number of low-income families increases with inequality, this mechanism explains the negative correlation between income inequality and social mobility, i.e., the Great Gatsby curve. However, this mechanism does not explain why this correlation is positive among individuals born into high-income families. To explain this finding, we introduce the effort disutility, which implies that individuals from affluent families exert less effort and obtain lower labor income as adults when they inherit a larger amount of wealth. This effect of inherited wealth on individuals' effort, that is known in the empirical labor literature as the Carnegie effect, explains the larger downward social mobility at the top of the income distribution when inequality is larger and, thus, affluent families are wealthier.[2]

The model presents clear implications regarding the effect of inherited wealth on individuals' decisions on both education and effort, which ultimately determine their labor income. When inherited wealth is low, individuals face the credit constraint and receiving a larger inheritance enables to increase the investment in education, which leads to higher labor income. Conversely, when inherited wealth is sufficiently high, individuals do not face a credit constraint and larger wealth results in a reduction of effort and, thus, on lower labor income. Therefore, we obtain a hump-shaped relationship between labor income and inherited wealth.

We empirically validate this relationship using microdata from the PSID and linking individual data on labor income, education, hours worked, and occupation with parental wealth during individuals' teenage years. Since the PSID does not provide data on inherited wealth, we use parental wealth as a proxy for inherited wealth. Our empirical analysis confirms the hump-shaped relationship and shows that it emerges due to the impact of wealth on education and effort levels.

An interesting implication of the model is that a larger inequality in the inherited wealth reduces income of individuals born into both low and high-income families. On the one hand,

[2] Holtz-Eakin, et al. (1993), Elinder, et al. (2012) and Brown, et al. (2010) have shown a negative effect of inheritances on labor supply.

larger wealth inequality reduces the inheritances received by individuals born in low-income families, which constrains their investment in education and results in lower income. On the other hand, larger inheritance inequality increases the inherited wealth of individuals born into affluent families. Since inherited wealth reduces effort, the average labor income of those individuals is also lower in more unequal economies. We provide empirical support to these results by showing that the income of individuals born in CZs characterized by larger inequality is lower at different percentiles of the income distribution.

In the last part of our paper, we use the model to perform a quantitative analysis. To this end, we assume that there are two sources of heterogeneity among individuals: inheritance and innate abilities. The distributions of inheritance and abilities are calibrated to match moments of the distributions of wealth and labor income of the US economy. The rest of parameters of the model are set to match several targets for the US economy, including measures of social mobility. We perform two quantitative exercises. First, we conduct simulations to compare CZs characterized by different levels of inherited wealth inequality. We show that the model generates a correlation between parental income inequality and several measures of social mobility that fits well with the observed correlations. Second, we show that a larger inequality in the wealth distribution reduces average labor income through a reduction in the average level of education of individuals born into low-income families and in the average effort exerted by individuals born into affluent families. An obvious question is then to measure the contribution of each mechanism in explaining the drop in average labor income resulting from a larger inequality. We find that 20% of this drop is explained by the reduction in effort due to the Carnegie effect.

The literature on social mobility has extended the seminal papers of Becker and Tomes (1979, 1986) to explain cross-country differences in social mobility. These extensions include social classes (Galor and Zeira 1993), the effect of parental human capital on the returns of education (Becker, et al. 2018), peer effects (Hassler and Mora, 2000), segregation (Durlauf and Seshadri, 2018), and cultural differences (Lekfuangfu and Odermatt, 2022), among other extensions. Our paper makes two contributions to this literature. First, we introduce the Carnegie effect in models of social mobility. While Degan and Thibault (2016) and Alonso-Carrera, et al. (2020) have introduced the Carnegie effect to analyze the long run dynamics of social classes in a model in which educational investment is indivisible, we introduce the Carnegie effect to explain the Great Gatsby curve. Second, and more important, we find that a larger income inequality has opposite effects on social mobility for individuals born in low-and in high-income families. We show that a model of social mobility that combines credit market imperfection and the Carnegie effect explains these opposite effects of inequality on social mobility.

The rest of the paper is organized as follows. Section 2 provides empirical evidence on the relationship between inequality and social mobility. Section 3 introduces the model and characterizes the two mechanisms that govern this relationship. Section 4 validates the model by analyzing the effect of parental wealth on labor income using microdata. Section 5 uses the model to conduct quantitative analysis that inform about the relative importance of the two aforementioned mechanisms governing the relationship between inequality and social mobility. Section 6 concludes the paper. Some technical details are relegated to an appendix.

2 Empirical evidence

In this section, we analyze the relationship between parental income inequality, social mobility, and the income that children obtain as adults, which we dub future income. To conduct the analysis, we use data from Chetty et al. (2014a), who use the federal income tax record to obtain information on the cohort born in the period 1980-1982. Specifically, they obtain the distribution of parental income in the period 1996-2000 when children were teenagers and the distribution of children's income in 2011-2012 when they were young adults. Income is defined as the average total taxable income in the two periods, 1996-2000 and 2011-2012.

Chetty et al. (2014a) provide these distributions for each CZ, facilitating the utilization of this regional data to examine the correlation between parental income inequality in the CZ and the future income of children raised in the CZ.[3] We conduct two analyses: the first focusses on the relation between inequality and social mobility whereas the second examines the association between inequality and future income.

2.1 Inequality and social mobility

Chetty et al. (2014a) provide the Gini index, transition matrices and the estimated coefficients of Rank-Rank regressions for 707 CZs of the US.[4] These Gini indexes are obtained from disposable income data for the period 1996-2000. Therefore, they provide information about income

[3] Individuals belong to the CZ where they grow up as teenagers, even if they live in a different CZ as adults.

[4] There are 741 CZs in the US but Chetty et al. (2014a) do not provide coefficients of Rank-Rank regressions when the number of children in the CZ is too low. A detailed explanation on Rank-Rank regressions and their coefficients can be found in Dahl and DeLeire (2008).

inequality among parents in each CZ when children were teenagers. Note that transition matrices and coefficients of Rank-Rank regressions are alternative measures of social mobility. We employ both measures to investigate the relationship between parental income inequality and social mobility.

We first use the transition matrices in which parents and children are grouped by quintiles of income, which are defined according to the national income distribution. The elements of these matrices are the conditional probabilities of the children's income falling into a quintile of the national income distribution given the parents' position in this distribution. Table 1 displays two transition matrices. One is generated as the average of the transition matrices of CZs with a Gini index below the median of all Gini indexes, while the other is obtained as the average of the matrices of CZs with a Gini index above the median. Observe that there is a substantial difference between the average values of the Gini index in these two groups of CZs.

Table 1. Transition matrices

	Children						Children				
Parents	Q1	Q2	Q3	Q4	Q5	Parents	Q1	Q2	Q3	Q4	Q5
Q1	0.33	0.28	0.18	0.13	0.08	Q1	0.28	0.23	0.20	0.17	0.12
Q2	0.24	0.25	0.21	0.17	0.12	Q2	0.19	0.20	0.22	0.22	0.18
Q3	0.17	0.20	0.22	0.22	0.19	Q3	0.13	0.17	0.21	0.25	0.24
Q4	0.13	0.16	0.20	0.24	0.26	Q4	0.10	0.14	0.20	0.26	0.30
Q5	0.12	0.13	0.17	0.24	034	Q5	0.09	0.12	0.17	0.25	0.37
Average Gini: 0.47. Mobility: 0.70						Average Gini: 0.34. Mobility: 0.74					

Source. Data is from Chetty et al. (2014a). Mobility is defined as one minus the second highest eigenvalue. The elements of these matrices are obtained as the average value of the probabilities in the transition matrices of commuting zones with a Gini index that is below or above the median of the Gini indexes.

These matrices show some common features. First, we observe upward and downward social mobility. The former implies that individuals born in low-income families move into higher income quintiles as adults and the latter implies that individuals born in affluent families move into lower income quintiles as adults. However, despite this evidence of social mobility, there is large intergenerational income persistence, especially in the lower and higher quintiles. The persistence of income inequality is illustrated by the high probability that individuals born into families in the bottom two quintiles remain in these quintiles, which significantly exceeds 20%. Similarly, individuals born into affluent families show persistence at the top, as probabilities of remaining in higher quintiles lie also above 20%.

Next, we compare the two transition matrices to analyze the connection between parental income inequality and social mobility. To perform this analysis, we use as a measure of social mobility one minus the second highest eigenvalue.[5] This measure equals zero when there is full intergenerational persistence and one when there is perfect mobility. We observe that this measure of mobility is 0.74 in low income inequality CZs and 0.70 in the more unequal CZs. This is consistent with the Great Gatsby curve, according to which more inequality is associated with less mobility.

In Table 1, we also observe that, in those CZs with a higher income inequality, the probabilities of falling into the first two quintiles given that the individual is born in a family whose income is in these quintiles are larger and, in contrast, the probabilities of falling into the last two quintiles given that the family's income is in these quintiles are smaller. Therefore, in CZs with higher inequality there is more persistence at the bottom of the distribution and there is more mobility at the top. It follows that the Great Gatsby curve is explained only by the relationship between inequality and social mobility of individuals born in low-income families.

Table 2. OLS coefficients

	descendants				
Parents	Q1	Q2	Q3	Q4	Q5
Q1	0.35	0.39	-0.09	-0.30	-0.34
Q2	0.36	0.35	-0.02	-0.30	-0.39
Q3	0.29	0.24	0.05	-0.23	-0.35
Q4	0.22	0.18	0.05	-0.15	-0.31
Q5	0.16	0.07	0.04	-0.08	-0.20

Note. OLS coefficients are obtained from regressing each element of the transition matrices of the commuting zones against a constant and the Gini index. Data is from Chetty et al. (2014a).

To confirm these findings, we pool the transition matrices of the 707 CZs and we regress each element of the transition matrices against a constant and the Gini index in each CZ. Table 2 provides the ordinary least square coefficient of these regressions. All the coefficients are significantly different from zero at the 1% significance level. Those coefficients provide information about the correlation between parental income inequality and each element of the transition matrix and, thus, they shed light on the link between income inequality and

[5] See Caballé (2016) for a discussion on different measures of social mobility.

social mobility across different parts of the income distribution. The coefficients in the main diagonal are especially informative. We observe a positive correlation between inequality and the probability that children from low-income families (the two lower quintiles) remain in the lower quintiles. Consequently, this suggests that income inequality is associated with larger persistence for individuals born into low-income families. In the third quintile, we observe a very small positive correlation. Finally, for the top two quintiles, the correlation is negative, indicating that inequality is associated with larger downward social mobility.

In what follows, we use the coefficients obtained in the Rank-Rank regressions to confirm the previous results regarding the relation between inequality and social mobility. For each CZ, Chetty et. al (2014a) regress children percentile rank in the national income distribution against a constant and parents percentile rank. The slope-coefficient measures the relative social mobility so that a smaller coefficient implies more mobility. The predicted children percentile rank, which is equal to the sum of the constant and the slope-coefficient times the parents percentile rank, measures the absolute social mobility at the parents' percentile rank. There is upward social mobility when the expected percentile rank of the children is higher than the percentile rank of the parents and downward social mobility otherwise. Chetty et al. (2014a) show that in all CZs there is upward social mobility for individuals born in families belonging to the low percentiles of the income distribution and downward for those other individuals born in affluent families. In fact, using the constant and the slope coefficients, we can easily obtain for each CZ the threshold of the percentile rank of the parents above which there is downward social mobility and below which there is upward social mobility.[6] We observe that these thresholds are between percentiles 25 and 75. As a consequence, in our analysis we consider families at the 25th percentile to be an example of a low-income family and those at the 75th percentile to be an example of a high-income family.

Table 3. Coefficients of Rank-Rank regressions and average income

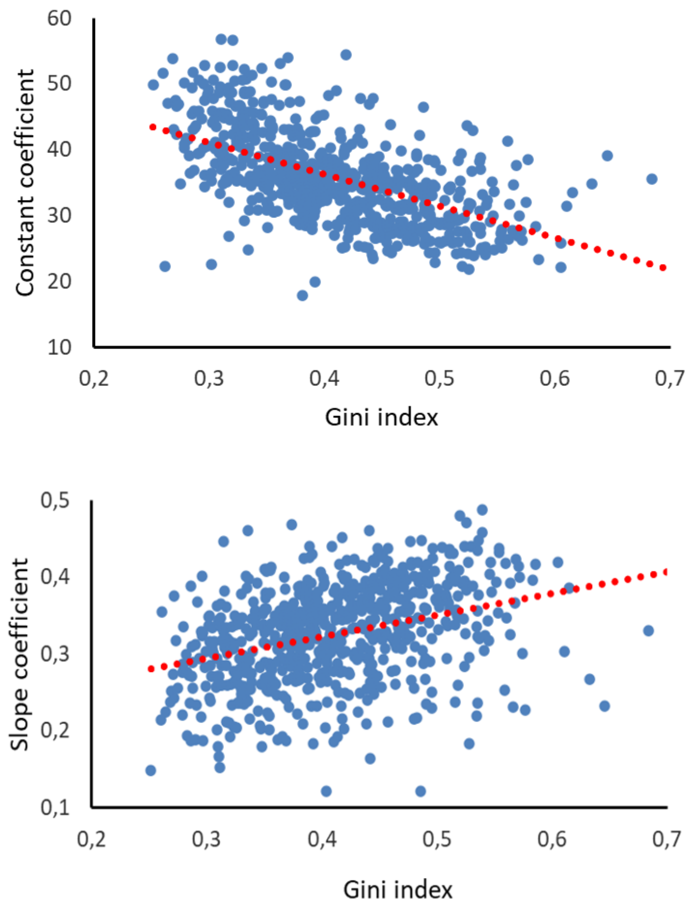
Gini	Constant	Slope	Threshold	Percent. 25	Percent. 75	Avg. Parent Inc.	Avg. Child Inc.
0.35	39.2	0.30	56.0	46.7	61.7	67,748	50,325
0.47	32.4	0.34	49.1	40.9	57.9	68,551	42,641

Source. Data is from Chetty et al. (2014a). The constant, the slope, the average parental and children income are obtained as the average value of these variables in CZs with a Gini index that is below or above the median of the Gini indexes. The threshold and the predicted rank percentiles are computed using the constant and the slope.

[6] This threshold is obtained as the constant divided by one minus the slope.

We next compare the coefficients of the Rank-Rank regressions to study the relationship between income inequality and social mobility. At this point, it is important to clarify that this analysis, based on the comparison between CZs, is meaningful because individuals are ranked in each CZ according to their position in the national income distribution.[7] Table 3 provides the average value of the Gini index, the two estimated coefficients and the threshold of parents' percentile rank of two groups of CZs: one group consisting of those CZs with a Gini index below the median of the Gini indices and another group consisting of those other CZs with a Gini index above the median. We observe that in those CZs with larger inequality the constant is substantially smaller and the slope is slightly larger.

Figure 1. Coefficients of Rank-Rank regressions and Gini index



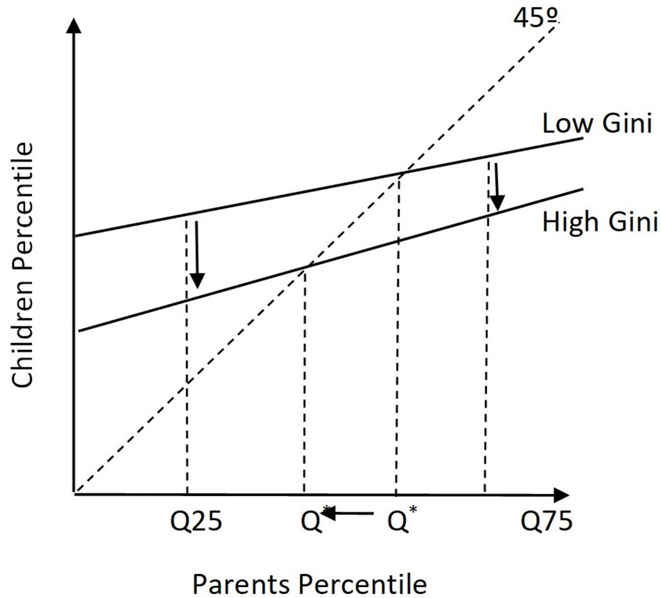
Source. Data is from Chetty et al. (2014a). Coefficients and the Gini index are of the US CZs.

The results in Table 3 are confirmed in Figure 1, which shows the relationship between the

[7]If individuals were ranked according to the income distribution of their commuting zones, then we would mechanically observe that when a group of individuals in a commuting zone has a higher rank, another group will necessarily have a lower rank.

two estimated coefficients, the constant and the slope, and the Gini index. This figure shows that inequality is negatively correlated with the constant coefficient (first panel) and positively correlated with the slope coefficient (second panel). Thus, the second panel displays the great Gatsby curve as the positive correlation between the Gini index and the slope implies that a larger inequality is associated with a smaller relative social mobility.

Figure 2. Children predicted percentile rank

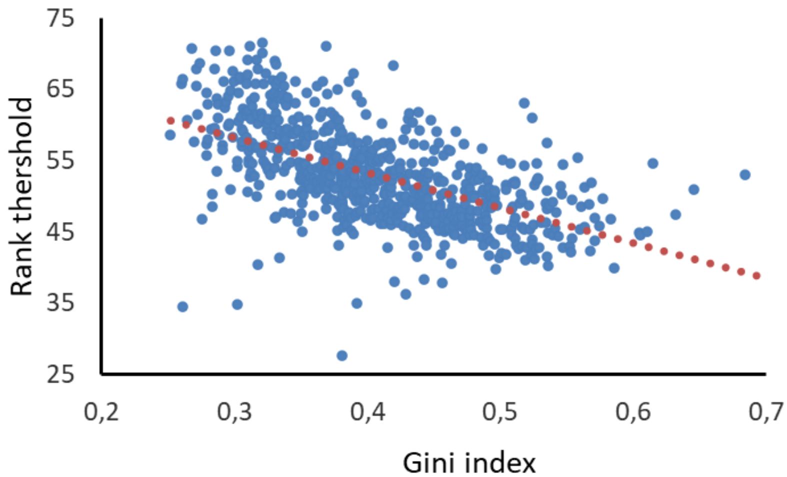


Note: Q* indicates the thershold of the percentile rank of parents.

Figure 2 illustrates the impact of a larger parental income inequality on social mobility in the CZs of the US economy. It plots the predicted children's percentile rank against parents' percentile rank using the coefficients from the Rank-Rank regressions of the two groups of CZs detailed in Table 3. Consistent with Table 3, the CZs with higher inequality exhibits a smaller constant and a slightly steeper slope. In the CZs with larger inequality, we first observe lower upward social mobility for individuals born into low-income families (with a parental rank percentile below the threshold), implied by the closer proximity of the predicted rank percentiles of children to those of their parents. Second, there is larger downward social mobility for individuals born into high-income families (above the threshold), as evidenced by the greater divergence between the rank of children and that of their parents. Additionally, the threshold value of the percentile rank is smaller in more unequal CZs. Note that these findings align with those obtained from the analysis of the transition matrices.

We can now use Table 3 to quantify the effects of a larger inequality. This table shows that a 12-points increase in the Gini index corresponds to a 6-point reduction in the rank percentile of children born into families at the 25th percentile, a 4-point reduction for those at the 75th percentile and almost a 7-point reduction for the threshold rank percentile. Notably, this last effect implies that a larger inequality moves from 56% to only 49% the fraction of families whose children are in a higher percentile rank than parents.

Figure 3. Threshold of percentile rank and Gini index



Source. Data is from Chetty et al. (2014a).

We perform two empirical exercises to confirm the insights obtained from Figure 2. First, we use the estimated coefficients to compute the thresholds for each CZ. We regress these thresholds against a constant and the Gini index. The results of this regression are shown in Figure 3 and confirm that inequality and the threshold are negatively correlated. Second, we regress the predicted percentile rank of the children born in families whose income falls into different rank percentiles against a constant and the Gini index. The results of these regressions are shown in Table 4. We obtain that higher inequality is associated with lower predicted percentile ranks, which is consistent with the implications obtained from Figure 2.

Table 4. Predicted percentile ranks of children

Parental percentile	10	25	50	75	90
Constant	57.74*** (1.044)	60.88*** (0.920)	66.12*** (0.766)	71.35*** (0.713)	74.50*** (0.740)
Gini index	-45.60*** (2.502)	-41.36*** (2.205)	-34.31*** (1.835)	-27.25*** (1.708)	-23.01*** (1.774)
Observations	707	707	707	707	707
R^2	0.32	0.33	0.33	0.27	0.19

Note: *** indicates p-value $<$ 0.01. Standard errors in parenthesis.

Summarizing, we have analyzed the effect of a higher parental income inequality on the conditional probabilities of the transition matrices and also on the coefficients of the Rank-Rank regressions. The first analysis shows that in more unequal CZs the probability that individuals born in low-income families earn low-income increases and the probability that individuals born in high-income families earn high-income decreases. The second analysis confirms these findings by showing that the predicted rank percentile of children born in low-income families is closer to the rank percentile of their parents when inequality is higher, while it has the opposite effect on children born in high-income families. Thus, we can safely conclude that in CZs with larger income inequality upward social mobility for individuals born in low-income families is smaller, whereas downward social mobility for individuals born in high-income families is larger.

2.2 Inequality and future income

We have just shown that the children's predicted rank percentiles are smaller in CZs with larger inequality. Since the rank percentile is a measure of absolute social mobility, this finding also indicates that higher inequality is associated with lower children's average income. Table 3 illustrates this effect by showing that although average parental income is slightly larger in more unequal CZs, average children income is substantially smaller.

The negative correlation between inequality and children's income or future income is shown in Table 5. The second column of this table provides the estimated coefficients obtained from the regression of children's average income against a constant, the Gini index and parents' average income. The negative and large coefficient associated to the Gini index confirms that a larger income inequality is associated with lower future average income. In the remaining columns of Table 5 we consider the children income at a particular percentile. We show that the negative correlation with inequality occurs at every percentile of the distribution of children's income .

Table 5. Income inequality and future income

Child. income	Average	P10	P25	P50	P75	P90
Constant	6.91*** (0.20)	-0.81*** (1.26)	4.40*** (0.44)	6.11*** (0.26)	7.49*** (0.19)	8.60*** (0.14)
Gini index	-1.21*** (0.05)	-6.70*** (0.35)	-2.52*** (0.12)	-2.00*** (0.07)	-1.27*** (0.05)	-0.63*** (0.04)
Parent income	0.39*** (0.01)	1.05*** (0.11)	0.57*** (0.04)	0.46*** (0.02)	0.37*** (0.02)	0.28*** (0.02)
Observations	707	688	707	707	707	707
R^2	0.55	0.39	0.44	0.60	0.57	0.48

Note: *** indicates p-value $<$ 0.01. Standard errors in parenthesis. We consider as dependent variable the natural logarithm of average children income and also the logarithm of children income at percentiles 10, 25, 50, 75 and 90. Parental income is the natural logarithm of average income of parents in each commuting zone. Children income in the 10th percentile is zero in some commuting zones. We eliminate these observations when the children income at the 10th percentile is the dependent variable.

We conclude that as income inequality increases future income decreases and this reduction occurs at every percentile of the income distribution. The following section aims to provide an explanation of these negative correlations.

Before turning to the next section, we address an important clarification of the empirical analysis. This analysis reports the relationship between parental income inequality within a CZ and the average future income of children who were teenagers in that same CZ. It is important to note that, since these children may move to a different CZ as adults, the correlation between inequality and the children' future average income could differ from the correlation between inequality and the future average income of this CZ. Therefore, our analysis cannot be used to study the correlation between inequality and income growth across CZs. Although in the US migration flows through the CZs are large, we show that the negative correlation between inequality and future income remains when future income is defined as GDP per capita. To perform this analysis, we regress GDP growth by county over the period 2001-2011, that we obtain from the Bureau of Economic Analysis, against the logarithm of GDP per capita in 2001 and the Gini index in 1996 that we obtain from Chetty (2014a). We find that the correlation between inequality and growth is negative, large, and highly significative.

3 The model

We proceed to build an analytically tractable model explaining the evidence discussed in the previous section. To keep the model simple, we consider a small open economy populated by individuals who live for two periods. In the first period, individuals are young. Every young individual i receives an inheritance b, and has an innate ability a_i. This ability is an idiosyncratic productivity shock that affects wages. Inheritances and abilities introduce heterogeneity among individuals. In the second period of life, individuals are adult and have a unique descendant.

Young individuals decide between investing the inheritance in education or saving it in financial assets.[8] This decision is subject to a credit constraint: savings s_i cannot be used to borrow; i.e. $s_i \geq 0$. We are thus introducing an imperfection in the financial markets that constrains the investment in education for those individuals who receive a small inheritance.

The return of savings is the exogenous interest factor R that is set in the world financial markets and the return of investment in education takes the form of a larger wage in the next period. More precisely, let $\mu_i \equiv \mu(h_i)$ be the investment in education necessary to obtain the education level h_i. We assume that education is a continuous variable defined in the interval $h_i \in (0, \infty)$ and the function $\mu(h_i)$ is continuous, increasing, and convex; i.e., $\mu_h > 0$ and $\mu_{hh} \geq 0$, where the subindex in the function indicates the argument with respect to which the partial derivative is taken.

Adult individuals work and obtain a wage, $w_i \equiv w(a_i, e_i, h_i)$, that depends on the innate ability a_i, the education h_i achieved when young, and the effort e_i exerted when adult. We assume that the wage function is increasing in all its arguments, jointly concave in education and effort and it exhibits complementarity between its three arguments. Therefore, we assume that $w_e > 0$, $w_h > 0$, $w_{ee} < 0$, $w_{hh} < 0$, $w_{e_ie_i}w_{hh} > (w_{e_ih})^2$, $w_{eh} > 0$, $w_{ae} > 0$ and $w_{ah} > 0$. In addition, we assume that the elasticity of substitution between h_i and e_i is smaller or equal to one.

Adult individuals use the wage and the return from savings to consume c_i and leave a bequest b'_i. It follows that the budget constraints in both periods of life are

$$\mu(h_i) + s_i = b_i, \tag{1}$$

$$c_i + b'_i = w(a_i, e_i, h_i) + s_i R. \tag{2}$$

Preferences satisfy

$$u_i = \ln c_i + \beta \ln b'_i - \phi(e_i),$$

where $\beta > 0$ is the altruism parameter and $\phi(e_i)$ is the disutility of effort, which is an increasing

[8] By assuming that individuals optimally decide to invest the inheritance in savings (financial assets) or in education (productive investment), we avoid the issues of overeducation or of strategic interaction between parents decision on education and children decisions on effort (Alonso-Carrera et al., 2018), which are not relevant for the purposes of this paper.

and convex function of effort; i.e., $\phi_e > 0$ and $\phi_{ee} \geq 0$.

3.1 Individual decisions

Individuals choose education, consumption, bequests and effort to maximize utility subject to the budget constraints (1) and (2) and the credit constraint $s \geq 0$. From the first order conditions, we obtain [9]

$$c = \frac{w + R(b-\mu)}{1+\beta}, \tag{3}$$

$$b' = \beta c, \tag{4}$$

$$w_h = R\mu_h \text{ if } b \geq \mu \;\text{ and }\; w_h > R\mu_h \text{ if } b < \mu, \tag{5}$$

$$\phi'(e) = \frac{w_e}{c} = w_e \left(\frac{1+\beta}{w + R(b-\mu)} \right). \tag{6}$$

Equations (3)-(6) determine the values of h, e, c, and b' as functions of the inheritance b and ability a. Equation (3) provides the optimal consumption level and (4) describes the intergenerational optimal allocation between consumption and bequests. These two equations determine the dynamics of bequests across generations given the life-time wage. This wage depends on education and effort decisions that are determined, respectively, by equations (5) and (6). These two decisions deserve some detailed explanation as they determine how the wage is affected by the inheritance received from the parents and, therefore, they will be crucial to explain the empirical findings of the previous section. In what follows, we study the effect of a larger inheritance on individuals decisions regarding effort and education.

3.1.1 Education decision

The education decision depends on the position of individuals in the asset market. When they are credit unconstrained lenders, $b \geq \mu$, the educational decision in (5) equalizes the marginal increase in the wage due to a larger investment in education with the marginal cost of this investment; i.e. $w_h = R\mu_h$. This equation implicitly defines the following function $h = H^u(e,a)$, with $H_e^u = -w_{eh}/(w_{hh} - R\mu_h) > 0$. This positive derivative shows that individuals who exert more effort invest more in education. Note this result is the consequence of the complementarity introduced by the wage function and that implies $w_{eh} > 0$.

When individuals are credit constrained, investment in education is limited by the inheritance. Education then satisfies the following equation: $\mu(h) = b$. This equation implicitly

[9] To keep the notation simple, in what follows we eliminate the subindex i.

defines the function $h = H^c(b)$, with $H_b^c = 1/\mu_h > 0$. A larger inheritance increases investment in education for credit constrained individuals. In contrast, investment in education does not depend on abilities or effort.

We denote the positive effect of inheritances on education as the imperfect financial market effect, since it is the consequence of the imperfection in the financial markets and it only affects credit constrained individuals. This effect was introduced in models of parental investment to explain social mobility since the seminal paper by Becker and Tomes (1976).

3.1.2 Effort decision

The the first-order condition (6) refers to the effort decision and equates the marginal cost in terms of utility of increasing effort with the marginal benefit. This benefit is obtained as the product between the marginal increase in the wage and the marginal utility of consumption. Using (4), we obtain that (6) defines a function $e = E^u(h, b, a)$ for unconstrained individuals and $e = E^c(h, a)$ for constrained individuals, with

$$\begin{aligned} E_h^u &= \frac{\frac{w_{eh}}{w_e}}{\frac{\phi_{ee}}{\phi_e} - \frac{w_{ee}}{w_e} + \frac{\phi_e}{1+\beta}} > 0, \\ E_h^c &= \frac{\frac{w_{eh}}{w_e} - \frac{w_h}{w}}{\frac{\phi_{ee}}{\phi_e} - \frac{w_{ee}}{w_e} + \frac{\phi_e}{1+\beta}} > 0, \end{aligned}$$

where the last inequality holds when $\frac{w_{eh}}{w_e} \geq \frac{w_h}{w}$. This condition is satisfied when the elasticity of substitution between h and e in the wage function is smaller than one. Therefore, the complementarity between education and effort explains why adult individuals who have accumulated more human capital exert more effort.

To study the effect of the inheritance on effort decisions, we consider the position of individuals in the financial market. For unconstrained individuals, the effect of a larger inheritance on effort is obtained from the following derivative:

$$E_b^u = -\frac{\frac{R}{1+\beta}\frac{\phi_e}{w_e}}{\frac{\phi_{ee}}{\phi_e} - \frac{w_{ee}}{w_e} + \frac{\phi_e}{1+\beta}} < 0.$$

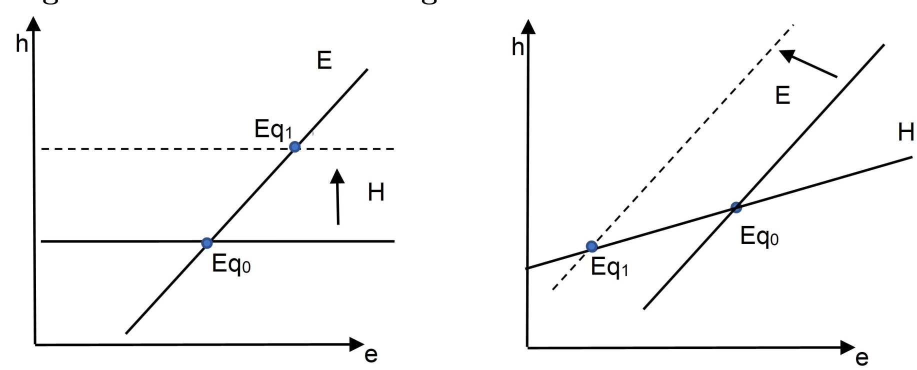
Therefore, a larger inheritance reduces effort for unconstrained individuals. As follows from (4), a larger inheritance increases consumption and, hence, reduces the marginal utility of consumption. As a result, the marginal benefit from effort decreases and (6) indicates that the marginal disutility of effort must also decrease. This explains the negative effect of inheritances on effort, which is known as the Carnegie effect.

For constrained individuals, the investment in education equals the inheritance. As a result, any increase in inheritances is used to increase education and does not change consumption, nor the marginal benefit of effort. This implies that the effort decision does not depend on the inheritance received for the credit-constrained individuals. In other words, these individuals are not affected by the Carnegie effect.

3.1.3 The effect of inheritances on education and effort

We next use the functions E and H to study the effect of a larger inheritance on the equilibrium values of effort and education. Figure 4 plots these two functions and distinguishes between constrained and unconstrained individuals. For the later, both functions are positively sloped. Therefore, a first necessary step is to determine which function is steeper at the point where they cross. In the appendix, it is shown that concavity of the wage function implies that the function E is steeper than the function H. Taking this into account and using the comparative static results that we have just discussed, Figure 4 shows the effect on individuals decisions of an increase in inheritances.

Figure 4. The effect of a larger inheritance on education and effort



Constrained individuals Unconstrained individuals

When individuals are credit constrained, the inheritance only affects the decisions on education through the imperfect financial market effect. A larger inheritance increases investment in education and, since the elasticity of substitution between education and effort is smaller than one, effort also increases.[10] Graphically, this is shown by the shift upwards in the function H. In contrast, when individuals are unconstrained, the inheritance only affects effort

[10] If the wage function is Cobb-Douglas, the elasticity of substitution equals one and the function E is vertical for constrained individuals. As a consequence, effort is constant and a larger inheritance only increases education.

decisions through the Carnegie effect. For these individuals, a larger inheritance reduces effort and, because of the complementarity between effort and education, investment in education also declines. Graphically, this is shown by the shift to the left in the function E.

We have seen that the effect of a larger inheritance on individuals decisions depends on the position in the asset market. Individuals who receive a low inheritance are constrained.[11] When these individuals receive a larger inheritance, they increase education, effort and labor income. In contrast, individuals who receive a large inheritance are unconstrained. For these individuals, a larger inheritance reduces effort, education and labor income. This suggests that as the inheritance received increases it has first a positive effect on effort, education and labor income but eventually, for a sufficiently large inheritance, individuals are unconstrained and the effect becomes negative. We conclude from the previous arguments that labor income, effort and education exhibit a hump-shaped relation with the inheritance received.

3.1.4 Inequality, income and social mobility

We proceed to intuitively discuss whether the model can explain the empirical results regarding the impact of a larger parental income inequality on income and social mobility when this larger inequality is associated to a more unequal distribution of inheritances.

We first consider the effect of a larger inequality on income. From the previous arguments, we easily infer that a more unequal distribution of inheritances reduces the income of both constrained and unconstrained individuals when a more unequal distribution implies that inheritances obtained by constrained individuals decrease, while inheritances received by unconstrained individuals increase. Therefore, this model can explain the results in Table 5, which shows that higher inequality reduces income at each percentile of the income distribution.

The model also explains the effect of inequality on social mobility, since they imply that a more unequal distribution of inheritances reduces income of both individuals born in low and high-income families. Therefore, a more unequal distribution of inheritances reduces upward social mobility for individuals born in low-income families and increases downward social mobility for individuals born in high-income families. It follows that the model can also explain the findings in Tables 2 and 4 that show the effect of inequality on social mobility.

[11] Credit-constrained individuals are those who receive low inheritance or a high innate ability. The complementarity between education and innate ability in the wage function implies that those individuals with high ability want to make a high investment in education and, as a result, will be credit constrained unless they obtain a high inheritance.

In Section 5, we complement this discussion with a quantitative analysis aimed at demonstrating that the model explains the correlations shown in the empirical section. To perform this quantitative analysis, we must first characterize the equilibrium. This characterization is the purpose of the following section.

3.2 Equilibrium

We assume the following functional forms that facilitate the analysis of the equilibrium and that we use in the quantitative analysis:

$$\begin{aligned}\mu &= Bh,\ B > 0,\\ \phi &= De^2,\ D > 0,\\ w &= Ah^{\alpha}e^{1-\alpha},\ A > 0.\end{aligned}$$

The parameter B measures the cost of education, D measures the disutility of effort and A measures the efficiency of technology. In this section, we assume that these parameters are identical for all individuals to keep the analysis simple. In the quantitative analysis of section 5, we introduce a distribution of abilities assuming that the value of parameter A is idiosyncratic.

3.2.1 Unconstrained individuals

Using (5), we obtain that the function H^u is $h_u = \Gamma e_u$, with $\Gamma = \left(\frac{\alpha A}{RB}\right)^{\frac{1}{1-\alpha}}$. The subindex u identifies optimal decisions of unconstrained individuals. We also deduce that the wage is $w_u = A\Gamma^{\alpha}e_u$. Using (6) and the function E^u, we obtain that effort is a function of inheritance $e_u = e\left(b\right)$ that solves

$$\left(\frac{1+\beta}{2De_u^2} - 1\right)\left(1-\alpha\right)A\Gamma^{\alpha}e_u = Rb. \tag{7}$$

We can easily see that $e\left(b\right) > 0$, $e'\left(b\right) < 0$, $e''\left(b\right) > 0$ and $e\left(0\right) = \sqrt{\frac{1+\beta}{2D}} \equiv \widehat{e}$. Therefore, effort satisfies that $e_u \in \left(0, \widehat{e}\right)$. The negative effect of the inheritance on effort is the Carnegie effect, which also implies that the wage declines with the inheritance.

Unconstrained individuals satisfy $b > \mu_u = B\Gamma e_u$, which implies that $Rb > BR\Gamma e_u$. Using (7), we obtain that unconstrained individuals exert effort $e_u < \overline{e}$ and receive $b > \overline{b} \equiv \alpha A\Gamma^{\alpha}\overline{e}/R$ with

$$\overline{e} = \sqrt{\frac{\left(1+\beta\right)\left(1-\alpha\right)}{2D}}.$$

We combine (3) and (4) to obtain that the bequests satisfy

$$b_u' = \beta\frac{\left(1-\alpha\right)A\Gamma^{\alpha}}{2De\left(b\right)}. \tag{8}$$

In the appendix, we use (7) and (8) to characterize the function $b_u'(b)$ that governs the transitional dynamics of the bequests of unconstrained individuals. We show that the function is increasing and convex with a slope smaller than one and $b_u'(b) > 0$ for all b. We also show that $b_u'(\overline{b}) < \overline{b}$ if and only if $R < \alpha(1+\beta)/\beta$. Equation (8) describes the transitional dynamics of bequests for families that initially receive $b > \overline{b}$. Given the properties of the function $b_u'(b)$, we deduce that bequests exhibit a monotonic transition towards the following steady state:

$$\begin{aligned} b_u^* &= \frac{\beta(1-\alpha)A\Gamma^\alpha}{2De_u^*}, \\ e_u^* &= \sqrt{\frac{1+\beta-\beta R}{2D}}. \end{aligned}$$

Note that this steady state is well-defined if $b_u^* > \overline{b}$ and $e_u^* > 0$. The first condition is satisfied when $R < \alpha(1+\beta)/\beta$ and the second when $R < (1+\beta)/\beta$. From now on, we assume that this second inequality is always satisfied.

3.2.2 Credit constrained individuals

Credit constrained individuals satisfy $\mu_c = b$, where the subindex c indicates the optimal decisions of a credit constrained individual. This equation implies that education satisfies $h_c = b/B$. Using (6), we deduce that $e_c = \overline{e}$. Note that effort is independent of inheritances, which implies that the Carnegie effect does not affect effort decisions. Instead, education increases with b as a consequence of the imperfections in the credit market. We next use the wage function to obtain $w_c = A(b/B)^\alpha \overline{e}^{1-\alpha}$. We observe that the inheritance increases labor income.

We combine (3) and (4) to get

$$b_c' = \beta\frac{A}{1+\beta}\left(\frac{b}{B}\right)^\alpha \overline{e}^{1-\alpha}. \tag{9}$$

Equation (9) describes the transitional dynamics of families that initially receive $b < \overline{b}$. In these families, bequests exhibit a monotonic transition towards the following steady state:

$$b_c^* = \left(\frac{\beta A}{(1+\beta)B^\alpha}\right)^{\frac{1}{1-\alpha}} \overline{e}.$$

The steady state for constrained individuals exists if $b_c^* < \overline{b}$, which happens when $R < \alpha(1+\beta)/\beta$.

3.2.3 Transitional dynamics

Given an initial distribution of inheritances, an equilibrium is a time path of $\{b', e, h, w\}$ for each family that satisfies (8), $e = e_u$, $h = h_u$ and $w = w_u$ if $b \geq \bar{b}$ and satisfies (9), $e = e_c$, $h = h_c$ and $w = w_c$ if $b < \bar{b}$.

The transitional dynamics are characterized by the dynamics of bequests between consecutive generations of individuals of the same family that are linked by altruism. This transitional dynamics are displayed in Figure 5 that plots bequest as a function of inheritance using the functions (8) and (9). This function is continuous as $b'_c\left(\bar{b}\right) = b'_u\left(\bar{b}\right)$.

Figure 5. Dynamics of bequests

$R > \alpha\left(1+\beta\right)/\beta$ $\qquad\qquad$ $R < \alpha\left(1+\beta\right)/\beta$

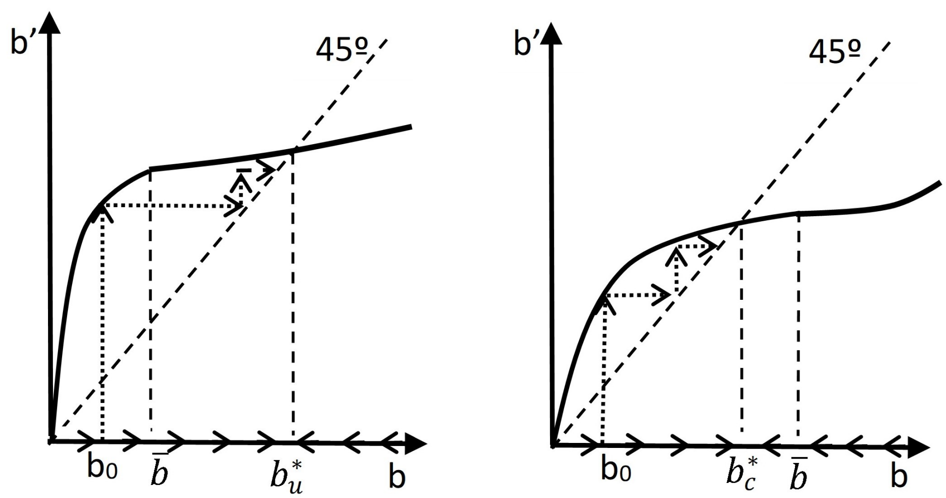


Figure 5 shows that we can distinguish between two types of transition in this economy. When $R \geq \alpha\left(1+\beta\right)/\beta$, we have that $b_c^* > b_u^* > \bar{b}$. In this case, individuals who receive an inheritance above $\bar{b}$ have descendants that will be unconstrained and the bequest will converge towards the steady state b_u^*. In contrast, individuals who receive an inheritance below $\bar{b}$ are credit constrained. Their descendants will be credit constrained during some generations until they inherit a quantity above $\bar{b}$ and become unconstrained individuals. In the long run, all individuals will be unconstrained lenders.

When $R < \alpha\left(1+\beta\right)/\beta$, we have $b_c^* < b_u^* < \bar{b}$. In this case, individuals who receive an inheritance below $\bar{b}$ are credit constrained. Their descendants will achieve the steady state b_c^*. In contrast, individuals who receive inheritances above $\bar{b}$ are unconstrained and their descendants will receive a lower inheritance. Eventually, these descendants will receive an inheritance lower than $\bar{b}$ and will be credit constrained. All individuals in the long run are credit constrained.

In the quantitative version of the model, the calibrated parameters satisfy $R > \alpha(1+\beta)/\beta$. However, since we introduce innate abilities as an individual productivity shock, varying across generations, the transitional dynamics of bequest in each family will not exhibit the monotonic and simple transition towards the steady state displayed in Figure 5. Nonetheless, monotonic transitions are still observable for the average values of the bequests.

4 Empirical validation of the model

The theoretical model outlined in the preceding section establishes that, for credit constrained individuals, there exists a positive correlation between education, effort, and labor income on the one hand, and inherited wealth on the other. Conversely, for unconstrained individuals, these correlations are negative. Given that constrained individuals receive a low inheritance, this model implies a hump-shaped relationship between education, effort, and labor income on one side, and inherited wealth on the other. In this section, we empirically examine these relationships using micro-data from the PSID (Panel Study of Income Dynamics). Specifically, we investigate how individual labor income and decision-making concerning education, hours worked, and occupation are influenced by parental wealth. Since we do not observe inheritances, we use parental wealth as a proxy of inherited wealth. The data used, except for parental wealth, corresponds to the year 2013 and is restricted to individuals aged between 25 and 54. All individuals are either head or wife of the family unit. We next clarify some aspects of the data used.

We ensure consistency with the methodology used in Chetty et al. (2014) by considering parental wealth during the individuals' teenage years, specifically between the ages of 15 and 25.[12] To this end, we split the sample of individuals into three age groups: 24-34, 35-44 and 45-54 years old. We obtain data on parental wealth from the years 2003, 1994, and 1984, which correspond to the years when individuals within each of these three age groups were teenagers. We use the GDP deflator to obtain wealth data in the different periods valued at 2003 US dollars.

The variables used are defined as follows. Individual labor income is the logarithm of annual labor income in the last year. It includes all last year income due to all the work for money, including jobs, businesses self-employment and part-time work.[13] When the time unit

[12] We consider parents both biological and adopted. When the members of the family (head and wife) report different wealth then we consider parental wealth is the wealth of the individual that reports the largest value.

[13] A remarkable difference with Chetty et al. (2014a) is that in this analysis we consider labor income, whereas

is not the entire year, we transform labor income into annual equivalent. Education refers to the highest grade or year of school completed and ranges between 1 and 17. Hours refer to average hours worked per week in all jobs and ranges between 1 and 112. Occupation is the most recent main job and the classification is based on the 3-digit occupation code from 2000 Census of population and housing of the Bureau of the Census. Wealth is the sum of 6 assets including home equity and net of debt value. We will use as control variables age of the individual in 2013, number of children in the family unit that were under 18 in 2013, a dummy variable to identify whether the individual is the head of the household and another dummy variable that identifies whether the individual is black.

We construct an effort index using the residuals of Mincer regressions. In particular, we group occupations into 25 groups that are defined in the PSID. We compute the average labor income and education level in each group of occupations and obtain the residuals that result from regressing average labor income against a constant and average education level in the occupation. We interpret these residuals as a measure of effort in each occupation and we normalize them to construct an index of effort by occupation ranging from zero to one. We assume that individuals choose effort by choosing the occupation. Therefore, the value of this index in a particular occupation measures the effort exerted by all individuals whose main last job is in that occupation.

Finally, we eliminate individuals who report an annual labor income below $1000, or who fail to provide information on education or hours worked. Following the analysis in Chetty et al. (2014), we eliminate individuals whose parental wealth exceeds the top 1 percent of the wealth distribution within the sample. This process results in a final sample size of 3.576 individuals.

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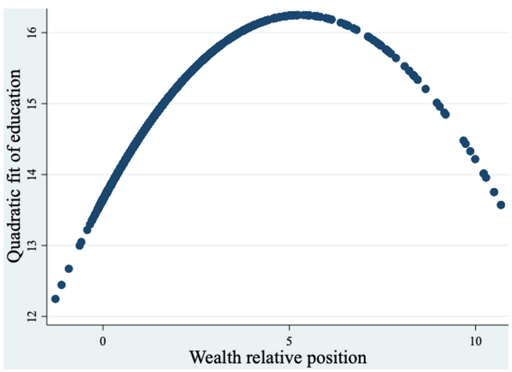
they use total income to study social mobility. Using labor income is motivated by the results of our model. In addition, Chetty et al. (2014a) measure children' income when children are very young adults and, therefore, total income is very close to labor income. This is confirmed by Chetty et al. (2014a), who show that the results of Rank-Rank regressions do not change when income is labor income instead of total income.

Figure 6. Quadratic fit

Panel a. Labor income

Panel b. Education



Panel c. Effort

Note. The horizontal axes is the ratio between parental net wealth and weigthed average net wealth, which is obtained using sample weights. The vertical axes in Panel a is the ratio between labor income and average labor income, in Panel b is the level of education as defined in PSID and in Panel c is the effort index ranging from zero to one.

As an illustrative exercise, Figure 6 plots the quadratic fit that is obtained by regressing labor income, education and effort against a quadratic polynomial of parental wealth. All coefficients of these regressions are significative at 1%. We clearly observe a hump-shaped relationship in the three panels, which provides empirical support to the results of the model. A remarkable finding of these analyses is that the maximum value of the three variables is attained at similar values of parental wealth. This finding is consistent with the mechanisms of the model, which suggests that for constrained individuals the effect of wealth on effort arises only because of the complementarity with education, for unconstrained individuals the effect of wealth on education arises because of the complementarity with effort and for both groups of individuals the effect of parental wealth on labor income is the consequence of the effect that wealth has on education and effort. The value of parental wealth for which labor income reaches a maximum value is 4.8 times the average wealth. In our sample, 4.9% of individuals have higher wealth, indicating the number of individuals who may be affected by the Carnegie effect.

Table 6. Three-stage least square estimation

Log labor income	[1]	[2]	[3]	[4]	[5]
Constant	5.76***	6.05***	6.1***	6.1***	6.23***
Education	0.20*	-0.11	0.17***	0.17***	0.17***
Effort	4.73	0.16***	4.55***	4.53***	4.57***
Education					
Constant	14.2***	14.2***	14.6***	14.7***	15.0***
Wealth	0.980***	0.986***	0.97***	0.884***	0.87***
Wealth^2	-0.092***	-0.094***	-0.091***	-0.083***	-0.084***
Age		-0.017***	-0.014***	-0.014***	-0.017***
Head			-0.62***	-0.58***	-0.62***
Black				-0.32***	-0.32***
Children					-0.11***
Effort index					
Constant	0.35***	0.32***	0.19***	0.22***	0.21***
Wealth	0.030***	0.028***	0.035***	0.020***	0.021***
Wealth^2	-0.0034***	-0.0028***	-0.0037***	-0.0023***	-0.0024***
Age		0.001***	0.003***	0.03***	0.003***
Head			0.07***	0.07***	0.07***
Black				-0.06***	-0.06***
Children					0.004***

Note:
[1] *** indicates p-value < 0.01, **indicates p-value < 0.05 and * indicates p-value < 0.1.
[2] Data is from PSID-2013. All Individuals are either head or wife in the family unit. Labor income is the logarithm of annual labor income in the last year. Parental wealth is parental net wealth divided by weighted average parental net wealth, which is constructed using sampling weights. Parental wealth is taken from the years 1984, 1994, and 2003 and measured in 2003 US dollars. Hours is the average hours worked per week. Education is the highest grade or year of school completed and ranges from 1 to 17. Children is the number of individuals in the family unit who in 2013 were under the age of 18. Black is the race of the family head and is constructed as a dummy variable that equals one when the race is "black, african american, or negro" and zero otherwise. Head is a dummy variable equal to one when the individual is family head and zero otherwise. Age is the actual age of the individual in 2013.

We provide further empirical support to the results of the model by performing a three stage least square estimation in which labor income depends on education and effort, and education and effort depend on a quadratic polynomial of wealth and different control variables. Table 6 provides five different regressions, which differ in the number of control variables. We observe that, once we control for the individual being the head of the family unit, all regressions yield very similar results and all coefficients are highly significative. In particular, the regressions show that labor income increases with education and effort and also corroborate the hump-shaped relationship of education and effort with parental wealth.

5 Quantitative analysis

The purpose of this section is two-fold. First, we show that the model introduced in Section 3 generates the correlations between inequality and social mobility documented in Section 2. Second, we use the model to measure the contribution of the Carnegie effect to explain the negative correlation between inequality and the labor income.

To perform these analyses, we introduce a distribution of abilities in the model of Section 3. This distribution is an additional source of heterogeneity among individuals necessary to generate the observed patterns of social mobility. We assume that abilities are an idiosyncratic productivity shock that determines the efficiency parameter in the wage function, according to the following function: $A_t^i = \exp(a_t^i)$. The intergenerational transmission of these abilities is set according to the following process:

$$a_t^i = \gamma + \rho a_{t-1}^i + \varepsilon_t,\ \rho \in (0,1)\,,$$

where ε_t is an i.i.d. normally distributed random variable with zero mean and variance σ^2 and the parameter ρ determines the intergenerational correlation of abilities.

5.1 Calibration

We calibrate the parameters to match several targets of the US economy in the period 1996-2000. Table 7 provides the value of parameters and targets.

Table 7. Calibration

Parameter	Value	Target	Data	Model
Initial distribution of inheritances and abilities				
η_a	1	Mean of parents abilities	-.-	1
σ_a	0.128	Third to first quartile of labor income[a]	2.4	2.4
η_b	0.112	Wealth to income ratio 1996-2000[b]	5.7	5.7
σ_b	0.077	Fraction of wealth between percentile 50-90[b]	35%	31%
Exogenous process for abilities				
ρ	0	Relative social mobility[c]	33%	33%
σ	0.8	Predicted rank percentile at 25P[c]	43%	42%
γ	-0.32	Mean of descendants abilities	-.-	1
Parameters of preferences and technology				
R	5.0477	Labor income share[d]	64%	64%
β	0.1179	Wealth to income ratio 2018-2022[b]	7.2	7.2
α	0.1	Gini index 1996-2000[e]	40%	40%
B	0.0088	Average h	-.-	1
D	0.3922	Average e	-.-	1

Source:
[a] US Bureau of labor statistics.
[b] FRED, St. Louis Fed. The wealth to income ratio is defined as the mean value of household and non-profit organization net worth as percentage of disposable personal income.
[c] Chetty et al. (2014a). Average values among commuting zones.
[d] Penn World Table.
[e] World Bank.

Two variables determine the initial heterogeneity among individuals: abilities and inheritances. We obtain the initial distribution of these two variables from two independent Gamma distributions that are calibrated as follows:

1. The number of individuals is set to 250.000.

2. The mean of abilities, η_a, is normalized to one and the mean of inheritances, η_b, is set to match the average wealth to income ratio in the period 1996-2000, which equals 5.7.[14]

[14] Data on social mobility is from Chetty et al. (2014a), who study cohorts born in the period 1980-1982 and measure parent's wealth in the period 1996-2000, when children were between 15 and 20 years old. To be consistent with this analysis, the wealth to income ratio is the average value of household and non-profit organization net worth as percentage of disposable personal income in the period 1996-2000. This variable is obtained from FRED, St. Louis Fed.

3. The variance of abilities, σ_a, is set to match the ratio between labor income in the third and first quartiles of the US labor income distribution over the period 2010-2020. Data on the distribution of labor income is from the US Bureau of Labor Statistics.

4. The variance of inheritances, σ_b, is set to match the share of net worth held by individuals who are between the 50th and 90th percentiles in the period 1996-2000, which is 35% according to the St. Louis Fed. The model generates a slightly lower value of this share, 31%. The wealth distribution generated by the model is closed to the observed wealth distribution. For example, the model allocates 68% of total wealth to the richest 10%, whereas, in the actual data, this group owns 62% of the wealth.

Three parameters, γ, ρ and σ, determine the distribution of future abilities and, therefore, will be key to determine social mobility. First, we set $\gamma = -\sigma^2/2$ to keep the mean of descendants abilities equal to one. This eliminates exogenous growth of wages. Second, ρ and σ are jointly set to match two measures of social mobility: the relative social mobility (slope of the Rank-Rank regression) and the predicted rank percentile of individuals born in families that are in the 25th percentile of the national income distribution. These two measures are obtained from Chetty, et al. (2014a). Specifically, the relative social mobility is obtained from the sample of children, which includes all current U.S. citizens born in birth cohorts 1980-82 and the predicted rank percentile is the median value among CZs. To find consistent measures, we use the data generated by the model to perform the same analysis than Chetty et al. (2014a). In particular, we rank parents and children income and conduct a Rank-Rank regression. We observe that when $\rho = 0$ and $\sigma = 0.8$ the model matches the relative mobility (the slope coefficient) in the data and slightly underestimates the predicted rank percentile. The calibrated value of ρ implies that we must assume that there is no intergenerational persistence in innate abilities to generate the social mobility reported in Chetty et al. (2014a).

We set the remaining parameters as follows. First, we set R to match the value of the labor income share in 2000, which is obtained from the Penn World Tables. This number implies an annual interest rate equal to 6.69%, when we assume that a period is 25 years. Second, the parameter β is set to match the value of the wealth to income ratio at the steady state. We approximate this value with the mean value of household and non-profit organization net worth as percentage of disposable personal income in the period 2018-2022, that we obtain from the St. Louis Fed.[15] Third, α is set to match the average value of the Gini index in disposable income post taxes and transfers in the US in the period 1996-2000. We obtain this number from the World Bank.[16] Finally, we set B and D so that the average level of education and effort are equal to one.

5.2 Inequality and social mobility

Chetty et al. (2014a) report huge differences in income inequality among CZs. These differences translate into significative differences in social mobility. To show these differences, in Table 8 we group CZs in quartiles, according to the value of the Gini index. The table provides relative mobility, measured by the slope coefficient of the Rank-Rank regression, and absolute mobility for individuals in families at the 25 and 75 percentiles of the income distribution. We observe that in all quartiles there is upward social mobility for individuals born in families that fall into the 25th percentile and downward social mobility for individuals born in families that fall into the 75th percentile. We also observe that in the last quartile, which groups the CZs with the largest inequality, there is a significant lower relative mobility (larger value of the slope coefficient in the Rank-Rank regressions), lower upward social mobility for individuals born in low-income families and larger downward social mobility for individuals born in high income families. We also provide average income of the parents and of the children. From the comparison between CZs, we do not observe any clear relationship between inequality and average parental income. In contrast, average children income is substantially lower in more unequal CZs. Note that the results in this table are in line with the evidence documented in Section 2.

[15] To obtain the value of β, we must take into account that income in the data is annual, whereas in the model is life-time income generated in a 25-years period. Assuming a constant annual income, we obtain the following relation between individual annual income (I_a^i) and life-time income (I^i):

$$I^i = \sum_{t=0}^{25} R_a^{25-t} I_a^i = \left(\frac{1 - R_a^{25}}{1 - 1R_a} \right) R_a I_a^i = 64.46 I_a^i,$$

when $R_a = 1.0669$ is the annual interest factor. Since wealth to income in the data is 7.2, the previous arguments imply that the wealth to income ratio in the model must satisfy $b/I = 7.2/64.46 = 0.111$.

[16] We can also obtain the Gini index as the average value of the Gini indexes in the CZs. This average Gini index is 41%, which almost coincides with the Gini index for the US reported by the World Bank.

Table 8. Data on social mobility and income for US CZs

	1st quartile	2on quartile	3rd quartile	4th quartile
Gini	0.317	0.377	0.431	0.515
Relative mobility	0.292	0.321	0.333	0.355
Expected rank 25th	49.1	44.6	42.1	39.8
Expected rank 75th	63.7	60.7	58.7	57.6
Average parental income	67,477	68,021	65,424	71,759
Average Child income	52,947	47,688	43,254	41,109
Child Inc. relative to 1st Qrtile.	1	90%	82%	77%

Note. Values in the table are elaborated using data from Chetty et. al. (2014).

We first investigate if the mechanisms introduced in the model can account for a significative part of the differences in social mobility shown in Table 8. To this end, we simulate the model assuming that the only difference between CZs is in the initial distribution of inheritances. Therefore, we simulate the different CZs using the value of the parameters in Table 7, with the exception of the parameters characterizing the distribution of inheritances. These parameters are set to generate a mean preserving spread of the distribution of inheritances introduced in Table 7. More precisely, the variance of the inheritance changes across CZs to generate different levels of income inequality. The results of this simulation are shown in Table 9.

Table 9. Simulation of inequality and social mobility

	1st quartile	2on quartile	3rd quartile	4th quartile
Variance of inheritance	0.047	0.068	0.091	0.133
Gini	0.316	0.376	0.431	0.516
Relative mobility	0.305	0.331	0.345	0.368
Expected rank 25th	43.7	42.1	40.7	38.4
Expected rank 75th	59.0	58.7	57.9	56.8

Note. The values in the table are obtained from the simulation of economies that have all the parameters in Table 7 except the variance of the inheritances.

Table 9 shows the value of the variance of inheritances needed to match the observed values of the Gini index in each quartile. We first observe that, as in the data, individuals born in families in the 25th percentile exhibit upward social mobility and individuals born in families in the 75th percentile exhibit downward social mobility. To see the effect of a larger inequality on social mobility, we compare the first and forth quartiles. We observe that, between these two quartiles, relative mobility falls by 6.3 points both in the model and in the data, expected rank at the 25th percentile drops by 5 points in the model and 9 in the data, and at the 75th percentile it drops by 2.2 points in the model and 6 in the data. We conclude that the model explains a large part of the correlation between inequality and measures of social mobility.

We next simulate two groups of counterfactual economies to show the contribution of the two mechanisms in driving the relationship between inequality and social mobility. In these simulations, parental income is taken from the simulations of the US economy and of the CZs shown in Table 9, and children's income is simulated by removing one of the two mechanisms from the model. More precisely, in one group of counterfactual economies we remove the imperfect financial market mechanism by assuming that children do not face credit constraints and in the other one we remove the Carnegie effect by assuming that unconstrained children exert the same level of effort than constrained children. Table 10 summarizes the main measures of social mobility for these counterfactual economies.

Table 10. Simulation of counterfactual economies

	US econ.	1st quartile	2on quartile	3rd quartile	4th quartile
Results. No Imperfect Market					
Relative mobility	0.323	0.303	0.323	0.329	0.327
Expected rank 25th	41.9	43.7	42.2	41.2	40.1
Expected rank 75th	58.1	58.8	58.4	57.7	56.4
Results. No Carnegie Effect					
Relative mobility	0.424	0.381	0.418	0.442	0.472
Expected rank 25th	39.4	42.7	40.1	37.9	34.7
Expected rank 75th	60.6	61.7	61.1	60.1	58.3

Note. The first column of the table shows the social mobility measures obtained using the parental income distribution of the calibrated U.S. economy and the children's income generated in the simulations of the counterfactual economies. The other columns show the social mobility measures obtained using the parental income distribution of the simulated economies in Table 10, the children's income distribution generated in the simulations of the counterfactual economies, and the children are ranked according to their position in the income distribution of the counterfactual U.S. economies.

The elimination of credit constraints increases social mobility, since individuals born in low-income families can borrow to finance education. The main effect is on the absolute mobility of individuals born in low-income families, which increases compared to the credit-constrained model, especially in the most unequal CZs, where the fraction of credit-constrained individuals is large and, hence, the removal of these constrains has a larger effect. From the comparison between the quartiles, we observe that inequality has a very small effect on the social mobility measures, which is due to the fact that inequality does not increase the fraction of credit constrained individuals in this counterfactual economy.

When we eliminate the Carnegie effect, wealthy individuals do not reduce effort and, as a result, social mobility decreases. The main effect occurs for individuals born in affluent families. We observe that the predicted rank percentile of these individuals is larger, which is due to the fact that these individuals do not reduce effort in this counterfactual economy. From the comparison between the different quartiles, we observe that higher inequality translates into a drop in relative mobility much larger than in the data. As inequality increases, there are more constrained individuals and less upward social mobility but, since we remove the Carnegie effect, unconstrained individuals do not experience a larger downward social mobility that limits the reduction in relative mobility. This explains why, as inequality increases, the drop in relative mobility generated by the counterfactual economy is larger than the drop observed in the data.

We conclude that the combination of both mechanisms, imperfections in the financial markets and the Carnegie effect, is necessary to explain the correlations between inequality and social mobility observed in the data.

5.3 The effect of wealth inequality on labor income

We next study the contribution of the different mechanisms to explain the effect on average labor income of a more unequal distribution of inheritances. The results of this analysis are summarized in Table 11, which provides the average values of effort, education and labor income for all individuals and also for credit-constrained and unconstrained individuals.

Table 11. The effect of inequality on income

	1st quartile	2on quartile	3rd quartile	4th quartile
Gini	0.316	0.376	0.431	0.516
Fraction of credit constrained	0.502	0.588	0.648	0.721
Average values for all ind.				
Bequests	0.112	0.112	0.112	0.112
Effort	0.978	0.992	1.004	1.021
Education	1.313	1.064	0.890	0.684
Labor income	0.919	0.844	0.774	0.668
Average values for constrained ind.				
Bequests	0.005	0.004	0.003	0.002
Effort	1.325	1.325	1.325	1.325
Education	0.612	0.446	0.348	0.244
Labor income	0.945	0.830	0.741	0.616
Average values for unconstrained ind.				
Bequests	0.221	0.267	0.315	0.396
Effort	0.821	0.793	0.768	0.736
Education	2.020	1.947	1.892	1.815
Labor income	0.893	0.861	0.836	0.802

We observe that a more unequal distribution of inheritances substantially reduces the average labor income. Table 11 shows the drivers of this reduction. The first one is the fraction of credit constrained individuals in the economy, that increases form 50% to 72% as we move from CZs in the lowest quartile of inequality to CZs in the highest quartile. Since the average labor income of credit-constrained individuals differs from that of unconstrained individuals, the rise of the fraction of credit constrained individuals explains part of the differences in labor income. The second driver is the lower average labor income that credit constrained individuals obtain in more unequal CZs. In these CZs, constrained individuals obtain lower inheritances and, therefore, invest less in education, which explains the lower labor income. Finally, the third driver is the lower average labor income that unconstrained individuals obtain in more unequal CZs. These individuals obtain larger inheritances when inequality is larger and, as a consequence, they exert less effort and obtain a lower labor income. This last driver is due to the Carnegie effect, while the first two are due to the imperfect financial market effect.

Therefore, we can use Table 11 to measure the contribution of each effect to the labor income gap between the CZs in the first quartile and CZs in the second, third and forth quartiles. We obtain that the contribution of the Carnegie effect to the labor income gap is 21.3%, 20% and 18.3%, respectively.

6 Conclusions

Commuting Zones with higher income inequality exhibit less upward social mobility for individuals born in low-income families, more downward social mobility for individuals born in high-income families and lower average income. We explain these findings combining two different mechanisms. One mechanism is based on a credit constraint affecting individuals born in low-income families and the other one is based on endogenous effort decisions affecting individuals born in high-income families.

We introduce these two mechanisms into an overlapping generations model, wherein individuals are heterogeneous due to both inherited wealth and innate abilities. In a rather general framework without particular functional forms, we argue that the model can account for the observed correlations between inequality and social mobility. Moreover, we show that the model implies that education, effort and labor income exhibit a hump-shaped relationship with inherited wealth. Using data from PSID on labor income, education, occupation, hours worked and parental wealth, we provide empirical support to these implications of the model.

We perform two quantitative exercises. We first show that the model explains the correlations between parental income inequality and social mobility observed between CZs. This analysis validates the model, since these correlations are not a target of the calibration. Finally, a remarkable finding is that a more unequal distribution reduces labor income, affecting individuals across the spectrum of family income levels. The model developed in this paper generates this negative effect on labor income, enabling us to quantify the contribution of the two mechanisms underlying it. In a second exercise, we find that the Carnegie effect accounts for 20% of the drop in average labor income due to a larger inequality.

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Appendix

A Slopes of the functions $\mathbf{E}^u$ and $\mathbf{H}^u$

The function $E^u(h,b,a)$ is steeper than $H^u(e,a)$ if the following inequality is satisfied:

$$\frac{\frac{\phi_{ee}}{\phi_e}-\frac{w_{ee}}{w_e}+\frac{\phi_e}{1+\beta}}{\frac{w_{eh}}{w_e}} > \frac{w_{eh}}{R\mu_h - w_{hh}}.$$

After some simple manipulation, we deduce that

$$\left(\frac{\phi_{ee}}{\phi_e}w_e - w_{ee} + \frac{\phi_e}{1+\beta}w_e\right)R\mu_h - \left(\frac{\phi_{ee}}{\phi_e}+\frac{\phi_e}{1+\beta}\right)w_e w_{hh} > (w_{eh})^2 - w_{hh}w_{ee}.$$

The left hand side is positive, whereas concavity of the wage function implies that the right side is negative. This proves that the inequality is satisfied and, hence, the function $E^u(h,b,a)$ is stepper than the function $H^u(e,a)$.

B Characterization of the function $b_u'(e(b))$

We use (7) and (8) to characterize the function $b_u'(e(b))$. First, we obtain the following derivatives

$$\frac{\partial b_u'}{\partial b} = \frac{\partial b_u'}{\partial e}\frac{\partial e}{\partial b} = -\beta\frac{(1-\alpha)}{2De_u}A\Gamma^\alpha\frac{\partial e}{\partial b},$$

with

$$\frac{\partial e}{\partial b} = -\frac{1}{\left(\frac{(1+\beta)}{\theta De_u}+1\right)\frac{(1-\alpha)A\Gamma^\alpha}{R}}.$$

Therefore, we obtain that

$$\frac{\partial b_u'}{\partial b} = \frac{\frac{\beta}{\theta De_u}R}{\frac{(1+\beta)}{\theta De_u}+1}.$$

Notice that $\frac{\partial b_u'}{\partial b}>0$ and, since $R<(1+\beta)/\beta$, we can easily deduce that $\frac{\partial b_u'}{\partial b}<1$.

Second, to show convexity of the function $b_u'(e(b))$ we calculate the following derivatives:

$$\begin{aligned}\frac{\partial^2 b_u'}{\partial b^2} &= \frac{\partial^2 b_u'}{\partial e^2}\left(\frac{\partial e}{\partial b}\right)^2 + \frac{\partial b_u'}{\partial e}\frac{\partial^2 e}{\partial b^2}\\ &= -\frac{\beta(1-\alpha)e_u^{-3}A\Gamma^\alpha}{2D}\left[-2\left(\frac{\partial e}{\partial b}\right)^2 + e_u\frac{\partial^2 e}{\partial b^2}\right],\end{aligned}$$

with

$$\frac{\partial^2 e}{\partial b^2} = \frac{-\frac{(1+\beta)}{D}e_u^{-3}\frac{(1-\alpha)A\Gamma^\alpha}{R}}{\left(\left(\frac{(1+\beta)}{2D}e_u^{-2}+1\right)\frac{(1-\alpha)A\Gamma^\alpha}{R}\right)^2}\frac{\partial e}{\partial b} = -\frac{(1+\beta)}{D}e_u^{-3}\frac{(1-\alpha)A\Gamma^\alpha}{R}\left(\frac{\partial e}{\partial b}\right)^3.$$

Therefore, we obtain

$$
\begin{aligned}
\frac{\partial^2 b_u'}{\partial b^2} &= \frac{\beta\left(1-\alpha\right)e_u^{-3}A\Gamma^{\alpha}\left[2\left(\frac{\partial e}{\partial b}\right)^2+e_u\frac{(1+\beta)}{D}e_u^{-2}\frac{(1-\alpha)A\Gamma^{\alpha}}{R}\left(\frac{\partial e}{\partial b}\right)^3\right]}{2D} \\
&= -\frac{\beta\left(1-\alpha\right)e_u^{-3}A\Gamma^{\alpha}}{2D}\left(\frac{\partial e}{\partial b}\right)^3\frac{(1-\alpha)A\Gamma^{\alpha}\left[2\left(\frac{(1+\beta)e_u^{-2}}{2D}+1\right)-\frac{(1+\beta)e_u^{-2}}{D}\right]}{R} \\
&= -\frac{\beta\left(1-\alpha\right)e_u^{-3}A\Gamma^{\alpha}}{D}\left(\frac{\partial e}{\partial b}\right)^3\frac{\left(1-\alpha\right)A\Gamma^{\alpha}}{R}>0,
\end{aligned}
$$

which is positive since $\frac{\partial e}{\partial b}<0$.

Third, we evaluate b_u' at $e=\overline{e}$ to obtain $b_u'\left(\overline{b}\right)$, which satisfies

$$
b_u'\left(\overline{b}\right)=\sqrt{\frac{\left(1-\alpha\right)}{\left(1+\beta\right)2D}}\beta A\Gamma^{\alpha}.
$$

It is immediate to show that $b_u'\left(\overline{b}\right)<\overline{b}$ if and only if $R<\alpha\left(1+\beta\right)/\beta$. These conditions imply that if $R>\alpha\left(1+\beta\right)/\beta$ then there is a steady state for unconstrained individuals.