Step by step: specificity-improved reliability with R

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Contents

This document provides the R syntax for calculating specificity-improved reliability in a longitudinal design setting with measurement invariance and specificity stability over time. This approach is based on Raykov (2007) and Sideridis et al. (2019).

To this end, we simulated 1000 responses for the three total scores (somatic anxiety, worry, and concentration disruption) across four waves in the Sport Anxiety Scale-2 (Smith et al., 2006; Ramis et al., 2010). Since the three subscales measure different components of anxiety, it is expected that they will share some true common variance while also diferentiating with some true specific variance. For each of the three subscales, the total score is defined as the sum of the responses to five measures rated on a 4-point scale (*1 = Not at all*, *4 = Very much*), consequently, possible values range between 5 and 20.

The data were simulated under ideal conditions: no missing values, a homogeneous group, and approximately normally distributed responses.

We will first compute specificity-improved reliability for two data collection points (T1, T2) following the standard test-retest design. Then, we will extend the approach to four time points (T1, T2, T3, and T4) as per follow-up studies.

In both cases, we will use the dataset *dades_ans.txt*.

1. Data import and exploration

1.1. Install packages

First, you will need to install the required libraries. This step is only necessary the first time you use them. If you already have them installed, you can skip this step.

```
install.packages(c("readr","tidyverse","psych",
                   "semTools","lavaan", "corrplot", "lavaanExtra"))
```

1.2. Load packages

Each time you start your working environment, you will need to activate the required libraries. To do this:

```
library(readr)
library(tidyverse)
library(psych)
library(lavaan)
library(semTools)
library(corrplot)
library(lavaanExtra)
```

1.3. Open txt file

The next step is to read the data. The provided dataset consists of 1000 rows and 12 columns. The measures are ordered so that the first three correspond to wave 1 (T1), the next three to wave 2 (T2), and so on. All variables are quantitative.

```
Data <- read_delim("dades_ans.txt", delim = "\t",   escape_double = FALSE, trim_ws = TRUE)
```

We can perform an initial inspection of the data to ensure that it is being read correctly, for example, using the *headTail* function from the *psych* package:

```r
headTail(Data)
```

```
##   Measure1_T1 Measure2_T1 Measure3_T1 Measure1_T2 Measure2_T2 Measure3_T2
## 1          12          13           6          10          16           6
## 2           7          17           6           9          13           9
## 3           5           6           5           5           8           5
## 4           5          13           8          10          16          11
## 5         ...         ...         ...         ...         ...         ...
## 6           8          12           8           5           7           5
## 7          13          17          12          11          15           9
## 8           8          11           9           7          12          10
## 9           8          17          10           7          15           9
##   Measure1_T3 Measure2_T3 Measure3_T3 Measure1_T4 Measure2_T4 Measure3_T4
## 1          11          14           9           6          19           8
## 2           6          13           7          10          19           7
## 3           8          15           5           6           8           8
## 4           5          14           8           7          12           7
## 5         ...         ...         ...         ...         ...         ...
## 6           9          12           8           8           8           7
## 7          11          13          11          12          14           8
## 8           6          15          10           8          17          13
## 9           5          14           7          10          14           7
```

2. Data analysis for two-wave designs

2.1. Create a dataset with measures for two waves

To work more comfortably, we create a subset of the data called "dat" by selecting the first six measures (Measure1_T1, Measure2_T1, Measure3_T1, Measure1_T2, Measure2_T2, Measure3_T2).

```r
dat <- Data %>%
  select(Measure1_T1:Measure3_T2)
```

2.2. Data description

First, we will conduct an initial inspection of the descriptive statistics of the variables. This step will help us become familiar with the dataset and make initial decisions about its distribution.

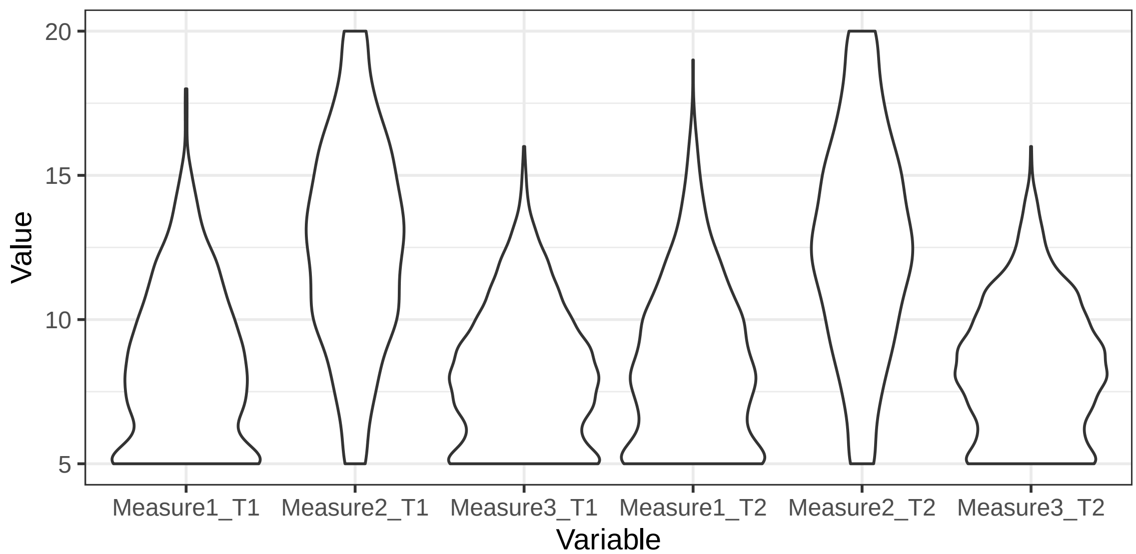
```r
describeBy(dat)
```

```
##             vars    n  mean   sd median trimmed  mad min max range  skew
## Measure1_T1    1 1000  8.35 2.71      8    8.14 2.97   5  18    13  0.54
## Measure2_T1    2 1000 12.59 3.63     13   12.60 4.45   5  20    15 -0.03
## Measure3_T1    3 1000  8.05 2.33      8    7.88 2.97   5  16    11  0.48
## Measure1_T2    4 1000  8.40 2.77      8    8.16 2.97   5  19    14  0.63
## Measure2_T2    5 1000 12.64 3.75     13   12.65 4.45   5  20    15 -0.03
## Measure3_T2    6 1000  8.27 2.30      8    8.16 2.97   5  16    11  0.34
##             kurtosis   se
## Measure1_T1    -0.34 0.09
## Measure2_T1    -0.56 0.11
## Measure3_T1    -0.42 0.07
## Measure1_T2    -0.14 0.09
## Measure2_T2    -0.58 0.12
## Measure3_T2    -0.50 0.07
```

As can be seen, the means and standard deviations for the measures across the two time points are similar. More importantly, the skewness and kurtosis values are less than 1 in absolute value for all measures, suggesting that the data approximate a normal distribution. Given these results, it may be reasonable to use a Maximum Likelihood (ML) estimator for the confirmatory factor analyses (see Figure 1 Doval, et al., 2023).

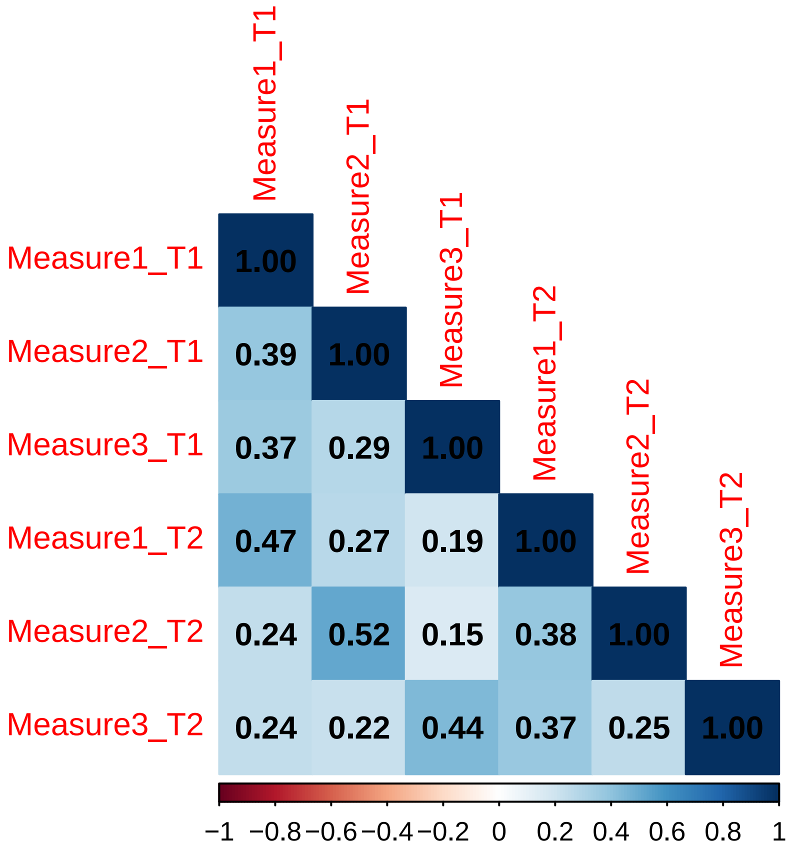
Additional inspection of the data can be done through graphs, for example, by using violin plots. These plots are useful for visualizing the distribution of the data and can help identify any potential outliers or skewness.

```
dat %>%
  pivot_longer(cols = everything()) %>%
  mutate(time = substr(name, nchar(name) - 1, nchar(name))) %>%
  mutate(name = factor(name, levels = names(Data))) %>%
  ggplot(aes(x = name, y = value)) +
  geom_violin() +
  theme_bw() +
  labs (x = "Variable", y = "Value")
```



Considering these distributions, calculating Pearson correlation coefficients is reasonable. The graph below displays the correlation data. As can be seen, the first three measures exhibit correlations ranging from low to high while measures from the second time point show moderate correlations. As expected, measurements of the same item across different time points yield high correlation coefficients.

```
corrplot(cor(dat), type = "lower",  method ="color", addCoef.col = 'black')
```



2.3 Measurement models and reliability

2.3.1 Unidimensional models

First, we examine the unidimensional measurement models for time 1 and time 2. Since each time point has 3 measures, the models are exactly identified, and their goodness of fit cannot be evaluated. As can be seen in the output, all parameter estimates show admissible values.

2.3.1.1 Measurement model and reliability for T1 .

```
model_t1 <- '
T1 =~ Measure1_T1 + Measure2_T1 + Measure3_T1'

f_t1 <- cfa(model = model_t1, data = Data)
summary(f_t1, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 31 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                         6
##
##   Number of observations                          1000
##
```

```
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Model Test Baseline Model:
##
##   Test statistic                               337.250
##   Degrees of freedom                                 3
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       1.000
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7217.992
##   Loglikelihood unrestricted model (H1)      -7217.992
##
##   Akaike (AIC)                               14447.985
##   Bayesian (BIC)                             14477.431
##   Sample-size adjusted Bayesian (SABIC)      14458.375
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.000
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.000
##   P-value H_0: RMSEA <= 0.050                       NA
##   P-value H_0: RMSEA >= 0.080                       NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.000
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   T1 =~
##     Measure1_T1       1.000                               1.907    0.705
##     Measure2_T1       1.048    0.116    9.061    0.000    1.998    0.550
##     Measure3_T1       0.637    0.070    9.062    0.000    1.215    0.522
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Measure1_T1       3.676    0.418    8.788    0.000    3.676    0.503
##    .Measure2_T1       9.195    0.589   15.602    0.000    9.195    0.697
```

```
##    .Measure3_T1       3.949    0.236   16.746    0.000    3.949    0.728
##     T1                3.635    0.477    7.616    0.000    1.000    1.000
```

Once the model is fitted, the coefficient omega can be obtained using the *compRelSEM* function. As can be seen in the output, the obtained value reflecting common true variance is quite low.

```r
compRelSEM(f_t1)
```

```
##    T1
## 0.609
```

2.3.1.2 Measurement model and reliability for T2 .

```r
model_t2 <- '
T2 =~ Measure1_T2 + Measure2_T2 + Measure3_T2'

f_t2 <- cfa(model = model_t2, data = Data)
summary(f_t2, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 31 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                         6
##
##   Number of observations                          1000
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Model Test Baseline Model:
##
##   Test statistic                               323.868
##   Degrees of freedom                                 3
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       1.000
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7266.685
##   Loglikelihood unrestricted model (H1)      -7266.685
##
##   Akaike (AIC)                               14545.370
##   Bayesian (BIC)                             14574.817
##   Sample-size adjusted Bayesian (SABIC)      14555.760
##
```

```
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.000
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.000
##   P-value H_0: RMSEA <= 0.050                       NA
##   P-value H_0: RMSEA >= 0.080                       NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.000
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   T2 =~
##     Measure1_T2       1.000                               2.089    0.755
##     Measure2_T2       0.913    0.112    8.175    0.000    1.907    0.509
##     Measure3_T2       0.542    0.066    8.156    0.000    1.131    0.491
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Measure1_T2       3.287    0.528    6.227    0.000    3.287    0.430
##    .Measure2_T2      10.397    0.628   16.545    0.000   10.397    0.741
##    .Measure3_T2       4.020    0.233   17.233    0.000    4.020    0.759
##     T2                4.362    0.594    7.348    0.000    1.000    1.000
```

For the second data collection, the internal consistency reliability value obtained is even lower.

```r
compRelSEM(f_t2)
```

```
##    T2
## 0.598
```

2.3.2 Two-wave models

According to Raykov (2007) obtaining improved-specificity reliability in a longitudinal design requires, at a minimum, metric invariance and specificity stability over time. Accordingly, we test three nested models across the two waves: congeneric measurement invariance (model_2w), metric measurement invariance (model_2wmi), and specificity stability (model_2wmis). Finally, we compute the specificity-improved reliability coefficients (model_2wmisR).

2.3.2.1 Testing the congeneric measurement model for 2 wave data . Measures load onto one factor per wave (F1 and F2), and correlations between item residuals across waves are freely estimated. The model shows a good fit. The reliability of each factor remains largely unchanged compared to the coefficients obtained from the unidimensional models.

```r
model_2w <-
'F1 =~ Measure1_T1 + Measure2_T1 + Measure3_T1
F2 =~ Measure1_T2 + Measure2_T2 + Measure3_T2
Measure1_T1 ~~ Measure1_T2
Measure2_T1 ~~ Measure2_T2
Measure3_T1 ~~ Measure3_T2'

f_2w <- cfa(model = model_2w, data = dat)
summary(f_2w, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 70 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        16
##
##   Number of observations                          1000
##
## Model Test User Model:
##
##   Test statistic                                 8.316
##   Degrees of freedom                                 5
##   P-value (Chi-square)                           0.140
##
## Model Test Baseline Model:
##
##   Test statistic                              1361.442
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.998
##   Tucker-Lewis Index (TLI)                       0.993
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -14138.673
##   Loglikelihood unrestricted model (H1)     -14134.515
##
##   Akaike (AIC)                               28309.346
##   Bayesian (BIC)                             28387.870
##   Sample-size adjusted Bayesian (SABIC)      28337.053
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.026
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.056
##   P-value H_0: RMSEA <= 0.050                    0.901
##   P-value H_0: RMSEA >= 0.080                    0.001
##
## Standardized Root Mean Square Residual:
```

```
##
##   SRMR                                           0.016
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   F1 =~
##     Measure1_T1       1.000                               1.956    0.722
##     Measure2_T1       1.012    0.103    9.786    0.000    1.980    0.547
##     Measure3_T1       0.592    0.062    9.574    0.000    1.158    0.498
##   F2 =~
##     Measure1_T2       1.000                               2.034    0.735
##     Measure2_T2       0.920    0.097    9.455    0.000    1.872    0.502
##     Measure3_T2       0.588    0.062    9.451    0.000    1.196    0.517
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .Measure1_T1 ~~
##    .Measure1_T2       1.058    0.272    3.882    0.000    1.058    0.301
##  .Measure2_T1 ~~
##    .Measure2_T2       4.475    0.419   10.683    0.000    4.475    0.458
##  .Measure3_T1 ~~
##    .Measure3_T2       1.531    0.164    9.365    0.000    1.531    0.384
##   F1 ~~
##     F2                2.543    0.313    8.124    0.000    0.639    0.639
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Measure1_T1       3.513    0.412    8.528    0.000    3.513    0.479
##    .Measure2_T1       9.202    0.566   16.249    0.000    9.202    0.701
##    .Measure3_T1       4.061    0.227   17.851    0.000    4.061    0.752
##    .Measure1_T2       3.513    0.447    7.859    0.000    3.513    0.459
##    .Measure2_T2      10.392    0.590   17.611    0.000   10.392    0.748
##    .Measure3_T2       3.915    0.229   17.079    0.000    3.915    0.732
##     F1                3.826    0.473    8.096    0.000    1.000    1.000
##     F2                4.138    0.511    8.093    0.000    1.000    1.000
```

```
compRelSEM(f_2w)
```

```
##    F1    F2
## 0.603 0.592
```

2.3.2.2 Testing the metric invariance across waves . Equal factor loadings across waves are specified for each measure (L1, L2, and L3), while allowing the loadings to be freely estimated across measures. The model shows good fit, with no appreciable difference compared to the fit of the congeneric model.

```r
model_2wmi <- '
# Factor loadings
F1 =~ L1*Measure1_T1 + L2*Measure2_T1 + L3*Measure3_T1
F2 =~ L1*Measure1_T2 + L2*Measure2_T2 + L3*Measure3_T2
Measure1_T1 ~~ Measure1_T2
Measure2_T1 ~~ Measure2_T2
Measure3_T1 ~~ Measure3_T2'

f_2wmi <- cfa(model = model_2wmi, data = dat)
summary(f_2wmi, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 67 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        16
##   Number of equality constraints                     2
##
##   Number of observations                          1000
##
## Model Test User Model:
##
##   Test statistic                                 9.259
##   Degrees of freedom                                 7
##   P-value (Chi-square)                           0.235
##
## Model Test Baseline Model:
##
##   Test statistic                              1361.442
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.998
##   Tucker-Lewis Index (TLI)                       0.996
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -14139.145
##   Loglikelihood unrestricted model (H1)     -14134.515
##
##   Akaike (AIC)                               28306.289
##   Bayesian (BIC)                             28374.998
##   Sample-size adjusted Bayesian (SABIC)      28330.533
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.018
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.045
##   P-value H_0: RMSEA <= 0.050                    0.977
##   P-value H_0: RMSEA >= 0.080                    0.000
```

```
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.018
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   F1 =~
##     Measr1_T1 (L1)    1.000                               1.992    0.734
##     Measr2_T1 (L2)    0.962    0.086   11.189    0.000    1.916    0.531
##     Measr3_T1 (L3)    0.588    0.052   11.205    0.000    1.171    0.503
##   F2 =~
##     Measr1_T2 (L1)    1.000                               2.009    0.727
##     Measr2_T2 (L2)    0.962    0.086   11.189    0.000    1.931    0.516
##     Measr3_T2 (L3)    0.588    0.052   11.205    0.000    1.181    0.512
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .Measure1_T1 ~~
##    .Measure1_T2       1.044    0.274    3.814    0.000    1.044    0.299
##  .Measure2_T1 ~~
##    .Measure2_T2       4.480    0.419   10.691    0.000    4.480    0.457
##  .Measure3_T1 ~~
##    .Measure3_T2       1.533    0.164    9.371    0.000    1.533    0.384
##   F1 ~~
##     F2               2.557    0.315    8.127    0.000    0.639    0.639
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Measure1_T1       3.398    0.395    8.610    0.000    3.398    0.461
##    .Measure2_T1       9.357    0.542   17.251    0.000    9.357    0.718
##    .Measure3_T1       4.056    0.224   18.128    0.000    4.056    0.747
##    .Measure1_T2       3.588    0.407    8.821    0.000    3.588    0.471
##    .Measure2_T2      10.276    0.582   17.666    0.000   10.276    0.734
##    .Measure3_T2       3.929    0.221   17.798    0.000    3.929    0.738
##     F1               3.968    0.443    8.951    0.000    1.000    1.000
##     F2               4.034    0.454    8.893    0.000    1.000    1.000
```

```r
anova(f_2w,f_2wmi)
```

```
##
## Chi-Squared Difference Test
##
##        Df   AIC   BIC  Chisq Chisq diff RMSEA Df diff Pr(>Chisq)
## f_2w    5 28309 28388 8.3156
## f_2wmi  7 28306 28375 9.2591    0.94346     0       2     0.6239
```

2.3.2.3 Testing metric invariance and specifity stability across waves . In addition to establishing metric invariance, a specific factor is defined for each measure (SP1, SP2, and SP3), with the factor loadings of the indicators (D1, D2, and D3) constrained to be invariant across waves, while still freely estimated. To ensure model identification, certain parameters are fixed: the variances of the specific factors are set to 1, and the correlations between each specific factor and all other factors are constrained to 0. Since each specific factor is defined by only two indicators, this portion of the model is exactly identified, and the overall model fit remains unchanged compared to the metric invariance model. In fact, the square of each specificity variance is equal to the corresponding error covariance in the metric invariance model (see Raykov (2007), equation 9)

```
model_2wmis <- '
# Factor loadings
F1 =~ L1*Measure1_T1 + L2*Measure2_T1 + L3*Measure3_T1
F2 =~ L1*Measure1_T2 + L2*Measure2_T2 + L3*Measure3_T2


# Loadings of item specificities
SP1 =~ NA*D1*Measure1_T1 + D1*Measure1_T2
SP2 =~ NA*D2*Measure2_T1 + D2*Measure2_T2
SP3 =~ NA*D3*Measure3_T1 + D3*Measure3_T2

# Fix variance of specific factors to 1 (for identification)
SP1 ~~ 1*SP1
SP2 ~~ 1*SP2
SP3 ~~ 1*SP3

# Specific factors constrained to be uncorrelated with all specific factors
SP1 ~~ 0*SP2 + 0*SP3
SP2 ~~ 0*SP3

#Specific factors constrained to be uncorrelated with all factors
SP1 ~~ 0*F1 + 0*F2
SP2 ~~ 0*F1 + 0*F2
SP3 ~~ 0*F1 + 0*F2
'
f_2wmis <- cfa(model = model_2wmis, data = dat)
summary(f_2wmis, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 44 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        19
##   Number of equality constraints                     5
##
##   Number of observations                          1000
##
## Model Test User Model:
##
##   Test statistic                                 9.259
##   Degrees of freedom                                 7
##   P-value (Chi-square)                           0.235
##
## Model Test Baseline Model:
```

```
##
##   Test statistic                              1361.442
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.998
##   Tucker-Lewis Index (TLI)                       0.996
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -14139.145
##   Loglikelihood unrestricted model (H1)     -14134.515
##
##   Akaike (AIC)                               28306.289
##   Bayesian (BIC)                             28374.998
##   Sample-size adjusted Bayesian (SABIC)      28330.533
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.018
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.045
##   P-value H_0: RMSEA <= 0.050                    0.977
##   P-value H_0: RMSEA >= 0.080                    0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.018
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   F1 =~
##     Measr1_T1 (L1)    1.000                               1.992    0.734
##     Measr2_T1 (L2)    0.962    0.086   11.189    0.000    1.916    0.531
##     Measr3_T1 (L3)    0.588    0.052   11.205    0.000    1.171    0.503
##   F2 =~
##     Measr1_T2 (L1)    1.000                               2.009    0.727
##     Measr2_T2 (L2)    0.962    0.086   11.189    0.000    1.931    0.516
##     Measr3_T2 (L3)    0.588    0.052   11.205    0.000    1.181    0.512
##   SP1 =~
##     Measr1_T1 (D1)    1.022    0.134    7.628    0.000    1.022    0.376
##     Measr1_T2 (D1)    1.022    0.134    7.628    0.000    1.022    0.370
##   SP2 =~
##     Measr2_T1 (D2)    2.117    0.099   21.383    0.000    2.117    0.586
##     Measr2_T2 (D2)    2.117    0.099   21.383    0.000    2.117    0.566
##   SP3 =~
```

```
##     Measr3_T1 (D3)    1.238    0.066   18.742    0.000    1.238    0.532
##     Measr3_T2 (D3)    1.238    0.066   18.742    0.000    1.238    0.537
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   SP1 ~~
##     SP2               0.000                               0.000    0.000
##     SP3               0.000                               0.000    0.000
##   SP2 ~~
##     SP3               0.000                               0.000    0.000
##   F1 ~~
##     SP1               0.000                               0.000    0.000
##   F2 ~~
##     SP1               0.000                               0.000    0.000
##   F1 ~~
##     SP2               0.000                               0.000    0.000
##   F2 ~~
##     SP2               0.000                               0.000    0.000
##   F1 ~~
##     SP3               0.000                               0.000    0.000
##   F2 ~~
##     SP3               0.000                               0.000    0.000
##   F1 ~~
##     F2                2.557    0.315    8.127    0.000    0.639    0.639
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     SP1               1.000                               1.000    1.000
##     SP2               1.000                               1.000    1.000
##     SP3               1.000                               1.000    1.000
##    .Measure1_T1       2.354    0.267    8.826    0.000    2.354    0.320
##    .Measure2_T1       4.876    0.418   11.655    0.000    4.876    0.374
##    .Measure3_T1       2.522    0.185   13.616    0.000    2.522    0.465
##    .Measure1_T2       2.544    0.276    9.211    0.000    2.544    0.334
##    .Measure2_T2       5.796    0.446   12.993    0.000    5.796    0.414
##    .Measure3_T2       2.395    0.183   13.103    0.000    2.395    0.450
##     F1                3.968    0.443    8.951    0.000    1.000    1.000
##     F2                4.034    0.454    8.893    0.000    1.000    1.000
```

2.3.2.4 Calculating specificty-improved reliability . To calculate the reliability values, the measurement errors (E1 to E6) and the variances of the latent factors (Fv1 and Fv2), which are freely estimated, are labeled for later reference. The reliability coefficients (OSR1 and OSR2) are then computed using the formula proposed by Raykov (2007).

```
model_2wmisR <- '
# Factor loadings
F1 =~ L1*Measure1_T1 + L2*Measure2_T1 + L3*Measure3_T1
F2 =~ L1*Measure1_T2 + L2*Measure2_T2 + L3*Measure3_T2


# Loadings of item specificities
SP1 =~ NA*D1*Measure1_T1 + D1*Measure1_T2
SP2 =~ NA*D2*Measure2_T1 + D2*Measure2_T2
```

```
SP3 =~ NA*D3*Measure3_T1 + D3*Measure3_T2

# Fix variance of specific factors to 1 (for identification)
SP1 ~~ 1*SP1
SP2 ~~ 1*SP2
SP3 ~~ 1*SP3

# Specific factors constrained to be uncorrelated with all specific factors
SP1 ~~ 0*SP2 + 0*SP3
SP2 ~~ 0*SP3

#Specific factors constrained to be uncorrelated with all factors
SP1 ~~ 0*F1 + 0*F2
SP2 ~~ 0*F1 + 0*F2
SP3 ~~ 0*F1 + 0*F2

# Label item errors
Measure1_T1 ~~ E1*Measure1_T1
Measure2_T1 ~~ E2*Measure2_T1
Measure3_T1 ~~ E3*Measure3_T1

Measure1_T2 ~~ E4*Measure1_T2
Measure2_T2 ~~ E5*Measure2_T2
Measure3_T2 ~~ E6*Measure3_T2

# Label latent factor variance
F1 ~~ Fv1*F1
F2 ~~ Fv2*F2

#Compute specificity-improved reliability
OSR1:= ((1 + L2 + L3 )^2*Fv1 + D1^2 + D2^2 + D3^2)/((1 + L2 + L3 )^2*Fv1 + D1^2 + D2^2 + D3^2 + E1 + E2
OSR2:= ((1 + L2 + L3 )^2*Fv2 + D1^2 + D2^2 + D3^2)/((1 + L2 + L3 )^2*Fv2 + D1^2 + D2^2 + D3^2 + E4 + E5
'
```

The specificity-improved reliability coefficient depends on the good fit of the model. As shown in the output, the fit indices remain unchanged compared to the previous models, as no additional constraints were imposed. All fit indices are satisfactory, and all estimated parameters and standard errors fall within acceptable ranges. In the final lines of the output, the specificity-improved reliability coefficients are reported: 0.771 for the first time point (OSR1) and 0.756 for the second time point (OSR2).

```
f_2wmisR <- cfa(model = model_2wmisR, data = dat)
summary(f_2wmisR, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 44 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        19
##   Number of equality constraints                     5
##
##   Number of observations                          1000
##
## Model Test User Model:
```

```
##
##   Test statistic                                 9.259
##   Degrees of freedom                                 7
##   P-value (Chi-square)                           0.235
##
## Model Test Baseline Model:
##
##   Test statistic                              1361.442
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.998
##   Tucker-Lewis Index (TLI)                       0.996
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -14139.145
##   Loglikelihood unrestricted model (H1)     -14134.515
##
##   Akaike (AIC)                               28306.289
##   Bayesian (BIC)                             28374.998
##   Sample-size adjusted Bayesian (SABIC)      28330.533
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.018
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.045
##   P-value H_0: RMSEA <= 0.050                    0.977
##   P-value H_0: RMSEA >= 0.080                    0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.018
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   F1 =~
##     Measr1_T1 (L1)    1.000                               1.992    0.734
##     Measr2_T1 (L2)    0.962    0.086   11.189    0.000    1.916    0.531
##     Measr3_T1 (L3)    0.588    0.052   11.205    0.000    1.171    0.503
##   F2 =~
##     Measr1_T2 (L1)    1.000                               2.009    0.727
##     Measr2_T2 (L2)    0.962    0.086   11.189    0.000    1.931    0.516
##     Measr3_T2 (L3)    0.588    0.052   11.205    0.000    1.181    0.512
##   SP1 =~
```

```
##     Measr1_T1 (D1)    1.022    0.134    7.628    0.000    1.022    0.376
##     Measr1_T2 (D1)    1.022    0.134    7.628    0.000    1.022    0.370
##   SP2 =~
##     Measr2_T1 (D2)    2.117    0.099   21.383    0.000    2.117    0.586
##     Measr2_T2 (D2)    2.117    0.099   21.383    0.000    2.117    0.566
##   SP3 =~
##     Measr3_T1 (D3)    1.238    0.066   18.742    0.000    1.238    0.532
##     Measr3_T2 (D3)    1.238    0.066   18.742    0.000    1.238    0.537
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   SP1 ~~
##     SP2               0.000                               0.000    0.000
##     SP3               0.000                               0.000    0.000
##   SP2 ~~
##     SP3               0.000                               0.000    0.000
##   F1 ~~
##     SP1               0.000                               0.000    0.000
##   F2 ~~
##     SP1               0.000                               0.000    0.000
##   F1 ~~
##     SP2               0.000                               0.000    0.000
##   F2 ~~
##     SP2               0.000                               0.000    0.000
##   F1 ~~
##     SP3               0.000                               0.000    0.000
##   F2 ~~
##     SP3               0.000                               0.000    0.000
##   F1 ~~
##     F2                2.557    0.315    8.127    0.000    0.639    0.639
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     SP1               1.000                               1.000    1.000
##     SP2               1.000                               1.000    1.000
##     SP3               1.000                               1.000    1.000
##    .Mesr1_T1  (E1)    2.354    0.267    8.826    0.000    2.354    0.320
##    .Mesr2_T1  (E2)    4.876    0.418   11.655    0.000    4.876    0.374
##    .Mesr3_T1  (E3)    2.522    0.185   13.616    0.000    2.522    0.465
##    .Mesr1_T2  (E4)    2.544    0.276    9.211    0.000    2.544    0.334
##    .Mesr2_T2  (E5)    5.796    0.446   12.993    0.000    5.796    0.414
##    .Mesr3_T2  (E6)    2.395    0.183   13.103    0.000    2.395    0.450
##     F1       (Fv1)    3.968    0.443    8.951    0.000    1.000    1.000
##     F2       (Fv2)    4.034    0.454    8.893    0.000    1.000    1.000
##
## Defined Parameters:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     OSR1              0.771    0.015   50.763    0.000    0.709    0.809
##     OSR2              0.756    0.016   48.265    0.000    0.689    0.804
```

We can obtain the bootstrapped confidence interval for the reliability coefficients

```r
set.seed(12345)
monteCarloCI(f_2wmisR)
```

```
##        est ci.lower ci.upper
## OSR1 0.771    0.739    0.799
## OSR2 0.756    0.723    0.785
```

2.3.2.4 Testing scalar invariance . An usual additional constraint for repeated measures is scalar invariance. To test scalar invariance the typical sequence of models involves testing congeneric (model_2w), metric (model_2wmi), and scalar (model_2wsi) invariance, followed by testing specificity stability (model_2wsis). In addition to equal factor loadings, the intercepts of each measure (i1, i2, and i3) are constrained to be equal across waves, while allowing them to be freely estimated. The mean of F1 is fixed at 0 for identification purposes, and the mean of F2 is freely estimated. The model shows good fit, with no appreciable difference compared to the fit of the metric invariance model.

```r
model_2wsi <- '
# Factor loadings
F1 =~ L1*Measure1_T1 + L2*Measure2_T1 + L3*Measure3_T1
F2 =~ L1*Measure1_T2 + L2*Measure2_T2 + L3*Measure3_T2
Measure1_T1 ~~ Measure1_T2
Measure2_T1 ~~ Measure2_T2
Measure3_T1 ~~ Measure3_T2

# Intercepts
Measure1_T1 + Measure1_T2  ~ i1*1
Measure2_T1 + Measure2_T2  ~ i2*1
Measure3_T1 + Measure3_T2  ~ i3*1


F1 ~ 0*1   # Fix mean of F1 to 0
F2 ~ 1     # Free F2 mean'

f_2wsi <- cfa(model = model_2wsi, data = dat)
summary(f_2wsi, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 73 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        23
##   Number of equality constraints                     5
##
##   Number of observations                          1000
##
## Model Test User Model:
##
##   Test statistic                                15.539
##   Degrees of freedom                                 9
##   P-value (Chi-square)                           0.077
##
## Model Test Baseline Model:
##
```

```
##   Test statistic                              1361.442
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.995
##   Tucker-Lewis Index (TLI)                       0.992
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -14142.285
##   Loglikelihood unrestricted model (H1)     -14134.515
##
##   Akaike (AIC)                               28320.570
##   Bayesian (BIC)                             28408.909
##   Sample-size adjusted Bayesian (SABIC)      28351.740
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.027
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.049
##   P-value H_0: RMSEA <= 0.050                    0.958
##   P-value H_0: RMSEA >= 0.080                    0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.019
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   F1 =~
##     Measr1_T1 (L1)    1.000                               1.983    0.731
##     Measr2_T1 (L2)    0.967    0.086   11.280    0.000    1.918    0.532
##     Measr3_T1 (L3)    0.595    0.053   11.296    0.000    1.180    0.506
##   F2 =~
##     Measr1_T2 (L1)    1.000                               1.998    0.724
##     Measr2_T2 (L2)    0.967    0.086   11.280    0.000    1.933    0.516
##     Measr3_T2 (L3)    0.595    0.053   11.296    0.000    1.189    0.515
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .Measure1_T1 ~~
##    .Measure1_T2       1.061    0.271    3.913    0.000    1.061    0.301
##  .Measure2_T1 ~~
##    .Measure2_T2       4.470    0.419   10.680    0.000    4.470    0.456
##  .Measure3_T1 ~~
```

```
##    .Measure3_T2       1.518    0.164    9.261    0.000    1.518    0.381
##   F1 ~~
##     F2               2.535    0.311    8.145    0.000    0.640    0.640
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Measr1_T1 (i1)    8.323    0.083  100.452    0.000    8.323    3.067
##    .Measr1_T2 (i1)    8.323    0.083  100.452    0.000    8.323    3.016
##    .Measr2_T1 (i2)   12.562    0.107  117.950    0.000   12.562    3.481
##    .Measr2_T2 (i2)   12.562    0.107  117.950    0.000   12.562    3.357
##    .Measr3_T1 (i3)    8.129    0.066  122.461    0.000    8.129    3.484
##    .Measr3_T2 (i3)    8.129    0.066  122.461    0.000    8.129    3.519
##     F1               0.000                               0.000    0.000
##     F2               0.116    0.075    1.540    0.124    0.058    0.058
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Measure1_T1       3.431    0.390    8.794    0.000    3.431    0.466
##    .Measure2_T1       9.343    0.541   17.258    0.000    9.343    0.717
##    .Measure3_T1       4.051    0.224   18.059    0.000    4.051    0.744
##    .Measure1_T2       3.626    0.402    9.016    0.000    3.626    0.476
##    .Measure2_T2      10.267    0.581   17.678    0.000   10.267    0.733
##    .Measure3_T2       3.923    0.221   17.727    0.000    3.923    0.735
##     F1               3.931    0.438    8.975    0.000    1.000    1.000
##     F2               3.992    0.448    8.914    0.000    1.000    1.000
```

2.3.2.5 Testing scalar invariance and specifity stability across waves .

In addition to scalar invariance, the restrictions for specificity stability across waves are specified for each measure, and certain values (see 2.3.2.3) are fixed to ensure model identification. Since each specific factor is defined by only two variables, this part of the model is exactly identified, and the goodness of fit remains unchanged compared to the scalar invariance model.

```
model_2wsis <- '
# Factor loadings
F1 =~ L1*Measure1_T1 + L2*Measure2_T1 + L3*Measure3_T1
F2 =~ L1*Measure1_T2 + L2*Measure2_T2 + L3*Measure3_T2

# Intercepts
Measure1_T1 + Measure1_T2  ~ i1*1
Measure2_T1 + Measure2_T2  ~ i2*1
Measure3_T1 + Measure3_T2  ~ i3*1

F1 ~ 0*1   # Fix mean of F1 to 0
F2 ~ 1     # Free F2 mean

# Loadings of item specificities
SP1 =~ NA*D1*Measure1_T1 + D1*Measure1_T2
SP2 =~ NA*D2*Measure2_T1 + D2*Measure2_T2
SP3 =~ NA*D3*Measure3_T1 + D3*Measure3_T2

# Fix variance of specific factors to 1 (for identification)
SP1 ~~ 1*SP1
SP2 ~~ 1*SP2
```

```r
SP3 ~~ 1*SP3

# Specific factors constrained to be uncorrelated with all specific factors
SP1 ~~ 0*SP2 + 0*SP3
SP2 ~~ 0*SP3

#Specific factors constrained to be uncorrelated with all factors
SP1 ~~ 0*F1 + 0*F2
SP2 ~~ 0*F1 + 0*F2
SP3 ~~ 0*F1 + 0*F2 '

f_2wsis <- cfa(model = model_2wsis, data = dat)
summary(f_2wsis, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 50 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        26
##   Number of equality constraints                     8
##
##   Number of observations                          1000
##
## Model Test User Model:
##
##   Test statistic                                15.539
##   Degrees of freedom                                 9
##   P-value (Chi-square)                           0.077
##
## Model Test Baseline Model:
##
##   Test statistic                              1361.442
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.995
##   Tucker-Lewis Index (TLI)                       0.992
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -14142.285
##   Loglikelihood unrestricted model (H1)     -14134.515
##
##   Akaike (AIC)                               28320.570
##   Bayesian (BIC)                             28408.909
##   Sample-size adjusted Bayesian (SABIC)      28351.740
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.027
##   90 Percent confidence interval - lower         0.000
```

```
##   90 Percent confidence interval - upper         0.049
##   P-value H_0: RMSEA <= 0.050                    0.958
##   P-value H_0: RMSEA >= 0.080                    0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.019
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   F1 =~
##     Measr1_T1 (L1)    1.000                               1.983    0.731
##     Measr2_T1 (L2)    0.967    0.086   11.280    0.000    1.918    0.532
##     Measr3_T1 (L3)    0.595    0.053   11.296    0.000    1.180    0.506
##   F2 =~
##     Measr1_T2 (L1)    1.000                               1.998    0.724
##     Measr2_T2 (L2)    0.967    0.086   11.280    0.000    1.933    0.516
##     Measr3_T2 (L3)    0.595    0.053   11.296    0.000    1.189    0.515
##   SP1 =~
##     Measr1_T1 (D1)    1.030    0.132    7.825    0.000    1.030    0.380
##     Measr1_T2 (D1)    1.030    0.132    7.825    0.000    1.030    0.373
##   SP2 =~
##     Measr2_T1 (D2)    2.114    0.099   21.360    0.000    2.114    0.586
##     Measr2_T2 (D2)    2.114    0.099   21.360    0.000    2.114    0.565
##   SP3 =~
##     Measr3_T1 (D3)    1.232    0.067   18.522    0.000    1.232    0.528
##     Measr3_T2 (D3)    1.232    0.067   18.522    0.000    1.232    0.533
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   SP1 ~~
##     SP2               0.000                               0.000    0.000
##     SP3               0.000                               0.000    0.000
##   SP2 ~~
##     SP3               0.000                               0.000    0.000
##   F1 ~~
##     SP1               0.000                               0.000    0.000
##   F2 ~~
##     SP1               0.000                               0.000    0.000
##   F1 ~~
##     SP2               0.000                               0.000    0.000
##   F2 ~~
##     SP2               0.000                               0.000    0.000
##   F1 ~~
##     SP3               0.000                               0.000    0.000
##   F2 ~~
##     SP3               0.000                               0.000    0.000
##   F1 ~~
```

```
##     F2                 2.535    0.311    8.145    0.000    0.640    0.640
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Measr1_T1 (i1)    8.323    0.083  100.452    0.000    8.323    3.067
##    .Measr1_T2 (i1)    8.323    0.083  100.452    0.000    8.323    3.016
##    .Measr2_T1 (i2)   12.562    0.107  117.950    0.000   12.562    3.481
##    .Measr2_T2 (i2)   12.562    0.107  117.950    0.000   12.562    3.357
##    .Measr3_T1 (i3)    8.129    0.066  122.461    0.000    8.129    3.484
##    .Measr3_T2 (i3)    8.129    0.066  122.461    0.000    8.129    3.519
##     F1                0.000                               0.000    0.000
##     F2                0.116    0.075    1.540    0.124    0.058    0.058
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     SP1               1.000                               1.000    1.000
##     SP2               1.000                               1.000    1.000
##     SP3               1.000                               1.000    1.000
##    .Measure1_T1       2.370    0.266    8.927    0.000    2.370    0.322
##    .Measure2_T1       4.873    0.418   11.648    0.000    4.873    0.374
##    .Measure3_T1       2.533    0.186   13.614    0.000    2.533    0.465
##    .Measure1_T2       2.564    0.275    9.330    0.000    2.564    0.337
##    .Measure2_T2       5.797    0.446   12.995    0.000    5.797    0.414
##    .Measure3_T2       2.406    0.184   13.100    0.000    2.406    0.451
##     F1                3.931    0.438    8.975    0.000    1.000    1.000
##     F2                3.992    0.448    8.914    0.000    1.000    1.000
```

2.3.2.6 Calculating specificty-improved reliability for scalar invariant measures . To calculate the reliability values, the errors of the measures (E1 to E6) and the variance of the latent factors (Fv1 and Fv2), which are freely estimated, are labeled for further reference. Finally, the reliability values (OSR1 and OSR2) are obtained by applying the formula by Raykov (2007).

```
model_2wsisR <- '
# Factor loadings
F1 =~ L1*Measure1_T1 + L2*Measure2_T1 + L3*Measure3_T1
F2 =~ L1*Measure1_T2 + L2*Measure2_T2 + L3*Measure3_T2

# Intercepts
Measure1_T1 + Measure1_T2  ~ i1*1
Measure2_T1 + Measure2_T2  ~ i2*1
Measure3_T1 + Measure3_T2  ~ i3*1

# Loadings of item specificities
SP1 =~ NA*D1*Measure1_T1 + D1*Measure1_T2
SP2 =~ NA*D2*Measure2_T1 + D2*Measure2_T2
SP3 =~ NA*D3*Measure3_T1 + D3*Measure3_T2

# Fix variance of specific factors to 1 (for identification)
SP1 ~~ 1*SP1
SP2 ~~ 1*SP2
SP3 ~~ 1*SP3

# Specific factors constrained to be uncorrelated with all specific factors
```

```
SP1 ~~ 0*SP2 + 0*SP3
SP2 ~~ 0*SP3

#Specific factors constrained to be uncorrelated with all factors
SP1 ~~ 0*F1 + 0*F2
SP2 ~~ 0*F1 + 0*F2
SP3 ~~ 0*F1 + 0*F2

F1 ~ 0*1   # Fix mean of F1 to 0
F2 ~ 1     # Free F2 mean

# Label item errors
Measure1_T1 ~~ E1*Measure1_T1
Measure2_T1 ~~ E2*Measure2_T1
Measure3_T1 ~~ E3*Measure3_T1

Measure1_T2 ~~ E4*Measure1_T2
Measure2_T2 ~~ E5*Measure2_T2
Measure3_T2 ~~ E6*Measure3_T2

# Label latent factor variance
F1 ~~ Fv1*F1
F2 ~~ Fv2*F2

#Compute specificity-improved reliability
OSR1:= ((1 + L2 + L3 )^2*Fv1 + D1^2 + D2^2 + D3^2)/((1 + L2 + L3 )^2*Fv1 + D1^2 + D2^2 + D3^2 + E1 + E2
OSR2:= ((1 + L2 + L3 )^2*Fv2 + D1^2 + D2^2 + D3^2)/((1 + L2 + L3 )^2*Fv2 + D1^2 + D2^2 + D3^2 + E4 + E5
'
```

The specificity-improved reliability coefficient depends on the good fit of the model. As can be seen in the output the fit indices remain unchanged with respect to the previous models. All of them are satisfactory and all estimated parameters and standard errors show acceptable values. The specificity-improved reliability values are also unchanged with respect section 2.3.2.3.

```
f_2wsisR <- cfa(model = model_2wsisR, data = dat)
summary(f_2wsisR, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 50 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        26
##   Number of equality constraints                     8
##
##   Number of observations                          1000
##
## Model Test User Model:
##
##   Test statistic                                15.539
##   Degrees of freedom                                 9
##   P-value (Chi-square)                           0.077
##
## Model Test Baseline Model:
```

```
##
##   Test statistic                              1361.442
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.995
##   Tucker-Lewis Index (TLI)                       0.992
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -14142.285
##   Loglikelihood unrestricted model (H1)     -14134.515
##
##   Akaike (AIC)                               28320.570
##   Bayesian (BIC)                             28408.909
##   Sample-size adjusted Bayesian (SABIC)      28351.740
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.027
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.049
##   P-value H_0: RMSEA <= 0.050                    0.958
##   P-value H_0: RMSEA >= 0.080                    0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.019
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   F1 =~
##     Measr1_T1 (L1)    1.000                               1.983    0.731
##     Measr2_T1 (L2)    0.967    0.086   11.280    0.000    1.918    0.532
##     Measr3_T1 (L3)    0.595    0.053   11.296    0.000    1.180    0.506
##   F2 =~
##     Measr1_T2 (L1)    1.000                               1.998    0.724
##     Measr2_T2 (L2)    0.967    0.086   11.280    0.000    1.933    0.516
##     Measr3_T2 (L3)    0.595    0.053   11.296    0.000    1.189    0.515
##   SP1 =~
##     Measr1_T1 (D1)    1.030    0.132    7.825    0.000    1.030    0.380
##     Measr1_T2 (D1)    1.030    0.132    7.825    0.000    1.030    0.373
##   SP2 =~
##     Measr2_T1 (D2)    2.114    0.099   21.360    0.000    2.114    0.586
##     Measr2_T2 (D2)    2.114    0.099   21.360    0.000    2.114    0.565
##   SP3 =~
```

```
##     Measr3_T1 (D3)    1.232    0.067   18.522    0.000    1.232    0.528
##     Measr3_T2 (D3)    1.232    0.067   18.522    0.000    1.232    0.533
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   SP1 ~~
##     SP2               0.000                               0.000    0.000
##     SP3               0.000                               0.000    0.000
##   SP2 ~~
##     SP3               0.000                               0.000    0.000
##   F1 ~~
##     SP1               0.000                               0.000    0.000
##   F2 ~~
##     SP1               0.000                               0.000    0.000
##   F1 ~~
##     SP2               0.000                               0.000    0.000
##   F2 ~~
##     SP2               0.000                               0.000    0.000
##   F1 ~~
##     SP3               0.000                               0.000    0.000
##   F2 ~~
##     SP3               0.000                               0.000    0.000
##   F1 ~~
##     F2                2.535    0.311    8.145    0.000    0.640    0.640
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Measr1_T1 (i1)    8.323    0.083  100.452    0.000    8.323    3.067
##    .Measr1_T2 (i1)    8.323    0.083  100.452    0.000    8.323    3.016
##    .Measr2_T1 (i2)   12.562    0.107  117.950    0.000   12.562    3.481
##    .Measr2_T2 (i2)   12.562    0.107  117.950    0.000   12.562    3.357
##    .Measr3_T1 (i3)    8.129    0.066  122.461    0.000    8.129    3.484
##    .Measr3_T2 (i3)    8.129    0.066  122.461    0.000    8.129    3.519
##     F1                0.000                               0.000    0.000
##     F2                0.116    0.075    1.540    0.124    0.058    0.058
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     SP1               1.000                               1.000    1.000
##     SP2               1.000                               1.000    1.000
##     SP3               1.000                               1.000    1.000
##    .Mesr1_T1  (E1)    2.370    0.266    8.927    0.000    2.370    0.322
##    .Mesr2_T1  (E2)    4.873    0.418   11.648    0.000    4.873    0.374
##    .Mesr3_T1  (E3)    2.533    0.186   13.614    0.000    2.533    0.465
##    .Mesr1_T2  (E4)    2.564    0.275    9.330    0.000    2.564    0.337
##    .Mesr2_T2  (E5)    5.797    0.446   12.995    0.000    5.797    0.414
##    .Mesr3_T2  (E6)    2.406    0.184   13.100    0.000    2.406    0.451
##     F1       (Fv1)    3.931    0.438    8.975    0.000    1.000    1.000
##     F2       (Fv2)    3.992    0.448    8.914    0.000    1.000    1.000
##
## Defined Parameters:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     OSR1              0.771    0.015   50.671    0.000    0.709    0.809
##     OSR2              0.755    0.016   48.118    0.000    0.689    0.804
```

We can obtain the bootstrapped confidence interval for the reliability coefficients

```
set.seed(12345)
monteCarloCI(f_2wsisR)
```

```
##        est ci.lower ci.upper
## OSR1 0.771    0.738    0.799
## OSR2 0.755    0.723    0.784
```

3. Extension to four-wave designs

In this section, the code necessary to perform the analysis with four data collections is presented, as an extension of the procedure already outlined, following Sideridis et al. (2019).

```
model_speR4 <- '
#Factor loadings
F1 =~ L1*Measure1_T1 + L2*Measure2_T1 + L3*Measure3_T1
F2 =~ L1*Measure1_T2 + L2*Measure2_T2 + L3*Measure3_T2
F3 =~ L1*Measure1_T3 + L2*Measure2_T3 + L3*Measure3_T3
F4 =~ L1*Measure1_T4 + L2*Measure2_T4 + L3*Measure3_T4

# Intercepts
Measure1_T1 + Measure1_T2 + Measure1_T3 + Measure1_T4 ~ i1*1
Measure2_T1 + Measure2_T2 + Measure2_T3 + Measure2_T4 ~ i2*1
Measure3_T1 + Measure3_T2 + Measure3_T3 + Measure3_T4 ~ i3*1

# Loadings of item specificities
SP1 =~ NA*D1*Measure1_T1 + D1*Measure1_T2 + D1*Measure1_T3 + D1*Measure1_T4
SP2 =~ NA*D2*Measure2_T1 + D2*Measure2_T2 + D2*Measure2_T3 + D2*Measure2_T4
SP3 =~ NA*D3*Measure3_T1 + D3*Measure3_T2 + D3*Measure3_T3 + D3*Measure3_T4

# Fix variance of specific factors to 1 (for identification)
SP1 ~~ 1*SP1
SP2 ~~ 1*SP2
SP3 ~~ 1*SP3

# Specific factors constrained to be uncorrelated with all factors
SP1 ~~ 0*SP2 + 0*SP3
SP2 ~~ 0*SP3

# Specific factors constrained to be uncorrelated with all factors
SP1 ~~ 0*F1 + 0*F2 + 0*F3 + 0*F4
SP2 ~~ 0*F1 + 0*F2 + 0*F3 + 0*F4
SP3 ~~ 0*F1 + 0*F2 + 0*F3 + 0*F4

F1 ~ 0*1   # Fix mean of F1 to 0
F2 ~ 1     # Free F2 mean
F3 ~ 1     # Free F3 mean
F4 ~ 1     # Free F4 mean

# Label item errors
Measure1_T1 ~~ E1*Measure1_T1
```

```r
Measure2_T1 ~~ E2*Measure2_T1
Measure3_T1 ~~ E3*Measure3_T1

Measure1_T2 ~~ E4*Measure1_T2
Measure2_T2 ~~ E5*Measure2_T2
Measure3_T2 ~~ E6*Measure3_T2

Measure1_T3 ~~ E7*Measure1_T3
Measure2_T3 ~~ E8*Measure2_T3
Measure3_T3 ~~ E9*Measure3_T3

Measure1_T4 ~~ E10*Measure1_T4
Measure2_T4 ~~ E11*Measure2_T4
Measure3_T4 ~~ E12*Measure3_T4

# Label latent factor variance
F1 ~~ Fv1*F1
F2 ~~ Fv2*F2
F3 ~~ Fv3*F3
F4 ~~ Fv4*F4

# Compute specificity-improved reliability
OSR1:= ((1 + L2 + L3 )^2*Fv1 + D1^2 + D2^2 + D3^2)/((1 + L2 + L3 )^2*Fv1 + D1^2 + D2^2 + D3^2 + E1 + E2
OSR2:= ((1 + L2 + L3 )^2*Fv2 + D1^2 + D2^2 + D3^2)/((1 + L2 + L3 )^2*Fv2 + D1^2 + D2^2 + D3^2 + E4 + E5
OSR3:= ((1 + L2 + L3 )^2*Fv3 + D1^2 + D2^2 + D3^2)/((1 + L2 + L3 )^2*Fv3 + D1^2 + D2^2 + D3^2 + E7 + E8
OSR4:= ((1 + L2 + L3 )^2*Fv4 + D1^2 + D2^2 + D3^2)/((1 + L2 + L3 )^2*Fv4 + D1^2 + D2^2 + D3^2 + E10 + E
'

f_speR4 <- cfa(model = model_speR4, data = Data)
summary(f_speR4, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 65 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        57
##   Number of equality constraints                    24
##
##   Number of observations                          1000
##
## Model Test User Model:
##
##   Test statistic                                79.786
##   Degrees of freedom                                57
##   P-value (Chi-square)                           0.025
##
## Model Test Baseline Model:
##
##   Test statistic                              4209.664
##   Degrees of freedom                                66
##   P-value                                        0.000
##
## User Model versus Baseline Model:
```

```
##
##   Comparative Fit Index (CFI)                    0.995
##   Tucker-Lewis Index (TLI)                       0.994
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -27683.388
##   Loglikelihood unrestricted model (H1)     -27643.495
##
##   Akaike (AIC)                               55432.777
##   Bayesian (BIC)                             55594.733
##   Sample-size adjusted Bayesian (SABIC)      55489.923
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.020
##   90 Percent confidence interval - lower         0.007
##   90 Percent confidence interval - upper         0.030
##   P-value H_0: RMSEA <= 0.050                    1.000
##   P-value H_0: RMSEA >= 0.080                    0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.026
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   F1 =~
##     Measr1_T1 (L1)    1.000                               1.977    0.723
##     Measr2_T1 (L2)    0.922    0.063   14.729    0.000    1.822    0.512
##     Measr3_T1 (L3)    0.603    0.041   14.749    0.000    1.192    0.506
##   F2 =~
##     Measr1_T2 (L1)    1.000                               2.023    0.734
##     Measr2_T2 (L2)    0.922    0.063   14.729    0.000    1.865    0.504
##     Measr3_T2 (L3)    0.603    0.041   14.749    0.000    1.220    0.519
##   F3 =~
##     Measr1_T3 (L1)    1.000                               2.098    0.730
##     Measr2_T3 (L2)    0.922    0.063   14.729    0.000    1.934    0.528
##     Measr3_T3 (L3)    0.603    0.041   14.749    0.000    1.265    0.522
##   F4 =~
##     Measr1_T4 (L1)    1.000                               1.971    0.733
##     Measr2_T4 (L2)    0.922    0.063   14.729    0.000    1.817    0.486
##     Measr3_T4 (L3)    0.603    0.041   14.749    0.000    1.189    0.503
##   SP1 =~
##     Measr1_T1 (D1)    1.083    0.094   11.475    0.000    1.083    0.396
##     Measr1_T2 (D1)    1.083    0.094   11.475    0.000    1.083    0.393
##     Measr1_T3 (D1)    1.083    0.094   11.475    0.000    1.083    0.377
##     Measr1_T4 (D1)    1.083    0.094   11.475    0.000    1.083    0.403
```

```
##   SP2 =~
##     Measr2_T1 (D2)    2.101    0.075   28.100    0.000    2.101    0.590
##     Measr2_T2 (D2)    2.101    0.075   28.100    0.000    2.101    0.568
##     Measr2_T3 (D2)    2.101    0.075   28.100    0.000    2.101    0.573
##     Measr2_T4 (D2)    2.101    0.075   28.100    0.000    2.101    0.562
##   SP3 =~
##     Measr3_T1 (D3)    1.327    0.049   27.137    0.000    1.327    0.563
##     Measr3_T2 (D3)    1.327    0.049   27.137    0.000    1.327    0.564
##     Measr3_T3 (D3)    1.327    0.049   27.137    0.000    1.327    0.547
##     Measr3_T4 (D3)    1.327    0.049   27.137    0.000    1.327    0.562
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   SP1 ~~
##     SP2               0.000                                0.000    0.000
##     SP3               0.000                                0.000    0.000
##   SP2 ~~
##     SP3               0.000                                0.000    0.000
##   F1 ~~
##     SP1               0.000                                0.000    0.000
##   F2 ~~
##     SP1               0.000                                0.000    0.000
##   F3 ~~
##     SP1               0.000                                0.000    0.000
##   F4 ~~
##     SP1               0.000                                0.000    0.000
##   F1 ~~
##     SP2               0.000                                0.000    0.000
##   F2 ~~
##     SP2               0.000                                0.000    0.000
##   F3 ~~
##     SP2               0.000                                0.000    0.000
##   F4 ~~
##     SP2               0.000                                0.000    0.000
##   F1 ~~
##     SP3               0.000                                0.000    0.000
##   F2 ~~
##     SP3               0.000                                0.000    0.000
##   F3 ~~
##     SP3               0.000                                0.000    0.000
##   F4 ~~
##     SP3               0.000                                0.000    0.000
##   F1 ~~
##     F2                2.495    0.277    8.999    0.000    0.624    0.624
##     F3                2.379    0.277    8.573    0.000    0.573    0.573
##     F4                2.214    0.264    8.380    0.000    0.568    0.568
##   F2 ~~
##     F3                2.638    0.290    9.107    0.000    0.621    0.621
##     F4                2.462    0.276    8.932    0.000    0.617    0.617
##   F3 ~~
##     F4                2.394    0.278    8.618    0.000    0.579    0.579
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
```

```
##    .Measr1_T1 (i1)    8.298    0.082  101.324    0.000    8.298    3.034
##    .Measr1_T2 (i1)    8.298    0.082  101.324    0.000    8.298    3.010
##    .Measr1_T3 (i1)    8.298    0.082  101.324    0.000    8.298    2.886
##    .Measr1_T4 (i1)    8.298    0.082  101.324    0.000    8.298    3.085
##    .Measr2_T1 (i2)   12.616    0.100  126.614    0.000   12.616    3.543
##    .Measr2_T2 (i2)   12.616    0.100  126.614    0.000   12.616    3.413
##    .Measr2_T3 (i2)   12.616    0.100  126.614    0.000   12.616    3.442
##    .Measr2_T4 (i2)   12.616    0.100  126.614    0.000   12.616    3.374
##    .Measr3_T1 (i3)    8.123    0.064  126.023    0.000    8.123    3.444
##    .Measr3_T2 (i3)    8.123    0.064  126.023    0.000    8.123    3.453
##    .Measr3_T3 (i3)    8.123    0.064  126.023    0.000    8.123    3.349
##    .Measr3_T4 (i3)    8.123    0.064  126.023    0.000    8.123    3.439
##     F1                0.000                               0.000    0.000
##     F2                0.122    0.076    1.608    0.108    0.060    0.060
##     F3                0.197    0.080    2.464    0.014    0.094    0.094
##     F4               -0.016    0.078   -0.202    0.840   -0.008   -0.008
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     SP1               1.000                               1.000    1.000
##     SP2               1.000                               1.000    1.000
##     SP3               1.000                               1.000    1.000
##    .Mesr1_T1  (E1)    2.396    0.227   10.575    0.000    2.396    0.320
##    .Mesr2_T1  (E2)    4.948    0.334   14.797    0.000    4.948    0.390
##    .Mesr3_T1  (E3)    2.381    0.153   15.556    0.000    2.381    0.428
##    .Mesr1_T2  (E4)    2.332    0.222   10.525    0.000    2.332    0.307
##    .Mesr2_T2  (E5)    5.771    0.364   15.845    0.000    5.771    0.422
##    .Mesr3_T2  (E6)    2.284    0.148   15.399    0.000    2.284    0.413
##    .Mesr1_T3  (E7)    2.692    0.250   10.771    0.000    2.692    0.326
##    .Mesr2_T3  (E8)    5.283    0.354   14.903    0.000    5.283    0.393
##    .Mesr3_T3  (E9)    2.520    0.162   15.603    0.000    2.520    0.428
##    .Mesr1_T4 (E10)    2.177    0.223    9.784    0.000    2.177    0.301
##    .Mesr2_T4 (E11)    6.269    0.387   16.203    0.000    6.269    0.448
##    .Mesr3_T4 (E12)    2.407    0.154   15.604    0.000    2.407    0.431
##     F1       (Fv1)    3.910    0.383   10.198    0.000    1.000    1.000
##     F2       (Fv2)    4.094    0.393   10.407    0.000    1.000    1.000
##     F3       (Fv3)    4.404    0.424   10.387    0.000    1.000    1.000
##     F4       (Fv4)    3.885    0.381   10.202    0.000    1.000    1.000
##
## Defined Parameters:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     OSR1              0.768    0.014   56.044    0.000    0.707    0.811
##     OSR2              0.763    0.014   55.095    0.000    0.693    0.811
##     OSR3              0.771    0.013   57.245    0.000    0.691    0.810
##     OSR4              0.747    0.015   51.454    0.000    0.684    0.806
```

```r
set.seed(12345)
monteCarloCI(f_speR4)
```

```
##        est ci.lower ci.upper
## OSR1 0.768    0.739    0.793
## OSR2 0.763    0.734    0.789
## OSR3 0.771    0.743    0.796
## OSR4 0.747    0.717    0.774
```