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Mixture-betweenness: Uncertainty and commitment ☆

Fernando Payró

Department of Economics and Economic History, Autonomous University of Barcelona and Barcelona School of Economics, Spain

A R T I C L E   I N F O

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A B S T R A C T

This paper develops axiomatic models of preference under uncertainty and preference for commitment that satisfy Mixture-Betweenness, a weakening of the Independence axiom originally proposed by Chew (1989) and Dekel (1986). A central contribution of the paper is a general representation theorem that can be applied across a wide range of domains.

1. Introduction

Ever since von Neumann and Morgenstern (1947), the Independence axiom has been a fundamental part of economic theory. Independence requires that the preference ranking between two objects remains unchanged when each is mixed (probabilistically) with a common third object. As a result, models that satisfy Independence must exhibit linear and parallel indifference curves. Although originally formulated in the domain of risk, Independence has since been extended to other areas of economic behavior. In seminal contributions, Anscombe and Aumann (1963) (henceforth AA) applies it to preferences under uncertainty, while Gul and Pesendorfer (2001) (henceforth GP) uses it to study preferences for commitment.

The main motivation of this paper is to argue that adopting the Independence axiom—or weakenings that preserve its parallelism implication—in broader domains such as uncertainty and commitment may lead to models that are overly restrictive and unintentionally rule out intuitive behavior.[1] As an alternative, we propose a weaker condition, Mixture-Betweenness, originally introduced by Chew (1989) and Dekel (1986) in the context of preferences over risk.

We present two examples of behavior inconsistent with Independence: one for preferences under uncertainty (Section 1.1) and one for preferences for commitment (Section 1.2). These examples are structurally similar to the failures of Independence in the risk domain that motivate the Chew–Dekel model and, in turn, motivate our adoption of their axiom in broader settings.

More specifically, the Chew-Dekel model is an implicit utility model in which the utility γ of a lottery p is the unique solution of

$$\gamma = u(p, \gamma), \tag{1}$$

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E-mail address: fernando.payro@uab.cat.

[1] Examples include C-Independence (Gilboa and Schmeidler (1989)) and Temptation Independence (Noor and Takeoka (2010)).

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where $u(\cdot,\gamma)$ is a von Neumann–Morgenstern (vNM) utility function over lotteries for each γ. The key axiom in their characterization of the model is *Mixture-Betweenness*: if a lottery p is preferred to another lottery q, then p should also be preferred to a mixture $\alpha p + (1-\alpha)q$, and that mixture should in turn be preferred to q for all α.[2] Because the axiom is silent about behavior that involves more than two objects, it permits *mixture-dependent* behavior: $p \succeq q$ and $\alpha p + (1-\alpha)r \prec \alpha q + (1-\alpha)r$ whenever $r \succ p$ or $q \succ r$.[3]

The axiom ensures that for each utility level γ, the indifference sets $\{p \mid u(p,\gamma)=\gamma\}$ are convex. Thus, the model features linear indifference curves. However, since $u(\cdot,\gamma)$ need not be a positive affine transformation of $u(\cdot,\gamma')$, the indifference curves are not required to be parallel as in expected utility theory.[4]

Our main contribution is to show that the central ideas of Chew (1989); Conlon (1995); Dekel (1986) can be extended (Theorem 3.1) by proving an analogous result in a more abstract domain. This result then serves as a foundation for constructing our models of behavior under uncertainty and commitment, which are capable of accommodating our motivating examples and can be combined to study menu-dependent non-Bayesian updating.

The introduction ends with Section 1.1 and Section 1.2, which present behavioral examples and models that motivate adopting Mixture-Betweenness for preferences under uncertainty and commitment. Section 2 contains a short literature review. Section 3 presents our characterization of Mixture-Betweenness over compact and convex sets. Section 4 studies Mixture-Betweenness under uncertainty, while Section 4.1 extends and reinterprets our beliefs model. Section 5 introduces axioms and the representation theorem for preference under commitment. Section 5.1 discusses the ex-post choice behavior implied by our model and relates it to a version of the Allais paradox. A broader discussion of mixture-dependent commitment preferences is provided in Section 5.2. Section 6 explores the implications of Mixture-Betweenness for updating, and Section 7 concludes with a few remarks on our view of Mixture-Betweenness in modeling uncertainty and commitment. All proofs are provided in the Appendix.

1.1. Uncertainty

Research in psychology and behavioral finance shows that people's likelihood assessments can be affected by an array of phenomena (framing, emotions, desirability, etc.).[5] Particularly interesting is the evidence of illusion of control and the affect heuristic.[6] Agents subject to the illusion of control tend to overestimate their ability to influence outcomes even when those outcomes are entirely determined by external factors. The affect heuristic, on the other hand, refers to the tendency for feelings to influence likelihood judgments and decisions. For example, a trader experiencing fear of missing out might invest heavily in an unproven stock due to the hype surrounding it, while overlooking more stable stocks with stronger fundamentals.

Illusion of control has been identified as a source of poor diversification in experimental studies (Fellner (2009)) and has been shown to lead to underperformance among traders (Fenton-O'Creevy et al. (2003)). Evidence for the affect heuristic helps explain a related pattern: fund managers often recommend companies with high perceived potential for rapid expansion, called "growth stocks", over companies that appear undervalued relative to their fundamentals, known as "value stocks" (Jegadeesh et al. (2004)). This leads to underperformance as in the medium to long run, value stocks tend to outperform growth stocks (Lakonishok et al. (1994) and Chan and Lakonishok (2004)).

Agents who exhibit an illusion of control or the affect heuristic may violate the Independence axiom. To illustrate this, consider a firm whose success depends on developing a technology. Suppose the firm offers the opportunity to invest t dollars in the company (f_t). If the technology succeeds, the firm pays three times the investment ($3t$\$), if it fails it pays 0. Suppose that an agent has to decide between investing in the firm or in a safe asset (g_t). Therefore, she needs to choose between f_t, g_t given by

$$f_t = \begin{bmatrix} 3t & \text{success} \\ 0 & \text{failure} \end{bmatrix} \text{ and } g_t = \begin{bmatrix} t & \text{success} \\ t & \text{failure} \end{bmatrix}.$$

Agents suffering from fear of missing out may not experience it for small t and thus, prefer g_t to f_t, but may experience it for large t and prefer f_t to g_t. Similarly, an investor suffering from illusion of control may believe that the more she invests, the more likely it is that the technology will succeed (fail)—and conversely, that investing less makes success less (more) likely. Thus, she would prefer $f_t(g_t)$ for large t and $g_t(f_t)$ for small t.

Under standard assumptions on the agent's utility over money, this behavior contradicts the Independence axiom.[7] Specifically, Independence implies that if the agent prefers g_t to f_t at large t, she must also prefer $g_{t'}$ to $f_{t'}$ at a smaller t'. The reason is that it delivers a single belief μ over the state space that is used to evaluate every single act. Hence, it rules out the possibility that as the stakes increase, the beliefs change. However, as we show in Theorem 4.1, Mixture-Betweenness delivers a representation that allows the belief to be act-dependent. The model is a natural counterpart of the Dekel (1986) model for preference under uncertainty. It generalizes the belief component of Subjective Expected Utility (SEU) in the same way that (1) generalizes vNM theory: it retains

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[2] Mixtures refer to convex combinations.

[3] This is the key to accommodate our motivating examples as both are forms of mixture-dependent behavior.

[4] This is precisely why the model can accommodate the Allais paradox: the observed violations of Independence are due to violations of the parallelism of indifference curves, not their linearity.

[5] For example, Nygren et al. (1996) argues that positive feelings promote overestimation of the likelihood of favorable events and underestimation of the likelihood of unfavorable events.

[6] See Baker and Nofsinger (2010) for a textbook treatment.

[7] Examples include CRRA, scale invariant utility and risk adverse utility.

the structure of SEU while allowing beliefs to vary with the overall utility level, leading to act-dependent beliefs. More formally, the utility $V(f)$ of an AA act f is the unique γ that solves:

$$\gamma = \sum_{\omega\in\Omega} u(f(\omega))\mu(\omega,\gamma), \tag{2}$$

where Ω is the state space, u is a vNM utility function over lotteries, and $\mu(\cdot,\gamma)$ is a probability measure over Ω for each γ.

We observe that (2) permits acts to affect beliefs, but not in an unconstrained manner. It imposes a form of internal consistency: the belief used to rationalize a preference $f \succeq g$ must also rationalize any other comparison involving f (see Section 4). For this reason, we refer to (2) as the rationalizable beliefs model. We find this requirement appealing, as belief distortions in economic settings are rarely unconstrained. For instance, a trader may not hold completely biased beliefs if she is accountable to a superior. This external pressure can serve as a moderating force, helping to keep some of her biases in check, as she may need to justify to her superior the beliefs used to rationalize her preferences over investment strategies.

1.2. Commitment

A key motivation for the literature on temptation and self-control problems comes from empirical evidence documenting a preference for commitment (Bryan et al. (2010), Gul and Pesendorfer (2007), and Laibson (1997)). GP provides an axiomatic model of temptation and self-control that characterizes a preference over menus of lotteries that permits such behavior. A distinct feature of GP is that it maintains the Independence axiom (appropriately adapted to the domain).

Empirical evidence (Burlacu et al. (2022)) suggests that individuals facing economic hardships find it easier to resist temptation because pressing needs become more salient. This implies that when the (subjective) stakes are high, agents experience less temptation than when the stakes are low, which can naturally lead to violations of the Independence axiom. More specifically, if stakes can affect temptation, then the preference for commitment must be sensitive to mixtures, which is ruled out by Independence.

To illustrate this, consider a risk-averse investor evaluating potential investments. Suppose she knows that she experiences fear of missing out only when presented with high-stakes opportunities that could affect her career, and thus feels tempted to choose them over safer alternatives. Consequently, she prefers to avoid situations where she faces risky investments with large potential payoffs. Specifically, if she is offered two investments, p and q, where p is less risky than q and both have sufficiently small payoffs, she would be indifferent between committing to p today or leaving herself the option to choose between p and q tomorrow.[8] Equivalently, she would be indifferent between dealing with a company that offers only p and a company that offers both p and q. Identifying a company with the set of alternatives it offers, her preferences can be expressed as:

$$\{p\} \sim \{p,q\}.$$

However, suppose instead that the companies offer investments with high potential payoffs, such as $\alpha\overline{p}+(1-\alpha)p$ and $\alpha\overline{p}+(1-\alpha)q$, where $\overline{p}$ represents an attractive high-stakes investment and α is sufficiently large. In this case, the investor would prefer the company that offers only $\alpha\overline{p}+(1-\alpha)p$, since, if given the choice, she may feel tempted to select $\alpha\overline{p}+(1-\alpha)q$. Her preference would thus reflect:

$$\{\alpha\overline{p}+(1-\alpha)p\} \succ \{\alpha\overline{p}+(1-\alpha)p,\ \alpha\overline{p}+(1-\alpha)q\}.$$

Hence, the agent's preference for commitment is sensitive to mixtures. This behavior is inconsistent with the Independence axiom, under which the preference for commitment must be mixture-independent:

$$\{p\} \sim \{p,q\} \implies \{\alpha\overline{p}+(1-\alpha)p\} \sim \{\alpha\overline{p}+(1-\alpha)p, \alpha\overline{p}+(1-\alpha)q\},$$
$$\{p\} \succ \{p,q\} \implies \{\alpha\overline{p}+(1-\alpha)p\} \succ \{\alpha\overline{p}+(1-\alpha)p, \alpha\overline{p}+(1-\alpha)q\}$$

for all $\alpha \in [0,1]$.

To accommodate mixture-dependent preference for commitment, we combine the Chew–Dekel model with the temptation and self-control model of GP. The result is an implicit utility model in which the utility $V(x)$ of a menu of lotteries x is defined as the unique γ that solves:

$$\gamma = \max_{p\in x}\left\{u(p) + v(p,\gamma) - \max_{q\in x} v(q,\gamma)\right\}, \tag{3}$$

where $u(\cdot)$ and $v(\cdot,\gamma)$ are vNM utility functions over lotteries for each γ.

Because singleton menus offer commitment and $V(\{p\}) = u(p)$, we interpret u as reflecting the agent's normative preference. The function v then captures the agent's temptation or urge at the moment of choice. The key distinction between our model and GP is that we allow the temptation preference to be non-linear and menu-dependent; it depends on the menu through the overall level of utility. This systematic dependence reflects the idea that the strength of temptation may be endogenous. This feature of the model is exactly what allows it to accommodate mixture-dependent preference for commitment.

[8] Here, "less risky" means that p second-order stochastically dominates q.

2. Literature review

This paper contributes to two different strands of the literature: preference under uncertainty and preference for commitment.

Uncertainty: This paper joins the large literature of non-expected utility behavior under uncertainty. It differs from most of the literature as it does not focus on ambiguity aversion. Indeed, Mixture-Betweenness rules out preference for hedging.

The closest papers to ours are Grant et al. (2000), Lehrer and Teper (2011), and Kopylov (2021). Grant, Kaji and Polak study a weakening of the Sure Thing Principle, called Decomposability, in the Savage domain. They show that it characterizes an implicit utility representation that generalizes Savage's subjective expected utility theory. We show that the same model is characterized by Mixture-Betweenness in the AA domain and study a special case of it. Kopylov, Lehrer and Tepper extend the multiple priors model of Gilboa and Schmeidler (1989) by weakening transitivity in two different directions. Lehrer and Teper (2011) drop transitivity in order to provide a behavioral definition of justifiable choice: f is preferred to g if for at least one belief the agent would rank f over g. Our model strengthens theirs by requiring the belief used to justify f over g be used to justify *any* decision involving f. Kopylov drops transitivity in order to accommodate Fox and Tversky (1995)'s comparative ignorance finding. A special case of his model assumes Mixture-Betweenness.[9] He characterizes a representation in which the (probabilistic) beliefs used to compare acts are not fixed and depend on the acts being evaluated. However, the dependence is different than in the current paper. Specifically, in Kopylov (2021), the beliefs used to rationalize $f \succeq g$ depend on both f and g via the smallest partition of Ω for which both acts are measurable, whereas in our model the beliefs depend on the stakes associated with the entire acts. To illustrate the difference, observe that in Kopylov's model uniformly increasing the payoffs of two acts does not affect the belief used to compare them, since it does not change the underlying partition of the state space on which both acts are measurable. In contrast, in our model, increasing the payoffs raises the stakes and can therefore affect the belief used to evaluate the acts. We observe that our models only overlap on the standard SEU model and thus, view our work as complementary to his.[10]

Finally, there is a large literature on belief distortions in which the belief is affected by endowments (Kovach (2020), Mayraz (2011)), status quo (Masatlioglu and Ok (2005), Ortoleva (2010)) and availability effects (Brunnermeier and Parker (2005), Tserenjigmid (2019)). We contribute to this literature by identifying novel behavior that is explicitly linked to mixtures.

Commitment: This paper also contributes to the axiomatic literature on temptation and self-control (see Gul and Pesendorfer (2001), Dekel et al. (2001); for a survey, see Lipman and Pesendorfer (2013)). The closest related works are Noor and Takeoka (2010, 2015) and Liang et al. (2019), both of which generalize GP by weakening the Independence axiom. Noor and Takeoka (2010) extends GP to a setting with convex self-control costs, a feature shared by the non-axiomatic model of Fudenberg and Levine (2006). Liang et al. (2019) enriches GP by introducing a finite stock of willpower, which constrains the agent's ability to exercise self-control.[11] In all of these models, both normative and temptation utilities satisfy the Independence axiom, thereby ruling out mixture-dependent preferences for commitment. Moreover, their behavioral motivations do not rely on the fact that the objects of choice are lotteries.

There are several generalizations of GP in the literature. Some prominent examples are: Dekel et al. (2009), Chatterjee and Krishna (2009), Stovall (2010) and Kopylov (2012). A key feature of these models is that they satisfy the Independence axiom. We believe that our characterization of Mixture-Betweenness in abstract domains and the arguments used in the proof of Theorem 3.1 can be used to generalize these models in the same way that we generalize GP.

Outside the temptation literature, but within the menus of lotteries literature, Ergin and Sarver (2010) provide a utility representation of costly contemplation. In their model, the agent chooses from a menu while facing uncertainty about her own preferences over lotteries. Specifically, she considers a set of possible preferences, each of which satisfies the standard Independence axiom. Their key axiom is a weakening of Mixture-Betweenness.

Finally, Dillenberger and Sadowski (2012) provide a model of shame for preferences over menus of monetary allocations between two agents. Their model is extended by Saito (2015), who proposes a model of impure shame and impure altruism, that is, shame and altruism driven by temptation, over menus of lotteries, while still assuming the Independence axiom.

3. Mixture-betweenness in abstract settings

Let $\mathcal{M}$ be a compact and convex subset of a linear space. Our primitive is a binary relation $\succeq$ over $\mathcal{M}$.

Consider the following adaptations of the basic axioms of expected utility:

Weak Order $\succeq$ is complete and transitive.
Continuity The sets $\{y | y \succeq x\}$ and $\{y | x \succeq y\}$ are closed for all $x \in \mathcal{M}$.
Independence $x \succeq y$ implies $\alpha x + (1-\alpha)z \succeq \alpha y + (1-\alpha)z$ for all $z \in \mathcal{M}$ and $\alpha \in [0,1]$.

These three axioms characterize the natural analog of the vNM representation in abstract settings. Specifically, Herstein and Milnor (1953) shows that they are equivalent to the existence of a *mixture-linear* utility representation: $\Phi : \mathcal{M} \to \mathbb{R}$ such that

[9] To the best of our knowledge, Kopylov (2021) is the only other paper that considers Mixture-Betweenness under uncertainty.

[10] His model satisfies C-Independence and under transitivity, Mixture-Betweenness and C-Independence imply Independence.

[11] A related contribution, Masatlioglu et al. (2020), studies menus of abstract alternatives rather than lotteries. An earlier version included lotteries and imposed linearity on temptation preferences.

$$\Phi(\alpha x+(1-\alpha)y)=\alpha\Phi(x)+(1-\alpha)\Phi(y)$$

for all $x, y \in \mathcal{M}$ and $\alpha \in [0,1]$.[12]

We generalize Herstein and Milnor (1953) by weakening the Independence axiom to an adaptation of Chew (1989)-Dekel (1986) Mixture-Betweenness:

Mixture-Betweenness

$x \succ y$ implies $x \succ \alpha x+(1-\alpha)y \succ y$ for all $\alpha \in (0,1)$, and
$x \sim y$ implies $x \sim \alpha x+(1-\alpha)y \sim y$ for all $\alpha \in (0,1)$.

The following theorem shows that just as Independence in abstract settings is characterized by an analog of the vNM utility model, Mixture-Betweenness is characterized by the corresponding analog of the Chew-Dekel model.

Theorem 3.1. *Let $\succeq$ be a non-trivial binary relation over $\mathcal{M}$. The following statements are equivalent:*

a) $\succeq$ satisfies Weak Order, Continuity, and Mixture-Betweenness.
b) There exists a function $\Phi : \mathcal{M} \times [0,1] \to \mathbb{R}$ such that:

1.- Φ is continuous in its first argument and in the second on $(0,1)$.
2.- Φ is mixture-linear in its first argument for all $\gamma \in [0,1]$.
3.- There exist $\overline{x}, \underline{x} \in \mathcal{M}$ such that $\Phi(\overline{x},\gamma)=1$ and $\Phi(\underline{x},\gamma)=0$ for all $\gamma \in [0,1]$.
4.- For each $x \in \mathcal{M}$, the equation $\Phi(x,\gamma)=\gamma$ has a unique solution in $[0,1]$.
5.- $\succeq$ can be represented by a continuous function $V : \mathcal{M} \to [0,1]$, where for each $x \in \mathcal{M}$, $V(x)$ is the unique $\gamma \in [0,1]$ that solves

$$\gamma=\Phi(x,\gamma).$$

The proof of Theorem 3.1 is similar *in spirit* to the proofs of Chew (1989), Dekel (1986) and Conlon (1995).[13] However, they exploit properties of the probability simplex that have no evident analog in our framework. More specifically, Chew (1989) and Dekel (1986) use its geometric properties, and Conlon (1995) combines the geometric and topological properties. Hence, to prove the theorem we were forced to employ different arguments.

Next, we provide the uniqueness properties of the representation defined by the Theorem. Say that Φ represents $\succeq$ if it satisfies the conditions of Theorem 3.1.

Theorem 3.2. *Let Φ, Φ' be such that $\Phi(\overline{x},\gamma)=\Phi'(\overline{x},\gamma)=1$ and $\Phi(\underline{x},\gamma)=\Phi'(\underline{x},\gamma)=0$ for all $\gamma \in [0,1]$. Then Φ, Φ' represent $\succeq$ if and only if $\Phi=\Phi'$.*

Theorem 3.2 establishes that a preference $\succeq$ has a unique representation Φ in which $\Phi(\overline{x},\gamma)=1$ and $\Phi(\underline{x},\gamma)=0$ for all γ. This is exactly the same uniqueness result as in Dekel (1986), where he fixes $\overline{x}$ and $\underline{x}$ exogenously.[14]

Due to its generality, Theorem 3.1 has several potential applications. In this paper, we fully develop an application to uncertainty (Section 4) and to preference for commitment (Section 5).

4. Mixture-betweenness under uncertainty

We consider the standard AA framework: Ω is a finite set of states, $\Delta(X)$ is the set of all lotteries over some finite set X and $\mathcal{F}=\{f : \Omega \to \Delta(X)\}$ the set of all AA acts.

We write p for the constant act f such that $f(\omega)=p$ for all $\omega \in \Omega$ and fAg for the act h such that $h(\omega)=f(\omega)$ for all $\omega \in A$ and $h(\omega)=g(\omega)$ for all $\omega \in \Omega \backslash A$. We say that a state ω is null if $f \sim g$ for all f, g such that $f(\omega')=g(\omega')$ for all $\omega' \neq \omega$. Given $f, g \in \mathcal{F}$ and $\alpha \in [0,1]$, we use $\alpha f+(1-\alpha)g$ to denote the act in $\mathcal{F}$ that equals $\alpha f(\omega)+(1-\alpha)g(\omega)$ in state ω.

A standard result in the literature is that a preference over $\mathcal{F}$ satisfies Weak Order, Continuity and Independence if and only if it admits a *state-dependent expected utility* representation:

$$U(f)=\sum_{\omega \in \Omega} u(f(\omega),\omega)$$

where $u(\cdot,\omega) : \Delta(X) \to \mathbb{R}$ is a vNM utility function for all $\omega \in \Omega$. An immediate implication of Theorem 3.1 is that replacing Independence with Mixture-Betweenness yields an *implicit state-dependent expected utility representation*:

[12] Herstein and Milnor's result applies to settings more general than convex sets.

[13] See the working paper version of this paper for a formulation of Theorem 3.1 that applies to settings more general than convex sets.

[14] A stronger uniqueness result can be established if one drops the requirement in Theorem 3.2 that $\Phi(\overline{x},\gamma)=\Phi'(\overline{x},\gamma)=1$ and $\Phi(\underline{x},\gamma)=\Phi'(\underline{x},\gamma)=0$. We provide the proof of such a result in the appendix.

Corollary 4.1. *A non-trivial preference $\succeq$ satisfies Weak Order, Continuity, and Mixture-Betweenness if and only if there exists a function $u : \Delta(X)\times\Omega\times[0,1]\to\mathbb{R}$ such that:*

1. *$u(\cdot,\omega,\gamma)$ is a vNM utility function for all $\omega\in\Omega$ and $\gamma\in[0,1]$.*
2. *u is continuous in γ on the interval $(0,1)$.*
3. *There exist constant acts $\overline{\delta},\underline{\delta}$ such that*

$$\sum_{\omega\in\Omega} u(\overline{\delta},\omega,\gamma)=1 \quad \text{and} \quad \sum_{\omega\in\Omega} u(\underline{\delta},\omega,\gamma)=0.$$

4. *$\succeq$ can be represented by a continuous utility function $V : \mathcal{F}\to[0,1]$, where for each $f\in\mathcal{F}$, $V(f)$ is the unique $\gamma\in[0,1]$ solving*

$$\gamma=\sum_{\omega\in\Omega} u(f(\omega),\omega,\gamma).$$

Grant et al. (2000) shows that weakening the Sure Thing Principle to the following axiom characterizes the implicit expected utility model in a version of the Savage domain.

Decomposability For all $A\subset\Omega$, and $f,g\in\mathcal{F}$,

$$fAg\succeq g \text{ and } gAf\succeq g \implies f\succeq g.$$

Intuitively, Decomposability requires that if an act f is preferred to g conditional on an event A, and f is preferred to g conditional on the complement of A, then f should be preferred to g unconditionally. Corollary 4.1 formally shows that under standard conditions, the behavioral implications of Decomposability on Savage acts are equivalent to the implications of Mixture-Betweenness on AA acts. Indeed, violations of Decomposability need to yield violations of Mixture-Betweenness. To see this, suppose that for some event we have $fAg\succeq g, gAf\succeq g$ and $g\succ f$. Then, under Mixture-Betweenness,

$$g\succ\frac{1}{2}g+\frac{1}{2}f\succ f \text{ and } \frac{1}{2}fAg+\frac{1}{2}gAf\succeq g.$$

However, $\frac{1}{2}g+\frac{1}{2}f=\frac{1}{2}fAg+\frac{1}{2}gAf$ an impossibility.

The implicit state-dependent model does not allow for a separation between tastes and beliefs as the model does not satisfy any sensible uniqueness properties. Moreover, it allows for non-expected utility behavior over lotteries which is unrelated to belief biases. This leads us to consider the following two axioms:

Independence over Lotteries: For all $p,q,r\in\Delta(X)$ and $\alpha\in[0,1]$,

$$p\succeq q \implies \alpha p+(1-\alpha)r\succeq\alpha q+(1-\alpha)r.$$

State-Independence: For all $f\in\mathcal{F}$, $p\in\Delta(X)$, and non-null $\omega'\in\Omega$,

$$p\omega f\succ q\omega f \implies p\omega' f\succ q\omega' f.$$

Independence over Lotteries guarantees that the agent does not deviate from expected utility theory when evaluating lotteries. Because any non-standard behavior implied by belief biases has to be over actions that have uncertain payoffs rather than actions with certain payoffs, it is natural to impose such a restriction. This is also our motivation to impose State-Independence. As its name suggests, it guarantees that tastes do not vary with states

Theorem 4.1. *A preference $\succeq$ satisfies Weak Order, Continuity, State-Independence, Independence over Lotteries and Mixture-Betweenness if and only if there exist a vNM utility function $u : \Delta(X)\to[0,1]$ and a mapping $\mu : \Omega\times[0,1]\to[0,1]$ such that $\mu(\cdot,\gamma)\in\Delta(\Omega)$ for all $\gamma\in[0,1]$, $\mu(\omega,\cdot)$ is continuous on $(0,1)$ for all $\omega\in\Omega$, $\mu(\omega,\gamma)=0$ for all null $\omega\in\Omega$ and $\gamma\in[0,1]$, and $\succeq$ can be represented by a continuous utility function $V : \mathcal{F}\to[0,1]$ where, for each $f\in\mathcal{F}$, $V(f)$ is the unique $\gamma\in[0,1]$ that solves*

$$\gamma=\sum_{\omega\in\Omega} u(f(\omega))\mu(\omega,\gamma).$$

Further, u is unique and the probability measure $\mu(\cdot,\gamma)$ is unique for each $\gamma\in(0,1)$.

The uniqueness properties of u are completely characterized by the restriction of $\succeq$ to $\Delta(X)$. Thus, the fact that u is normalized implies that it is unique. Similarly, observe that for each γ, $\sum_{\omega\in\Omega}u(\cdot)\mu(\omega,\gamma)$ is a SEU functional and therefore $\mu(\cdot)$ is unique. The lack of uniqueness of $\mu(\cdot,1)$ and $\mu(\cdot,0)$ comes from the fact that our axioms imply the existence of a best ($\overline{\delta}$) and a worst ($\underline{\delta}$) act that are constant and degenerate and thus, no constraints are imposed on $\mu(\cdot,1)$ and $\mu(\cdot,0)$ other than placing probability zero on null states.

The key property of the above model is that $f \succsim g$ if and only if

$$\sum_{\omega\in\Omega} u(f(\omega))\mu(\omega, V(f)) \geq \sum_{\omega\in\Omega} u(g(\omega))\mu(\omega, V(f)) \quad \text{and} \quad \sum_{\omega\in\Omega} u(f(\omega))\mu(\omega, V(g)) \geq \sum_{\omega\in\Omega} u(g(\omega))\mu(\omega, V(g)). \tag{4}$$

That the rationalizable model satisfies (4) follows from continuity of μ and the unique solution requirement in Theorem 4.1. Under the interpretation that $\mu(\cdot, V(f))$ is the belief the agent holds when evaluating f, (4) can be understood as stating that the agent prefers f over g if and only if f yields a higher utility under the beliefs induced by both f and g. Therefore, the belief used to evaluate f supports any decision involving f. We view this as an attractive feature of the model, as in many settings belief biases cannot be entirely unconstrained and some degree of internal consistency should hold.

Finally, we conclude by elaborating on our motivating example. Recall the rankings:

$$f_t = \begin{bmatrix} 3t & \text{success} \\ 0 & \text{failure} \end{bmatrix} \succ g_t = \begin{bmatrix} t & \text{success} \\ t & \text{failure} \end{bmatrix}$$

for large t, while $g_t \succ f_t$ for small t. Observe that under SEU, $f_t \succ g_t$ if and only if $\mu(\text{success}) > \frac{u(t)}{u(3t)}$.[15] Under standard assumptions on u, that ratio cannot vary too much between small and large scales. For instance, for concave u, $u(3t) \leq 3u(t)$, so $\frac{u(t)}{u(3t)} \geq \frac{1}{3}$ for all $t > 0$. Hence, if $\mu(\text{success}) \leq \frac{1}{3}$, it is impossible for the agent to prefer f_t at any scale. If $\mu(\text{success}) > \frac{1}{3}$, then for small enough t the agent has a strict preference for f_t. For homothetic u (e.g., CRRA), the ratio $\frac{u(t)}{u(3t)}$ is constant in t, so the ranking of f_t vs. g_t is independent of stakes.

Intuitively, under Independence the geometry of indifference is parallel across scales, which rules out the pattern $g_t \succ f_t$ for small t but $f_{t^*} \succ g_{t^*}$ for large t^*. For example, with CRRA utility for any $t < t^*$ there exist $\underline{\delta}\alpha$ such that

$$\alpha f_{t^*} + (1-\alpha)\delta \sim f_t, \quad \alpha g_{t^*} + (1-\alpha)\delta \sim g_t,$$

which implies that $f_{t^*} \succsim g_{t^*}$ if and only if $f_t \succsim g_t$. Since Mixture-Betweenness relaxes Independence by dropping parallelism, it is particularly well suited to accommodate such behavior.

4.1. An extension and a reinterpretation

It is worth pointing out that there is an intermediate special case between the rationalizable beliefs model and the implicit state-dependent model. Specifically, the utility $V(f)$ of an act f is the unique γ that solves

$$\gamma = \sum_{\omega\in\Omega} u(f(\omega), \gamma)\mu(\omega, \gamma), \tag{5}$$

where $u(\cdot, \gamma)$ is a vNM utility function and $\mu(\cdot, \gamma)$ is a probability measure over Ω.[16]

An interesting feature of the model is that it cannot generate the three-color Ellsberg Paradox.[17] Given that the implicit state-dependent model can accommodate the three-color Ellsberg Paradox (see Grant et al. (2000)), (5) can be seen as the most general model within the Mixture-Betweenness class that maintains a separation between tastes and beliefs but cannot capture Ellsberg-type behavior. As such, it provides a natural boundary for classifying behavior under uncertainty as either *mixture-dependent* or ambiguity-averse.

Finally, we conclude the application to uncertainty by reinterpreting our results. Let $\succsim$ be a preference over streams of lotteries of length T: $C = \{(p_t)_{t=1}^T | p_t \in \Delta(X) \text{ for all } t\}$. Because this setting has an identical structure to the AA setting, our axioms yield a representation for $\succsim$ in which the utility of the stream $(p_t)_{t=1}^T$ is the unique γ that solves

$$\gamma = \sum_t \beta(t, \gamma) u(p_t),$$

where u is a vNM utility function over lotteries and $\beta(\cdot, \gamma) \in (0, 1)$ for all γ and all t. Hence, our axioms also characterize an implicit discount model. Such a model is a special case of Epstein (1986). He shows that it satisfies the turnpike properties and argues for its tractability.

[15] Here $u(0)$ is normalized to 0.

[16] See the working paper version for an axiomatization of (5).

[17] In the three-color Ellsberg Paradox, the agent is told that there are 90 balls in an urn: 30 are red, and the remaining 60 are either blue or green in unknown proportion. Under the model (5), it must be that $\mu(\text{Red}, \gamma) = \frac{1}{3}$ for all γ, reflecting the known proportion. Moreover, if the agent is indifferent between a bet on Blue and a bet on Green, the model requires that $\mu(\text{Blue}, \gamma) = \mu(\text{Green}, \gamma) = \frac{1}{3}$, for all γ. Therefore, the agent cannot strictly prefer a bet on Red to a bet on Blue.

5. Mixture-betweenness under commitment

We adopt the same setting as Dekel et al. (2001). Recall X is a finite set and let $\mathcal{X}$ be the set of all non-empty closed subsets of $\Delta(X)$. We endow $\mathcal{X}$ with the topology generated by the Hausdorff metric.[18] A menu is an element of $\mathcal{X}$. Generic menus will be denoted by x, y, and z and generic lotteries will be denoted as in the uncertainty section. For any two menus x, y and $\alpha \in [0,1]$, define the mixture $\alpha x + (1-\alpha)y$ as the menu generated by the point-wise mixtures:

$$\alpha x + (1-\alpha)y = \{r \in \Delta(X) | r = \alpha p + (1-\alpha)q, p \in x, q \in y\}.$$

In this setting, our primitive is a preference $\succeq$ over $\mathcal{X}$. Observe that $\mathcal{X}$ is not a convex subset of a linear space. Indeed, a mixture of a menu x with itself can yield a different menu.[19] Nevertheless, the collection of all closed and convex subsets does form a compact and convex set (See Appendix C). This suffices to apply the techniques developed in Section 3.

We impose five axioms on $\succeq$ of which the first three are from GP. The first two are the Weak Order and Continuity axioms of Section 3. The third is their key axiom:

Set-Betweenness $x \succeq y$ implies $x \succeq x \cup y \succeq y$.

Set-Betweenness admits an interpretation in terms of temptation and self-control. To illustrate this, consider the ranking $\{p\} \succ \{p,q\} \succ \{q\}$. The ranking $\{p\} \succ \{p,q\}$ is referred to as *preference for commitment*, it suggests that the agent expects to be tempted by q if she faces $\{p,q\}$. Thus, $\{p,q\} \succ \{q\}$ implies that the agent expects to be able to resist temptation if she faces $\{p,q\}$, but it will require costly self-control. Similarly, $\{p\} \succ \{p,q\} \sim \{q\}$ suggests that the agent expects to be overwhelmed by temptation if she faces $\{p,q\}$. Finally, the lack of preference for commitment in $\{p\} \sim \{p,q\} \succ \{q\}$ suggests that the agent does not expect to be tempted by q if she faces $\{p,q\}$.

Whenever $x \subset y$ and $x \succ y$ we say $\succeq$ *has preference for commitment at* y. Under the temptation and self-control interpretation, preference for commitment at y reveals that there is some element in y that the agent expects to be tempted by and thus, would like to remove from the feasible set she will face in the second stage.

GP's fourth axiom is the standard Independence axiom of Section 3. GP, Dekel et al. (2001) and the literature that followed them adopt this axiom because of its normative appeal and the analytical convenience it offers. To understand their motivation consider an extension of the preference to the set of lotteries over $\mathcal{X}$, the interpretation being that randomization over menus is resolved before the second stage. Suppose that this extended preference satisfies the standard vNM Independence axiom: the preference between a lottery that yields with probability α a menu x and with probability $1-\alpha$ a menu z, denoted by $\alpha \circ x + (1-\alpha) \circ z$, and $\alpha \circ y + (1-\alpha) \circ z$ is the same as the preference between x and y. If the agent is indifferent between uncertainty being resolved before the second stage or after the second stage, then she satisfies *Reduction*:

$$\alpha \circ x + (1-\alpha) \circ y \sim \alpha x + (1-\alpha)y \quad \text{for all } x, y \in \mathcal{X} \text{ and } \alpha \in [0,1].$$

GP observe that vNM Independence and Reduction imply Independence and thus, find it normatively appealing. However, once we consider stake-dependent temptation, then Reduction is not appropriate. Recall the rankings in our example:

$$\begin{aligned} &\{p\} \sim \{p,q\}, \\ &\{\alpha\overline{p} + (1-\alpha)p\} \succ \{\alpha\overline{p} + (1-\alpha)p, \alpha\overline{p} + (1-\alpha)q\}. \end{aligned} \tag{6}$$

The intuition behind these rankings implies that the agent would strictly prefer $\{\alpha\overline{p}+(1-\alpha)p\}$ to $\{\alpha\overline{p}+(1-\alpha)p, \alpha\overline{p}+(1-\alpha)q\}$ because for high-stakes lotteries she is tempted to take more risk than what her ex-ante persona would do. Hence, if instead of considering $\{\alpha\overline{p}+(1-\alpha)p\}$ and $\{\alpha\overline{p}+(1-\alpha)p, \alpha\overline{p}+(1-\alpha)q\}$ we consider $\alpha \circ \{\overline{p}\} + (1-\alpha) \circ \{p\}$ and $\alpha \circ \{\overline{p}\} + (1-\alpha) \circ \{p,q\}$, the agent would not expect to feel any temptation. Indeed, her ex-post choices from $\{p,q\}$ do not involve high-stakes.

Independence as adopted by GP implies that preference for commitment is mixture-independent: if $\succeq$ has preference for commitment at y, then $\succeq$ also has preference for commitment at $\alpha y + (1-\alpha)z$ for all $\alpha \in (0,1]$ and $z \in \mathcal{X}$. This follows from the fact that under Independence,

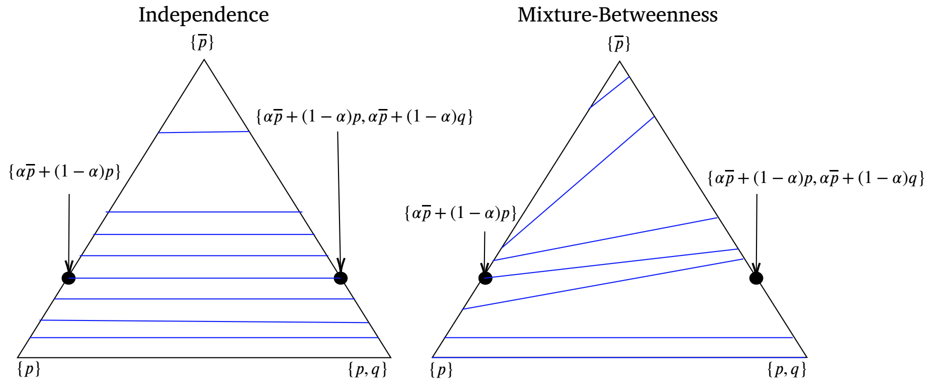
$$\begin{aligned} &x \succ y \implies \alpha x + (1-\alpha)z \succ \alpha y + (1-\alpha)z, \\ &x \subset y \implies \alpha x + (1-\alpha)z \subset \alpha y + (1-\alpha)z. \end{aligned}$$

Hence, Independence needs to be weakened. We weaken it to Mixture-Betweenness. As described in the uncertainty section, Mixture-Betweenness requires the indifference curves be linear but allows them to not be parallel. Thus, the ranking of two menus may change when each is mixed with a common third menu. This is necessary to allow for temptation to be *sensitive* to risk.

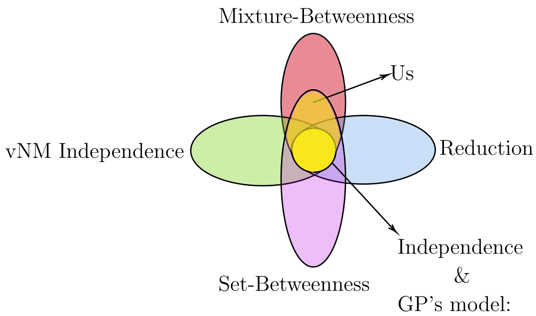
[18] Let d be any metric on $\Delta(X)$. For any $x, y \in \mathcal{X}$ and $p, q \in \Delta(X)$, define $d(p,y) \equiv \inf_{q \in y} d(p,q)$ and $d_h(x,y) = \max\{\sup_{p\in x} d(p,y), \sup_{q \in y} d(q,x)\}$. The topology generated by d_h is the Hausdorff metric topology.

[19] For instance, $\frac{1}{2}\{p,q\} + \frac{1}{2}\{p,q\} \neq \{p,q\}$ if $p \neq q$.

To build intuition, we can provide a visual representation of the indifference curves implied by the axiom and contrast them with those under the standard Independence axiom. Consider the rankings in (6). The figure below illustrates how the convexity of indifference curves permits mixture-dependent preferences for commitment.



To understand how Mixture-Betweenness relates to GP's key axioms, observe that without Reduction, it can coexist with vNM Independence while failing to satisfy Independence. Indeed, both axioms impose structure on different objects. However, under Reduction, vNM Independence implies Independence and thus, is a special case of Mixture-Betweenness. Moreover, GP's Set-Betweenness is unrelated to mixtures. A visual representation of the relationships is provided by the following diagram.



It is important to note that in the context of preference over lotteries, the experimental literature has found that Mixture-Betweenness over lotteries is often violated. We argue that such evidence cannot be applied to our setting and that the evidence actually suggests it is a good descriptive axiom for preference over menus (see Section 5.1).

Our final axiom requires that the agent's commitment/normative preference satisfy the standard Independence axiom.[20]

Commitment Independence

$\{p\} \succeq \{q\}$ implies $\alpha\{p\}+(1-\alpha)\{r\} \succeq \alpha\{q\}+(1-\alpha)\{r\}$
for all $\alpha \in [0,1]$ and $r \in \Delta(X)$.

The central result of the mixture-dependent section is the following axiomatization of utility over menus.

Theorem 5.1. *A non-trivial preference $\succeq$ satisfies Weak Order, Hausdorff Continuity, Set-Betweenness, Commitment Independence, and Mixture-Betweenness if and only if there exist $u : \Delta(X) \to [0,1]$ and $v : \Delta(X) \times [0,1] \to \mathbb{R}$ such that:*

1. *u and $v(\cdot,\gamma)$ are vNM utility functions for all $\gamma \in [0,1]$.*
2. *$\succeq$ can be represented by a continuous utility function $V : \mathcal{X} \to [0,1]$ where, for each $x \in \mathcal{X}$, $V(x)$ is the unique $\gamma \in [0,1]$ that solves*

$$\gamma = \max_{p\in x}\{u(p)+v(p,\gamma)-\max_{q\in x} v(q,\gamma)\}.$$

The fact that the temptation preferences depend on the overall level of utility does not imply that v defines a Chew-Dekel utility over $\Delta(X)$ and thus, the ex-post choice need not be Chew-Dekel. To illustrate this, consider the case in which $v(\cdot,\gamma) = -\gamma u(\cdot)$. Then, v does not define an implicit utility function over $\Delta(X)$. However, (u, v) define the following explicit utility function over $\mathcal{X}$:

[20] See the working paper version of this paper for the representation result that does not assume Commitment Independence.

$$V(x) = \frac{\max_{p\in x} u(p)}{1 + \max_{p\in x} u(p) - \min_{q\in x} u(q)}.$$

More generally, the ex-post choice induced by our model need not be Chew-Dekel. This is the focus of Section 5.1.

Next, we establish the uniqueness properties of the model. To do so, we require some additional terminology. Given any pair of functions $f, g : \Delta(X) \to \mathbb{R}$, we say that f is a *positive (resp. negative) affine transformation of* g if there exist $a, b \in \mathbb{R}$ such that $a > 0$ (resp. $a < 0$) and $f = ag + b$.

Say that (u, v) *represents* $\succeq$ if it satisfies the conditions of Theorem 5.1.

Theorem 5.2. *(u, v) and (u', v') represent $\succeq$ if and only if $u(\cdot) = u'(\cdot)$, and for all $\gamma \in (0, 1)$:*

1. *If $v(\cdot, \gamma)$ is a positive affine transformation of $u(\cdot)$ or a constant, then $v'(\cdot, \gamma)$ is a positive affine transformation of $v(\cdot, \gamma)$ or a constant.*
2. *If $v(\cdot, \gamma) = -a_\gamma u(\cdot) + b_\gamma$ for some $a_\gamma \geq 1$ and $b_\gamma \in \mathbb{R}$, then $v'(\cdot, \gamma) = a'_\gamma v(\cdot, \gamma) + b'_\gamma$ for some $a'_\gamma \geq \frac{1}{a_\gamma}$ and $b'_\gamma \in \mathbb{R}$.*
3. *If $v(\cdot, \gamma)$ is not a constant or a positive affine transformation of $u(\cdot)$ and the condition in 2 does not hold, then $v'(\cdot, \gamma) = v(\cdot, \gamma) + b_\gamma$ for some $b_\gamma \in \mathbb{R}$.*

The uniqueness properties of u are completely characterized by the restriction of $\succeq$ to $\Delta(X)$. Thus, the standard vNM uniqueness result guarantees that there exists a unique normalized representation.

To aid intuition for the uniqueness properties of v, we describe why the conditions in Proposition 5.2 are sufficient. Assume (u, v) represents $\succeq$ and fix $\gamma \in (0, 1)$. If $v(\cdot, \gamma)$ is a constant or a positive affine transformation of $u(\cdot)$, then for all $x \in \mathcal{X}$,

$$\arg\max_{p\in x} v(p, \gamma) = \arg\max_{p\in x} \{u(p) + v(p, \gamma)\},$$

and

$$\max_{p\in x} \{u(p) + v(p, \gamma) - \max_{p\in x} v(p, \gamma)\} = \max_{p\in x} u(p).$$

Thus, replacing $v(\cdot, \gamma)$ with a constant or one of its positive affine transformations does not affect the representation. If $v(\cdot, \gamma) = -a_\gamma u(\cdot) + b_\gamma$ for some $a_\gamma \geq 1$, then for all $x \in \mathcal{X}$,

$$\arg\max_{p\in x} \{v(p, \gamma)\} = \arg\min_{p\in x} \{u(p)\},$$
$$\arg\min_{p\in x} \{u(p)\} \subseteq \arg\max_{p\in x} \{u(p) + v(p, \gamma)\}.$$

To see this, note that for any $a_\gamma \geq 1$ and $p, q \in \Delta(X)$ such that $u(p) \geq u(q)$,

$$u(p) - u(q) \leq a_\gamma(u(p) - u(q)),$$
$$u(p) - a_\gamma u(p) + b_\gamma \leq u(q) - a_\gamma u(q) + b_\gamma,$$
$$u(p) + v(p, \gamma) \leq u(q) + v(q, \gamma).$$

Hence,

$$q \in \arg\min_{p\in x} \{u(p)\} \implies q \in \arg\max_{p\in x} \{u(p) + v(p, \gamma)\},$$
$$\implies \max_{p\in x} \{u(p) + v(p, \gamma) - \max_{q\in x} v(q, \gamma)\} = \min_{p\in x} \{u(p)\},$$

for all $x \in \mathcal{X}$. Thus, if we replace $v(\cdot, \gamma)$ with $v(\cdot, \gamma) = a'_\gamma v(\cdot, \gamma) + b'_\gamma$ for some $a'_\gamma \geq \frac{1}{a_\gamma}$, then $v'(\cdot, \gamma)$ is also a negative affine transformation of $u(\cdot)$ in which the coefficient multiplying $-u(\cdot)$ is greater than or equal to one. Thus, the representation is not affected. Finally, note that if $v(\cdot, \gamma)$ is not a constant or a positive affine transformation of $u(\cdot)$ and the condition in 2 does not hold, then for any $b_\gamma \in \mathbb{R}$,

$$\max_{p\in x} \{u(p) + v(p, \gamma) + b_\gamma - \max_{q\in x} \{v(q, \gamma) + b_\gamma\}\} = \max_{p\in x} \{u(p) + v(p, \gamma) - \max_{q\in x} v(q, \gamma)\}.$$

Hence, replacing $v(\cdot, \gamma)$ with $v(\cdot, \gamma) + b_\gamma$ does not affect the representation.

5.1. Ex-post choice

The model implies the choice correspondence defined by

$$c(x) = \arg\max_{p\in x} \{u(p) + v(p, V(x))\}.$$

Although similar in functional form to the Chew–Dekel model, it allows for behavior that is inconsistent with Mixture-Betweenness over lotteries. For example, it permits:

$$c(\{a,b\}) = \{a,b\} \text{ and } c(\{a, \frac{1}{2}a + \frac{1}{2}b\}) = \{a\}.$$

Such choices imply the agent is indifferent between a and b but strictly prefers a to $\frac{1}{2}a + \frac{1}{2}b$.[21] A Chew-Dekel model cannot possibly accommodate this: Mixture-Betweenness over lotteries requires that $\frac{1}{2}b + \frac{1}{2}a$ be indifferent to a if a is indifferent to b.

We observe that our model offers a novel perspective on the Allais paradox. Specifically, Allais shows that individuals tend to violate vNM Independence when lotteries are realized immediately after being chosen. However, Weber and Chapman (2005) and Baucells and Heukamp (2010) provide experimental evidence that subjects are more likely to satisfy vNM Independence when there is a delay between the time of choice and the realization of outcomes. In our model, this corresponds to choosing among singleton menus, where normative preferences dominate and are linear. Thus, the model provides a unified explanation for both findings: non-linear temptation preferences account for violations of Independence in lotteries that realize immediately after the choice (choices out of a menu), while linear normative preferences govern behavior when choices concern(s) delayed outcomes (choices between singleton menus).[22] This suggests that agents may actively demand commitment in situations in which they are worried about falling prey to biases such as the certainty effect.

5.2. *Mixture-dependent preference for commitment*

Accommodating mixture-dependent preference for commitment necessitates a violation of the linearity of v. To see this, consider the following possible rankings involving binary menus:

$$(*)\ \ \{p\} \succ \{p,q\} \succsim \{q\} \text{ and } \{\alpha p + (1-\alpha)r\} \sim \{\alpha p + (1-\alpha)r, \alpha q + (1-\alpha)r\}$$

$$(**)\ \ \{p\} \sim \{p,q\} \succ \{q\} \text{ and } \{\alpha p + (1-\alpha)r\} \succ \{\alpha p + (1-\alpha)r, \alpha q + (1-\alpha)r\}.$$

In the GP model ($v(\cdot,\gamma) = v(\cdot)$), such rankings impose the following restrictions on v:

$$\{p\ \} \succ \{p\ , q\ \} \succsim \{q\ \} \implies v(q) > v(p)$$

$$\{p'\} \sim \{p', q'\} \succ \{q'\} \implies v(p') \geq v(q').$$

Hence, the rankings in $(*)$ imply

$$v(q) > v(p) \text{ and } v(\alpha p + (1-\alpha)r) \geq v(\alpha q + (1-\alpha)r).$$

Thus, by linearity of v, $v(q) > v(p)$ and $v(q) \leq v(p)$: an impossibility. Similarly, the rankings in $(**)$ imply

$$v(p) \geq v(q) \text{ and } v(\alpha q + (1-\alpha)r) \geq v(\alpha p + (1-\alpha)r).$$

Hence, by linearity of v, $v(q) \geq v(p)$ and $v(q) < v(p)$: another impossibility.

6. Updating

Epstein (2006) combines GP's theory of temptation and self-control with the SEU model to deliver an axiomatic model of non-Bayesian updating. His main contribution is to show that GP's temptation theory goes beyond "tastes" and also applies to beliefs.

He takes as objects of choice news-contingent menus: Let S_1, S_2 be two finite state spaces and $\mathcal{F}$ the set of all AA acts on S_2. The set of all menus of acts is denoted by $\mathcal{M}(\mathcal{F})$. A contingent menu is a function $F : S_1 \to \mathcal{K}(\mathcal{M})$. To interpret, think of an agent who lives 3 periods. In period 1 she will observe news $s_1 \in S_1$ about the state of the market $s_2 \in S_2$ which she will know in period 2. Suppose further that at time 0 she can restrict the investment strategies that will be available to her, conditional on the news she will observe.

Epstein takes a preference $\succsim$ over the set of all contingent menus and axiomatizes the following utility function for $\succsim$

$$V(F) = \int_{S_1} \mathcal{U}(F(s_1), s_1)\, \mu_1(ds_1),$$

where

[21] Taking $u(a) = \frac{1}{2}, u(b) = \frac{1}{4}, v(a,\gamma) = \gamma - \frac{1}{4}$ and $v(b,\gamma) = \gamma$ generates the above choices.

[22] An alternative explanation is proposed by Noor and Takeoka (2010, 2015), who conjecture that self-control costs are the driving force behind these behavioral patterns.

$$\mathcal{U}(F(s_1), s_1) = \max_{f \in F(s_1)} \left\{ \int_{S_2} u(f(\omega))\, d\mu(ds_2 \mid s_1) + \alpha(s_1) \int_{S_2} u(f(\omega))\, d\nu(ds_2 \mid s_1) \right\}$$
$$- a(s_1) \max_{g \in F(s_1)} \left\{ \int_{S_2} u(g(\omega))\, d\nu(ds_2 \mid s_1) \right\},$$

u is a vNM utility function over lotteries, μ and ν are probability measures over $S_1 \times S_2$ such that μ_1 is the marginal of μ and ν over S_1, $\mu(\cdot|s_1)$ is the conditional of μ on S_2 given s_1, $\nu(\cdot|s_1)$ is the conditional of ν on S_2 given s_1 and a is a positive function.

His model admits an identical interpretation to GP's but in beliefs: μ is the normative belief whereas ν is the temptation belief. The intuition being that in the absence of commitment, the agent may overreact (or under-react) to information and be tempted to choose according to ν as opposed to μ.

Because his model satisfies Independence, it rules out the possibility that overreaction depends on the menu. Intuitively, an investor may only overreact to news if she is considering a large investment and thus, only demand commitment for large enough investments.

In order for the model to accommodate such behavior, $\mathcal{U}(\cdot, s_1)$ would need to be non-linear. Given Theorem 3.1 and Kreps (2018) (Proposition 7.4), this can be achieved by replacing Epstein's adaptation of the Independence axiom with Mixture-Betweenness. This will deliver an implicit utility representation for $\succeq$ in which the utility of a contingent menu F is the unique γ that solves

$$\gamma = \int_{S_1} \mathcal{U}(F(s_1), s_1, \gamma)\, ds_1,$$

where

$$\mathcal{U}(F(s_1), s_1, \gamma) = \max_{f \in F(s_1)} \left\{ U(f, s_1, \gamma) + V(f, s_1, \gamma) \right\} - \max_{g \in F(s_1)} \left\{ V(g, s_1, \gamma) \right\},$$

where $U(\cdot, s_1, \gamma)$ and $V(\cdot, s_1, \gamma)$ are mixture-linear. Such model can accommodate menu-dependent reactions but is unsatisfactory since it does not have a sensible interpretation. However, it can be further specialized into a model that generalizes Epstein's by allowing the temptation belief to be menu-dependent. Thus, it would allow the agent to only overreact to news only if she will have the opportunity to perform large investments after observing the signal.

More formally, by taking the "commitment" utility U to be independent of γ and consistent with standard SEU and imposing that the temptation utility V adopts the functional form of our rationalizable beliefs model, we obtain a model that allows the temptation beliefs to be menu-dependent:

$$U(f, s_1, \gamma) = \int_{S_2} u(f(\omega)) d\mu(ds_2|s_1)$$

$$V(g, s_1, \gamma) = a(s_1, \gamma) \int_{S_2} u(g(\omega)) d\nu(ds_2|s_1, \gamma).$$

Observe that ν varies with the menu and thus delivers a theory of menu-dependent non-Bayesian updating. We leave its analysis for future research.

7. Conclusion

In this paper, we propose axiomatic models of preference under uncertainty and commitment, grounded in the Mixture-Betweenness axiom. We conclude with a few remarks on why we view our models as both conceptually and practically attractive.

We discuss the conceptual appeal of each model separately. However, since tractability is a shared feature, we address it here: Any optimization problem involving a Mixture-Betweenness satisfying model (Φ) will have the property that the solution (x^*) will only depend on the constraint set and on the indifference set that "passes" through the optimum.[23] Moreover, the function $\Phi(\cdot, V(x^*))$ satisfies Independence and its analysis is enough to describe the properties of x^*. Hence, any tool developed for analyzing models with Independence can be directly applied to it.

Uncertainty: Models of preference under uncertainty based on Mixture-Betweenness imply indifference to hedging. This property makes them particularly well-suited for delineating the boundary between behavior driven by preference for risk over uncertainty (ambiguity aversion) and behavior unrelated to such preference. Thus, they can be used as a benchmark for defining ambiguity aversion within the AA framework. More specifically, definitions of ambiguity aversion are comparative. For instance, Epstein (2004) defines it relative to probabilistic sophistication, while Ghirardato and Marinacci (2002) does so with respect to expected utility.

[23] This is guaranteed by the fact that the indifference sets are convex.

Moreover, its connection to Decomposability in the Savage framework further implies that the implicit state-dependent model be used as a benchmark there.

In addition to serving as tools for the study of classical ambiguity aversion, the fact that Mixture-Betweenness satisfying models yield predictions that diverge from those of standard SEU theory in contexts where ambiguity-averse models typically align with SEU makes them interesting in their own right. For instance, they suggest that liquidity constraints may play a beneficial role in financial markets. Traders facing such constraints may be less susceptible to fear of missing out, simply because they are unable to over-allocate resources. In contrast, unconstrained traders may overinvest in highly risky assets, exacerbating volatility and contributing to market bubbles.

Commitment: Models of commitment based on Mixture-Betweenness allow temptation to be sensitive to risk. We feel the implications of this feature need to be taken seriously as they can yield predictions, and thus policy recommendations, that differ from those implied by Independence.

To illustrate this, consider the standard precautionary savings setting (Kimball (1990)) in which an agent lives for two periods and cannot borrow against second-period income. First-period income is deterministic and known, while second-period income may be stochastic. By choosing how much to consume and save in the first period, the agent effectively selects a menu of second-period consumption options. Models that satisfy Independence predict that the temptation to overconsume and undersave is unaffected by the presence of second-period income risk. In contrast, our model predicts that such temptation may arise precisely because future income is risky.

We find this prediction both intuitive and policy-relevant. It suggests that government programs aimed at reducing income risk may have an unintended yet beneficial side effect: they reduce the agent's demand for commitment by dampening temptation. Moreover, this offers a new perspective on precautionary savings: In the absence of commitment devices, the precautionary motive to save may be weakened by the temptation to overconsume in the face of future income uncertainty. The fact that our model offers novel perspectives on the Allais paradox (Section 5.1) and the precautionary savings motive suggests that it may also yield new insights in other domains.

Declaration of competing interest

None.

Appendix A. Mixture-betweenness in abstract settings

This section contains the proofs for the results in Section 3. The section is divided into three subsections: the proof of the sufficiency part of Theorem 3.1, the necessity part, and the uniqueness part.

A.1. Sufficiency

Lemma A.1. *Let $\succeq$ be a binary relation over a compact and convex subset of a linear space $\mathcal{M}$ that satisfies Weak Order, Continuity and Mixture-Betweenness. Then:*

1. $x \succ y$ *and* $0 \le \alpha < \beta \le 1$ *implies* $\beta x + (1-\beta)y \succ \alpha x + (1-\alpha)y$.
2. *For all* x, y *and* z *the following sets are closed:* $\{\alpha | \alpha x + (1-\alpha)y \succeq z\}$ *and* $\{\alpha | z \succeq \alpha x + (1-\alpha)y\}$.
3. *If* $x \succ z \succ y$*, then there exists a unique* $\alpha \in (0,1)$ *such that* $z \sim \alpha x + (1-\alpha)y$.
4. *There exist* $\overline{x}, \underline{x} \in \mathcal{M}$ *such that* $\overline{x} \succeq x \succeq \underline{x}$ *for all* $x \in \mathcal{M}$.

The proof of Part 1 is straightforward. The argument for Part 2 is identical to the one in Theorem 1 of Karni and Safra (2015). Part 3 follows immediately from Part 1 and 2. Part 4 is an immediate implication of Weak Order, Continuity and the fact that $\mathcal{M}$ is compact.

Lemma A.1 shows that for each $x \in \mathcal{M}$ there exists a unique $\gamma(x) \in [0,1]$ such that $x \sim \gamma(x)\overline{x} + (1-\gamma(x))\underline{x}$. Define $V(x) \equiv \gamma(x)$. Then, V is mixture-continuous and represents $\succeq$. Now we proceed with the construction of Φ.

For each $\gamma \in (0,1)$, let x_γ denote $\gamma\overline{x} + (1-\gamma)\underline{x}$ and $I(\gamma) = \{x | x \sim x_\gamma\}$. Our goal is to construct a mixture-linear $\Phi(\cdot, \gamma)$ that represents an artificial preference that has indifference curves that are "parallel" to $I(\gamma)$.

Consider the mapping $\lambda : \mathcal{M} \times (0,1) \to [0,1]$ and $\Phi : \mathcal{M} \times (0,1) \to [0,1]$ given by

$$\lambda(x,\gamma) = \begin{cases} \alpha | \alpha x + (1-\alpha)\underline{x} \sim x_\gamma & V(x) > \gamma \\ 1 & V(x) = \gamma \\ \beta | \beta x + (1-\beta)\bar{x} \sim x_\gamma & V(x) < \gamma. \end{cases}, \qquad \Phi(x,\gamma) = \begin{cases} \frac{\gamma}{\lambda(x,\gamma)} & V(x) > \gamma \\ \gamma & V(x) = \gamma \\ 1 - \frac{1-\gamma}{\lambda(x,\gamma)} & V(x) < \gamma. \end{cases}$$

By Lemma A.1, λ is well defined and thus, so is Φ.

Observe that $\Phi(x,\gamma) = \gamma$ if and only if $V(x) = \gamma$. Further,

$$V(x), V(y) \ge \gamma \implies \Phi(x,\gamma) \ge \Phi(y,\gamma) \iff \lambda(x,\gamma) \le \lambda(y,\gamma)$$

$$V(x) \geq \gamma \geq V(y) \implies \Phi(x,\gamma) \geq \Phi(y,\gamma)$$

$$V(x), V(y) \leq \gamma \implies \Phi(x,\gamma) \geq \Phi(y,\gamma) \iff \lambda(x,\gamma) \geq \lambda(y,\gamma).$$

We will show that $\Phi(\cdot,\gamma)$ is mixture linear. Continuity in its second argument follows from the following lemma that describes several properties of λ.

Lemma A.2. *If $\succeq$ satisfies the axioms of Theorem 3.1, then λ satisfies the following properties:*

1. $V(x), V(y) \geq \gamma \implies \lambda(\alpha x + (1-\alpha)y, \gamma) = \frac{\lambda(x,\gamma)\lambda(y,\gamma)}{\alpha\lambda(y,\gamma)+(1-\alpha)\lambda(x,\gamma)}$ *for all* $\alpha \in [0,1]$.
2. $V(x), V(y) \leq \gamma \implies \lambda(\alpha x + (1-\alpha)y, \gamma) = \frac{\lambda(x,\gamma)\lambda(y,\gamma)}{\alpha\lambda(y,\gamma)+(1-\alpha)\lambda(x,\gamma)}$ *for all* $\alpha \in [0,1]$.
3. $V(x), V(y) > \gamma > V(z)$ *and* $\lambda(x,\gamma) < \lambda(y,\gamma) \implies \lambda(\alpha x + (1-\alpha)z, \gamma) = \lambda(y,\gamma)$ *for a unique* $\alpha \in (0,1)$.
4. $V(x), V(y) < \gamma < V(z)$ *and* $\lambda(x,\gamma) < \lambda(y,\gamma) \implies \lambda(\alpha x + (1-\alpha)z, \gamma) = \lambda(y,\gamma)$ *for a unique* $\alpha \in (0,1)$.
5. $V(x), V(y) > \gamma > V(z)$, $\lambda(x,\gamma) = \lambda(y,\gamma)$ *and* $\alpha x + (1-\alpha)z \sim x_\gamma \implies \alpha y + (1-\alpha)z \sim x_\gamma$.
6. $V(x), V(y) < \gamma < V(z)$, $\lambda(x,\gamma) = \lambda(y,\gamma)$ *and* $\alpha x + (1-\alpha)z \sim x_\gamma \implies \alpha y + (1-\alpha)z \sim x_\gamma$.
7. $\lambda(x,\cdot)$ *is continuous in its second argument for all* $x \in \mathcal{M}\backslash\{\overline{x}, \underline{x}\}$.

Proof. The proof of Part 3-7 is straightforward and therefore, omitted. The proof of Part 2 is analogous to the proof of Part 1 and also omitted. Thus, we only provide the proof for Part 1.

Fix x, y such that $V(x), V(y) \geq \gamma$ and $\alpha \in (0,1)$. Then, by Mixture-Betweenness, $V(\alpha x + (1-\alpha)y)) \geq \gamma$. Further, $\lambda(x,\gamma)x + (1-\lambda(x,\gamma))\underline{x} \sim x_\gamma \sim \lambda(y,\gamma)y + (1-\lambda(y,\gamma))\underline{x}$. Observe that

$$\begin{aligned}&\frac{\lambda(x,\gamma)\lambda(y,\gamma)}{a\lambda(y,\gamma)+(1-a)\lambda(x,\gamma)}(\alpha x + (1-\alpha)y) + (1 - \frac{\lambda(x,\gamma)\lambda(y,\gamma)}{a\lambda(y,\gamma)+(1-a)\lambda(x,\gamma)})\underline{x} \\ &= c(\lambda(x,\gamma)x + (1-\lambda(x,\gamma))\underline{x}) + (1-c)(\lambda(y,\gamma)y + (1-\lambda(y,\gamma))\underline{x})\end{aligned}$$

where $c = \frac{\alpha\lambda(y,\gamma)}{\alpha\lambda(y,\gamma)+(1-\alpha)\lambda(x,\gamma)}$. Hence, $\lambda(\alpha x + (1-\alpha)y, \gamma) = \frac{\lambda(x,\gamma)\lambda(y,\gamma)}{\alpha\lambda(y,\gamma)+(1-\alpha)\lambda(x,\gamma)}$. ☐

A.1.1. $\Phi(\cdot,\gamma)$ is mixture linear

Fix $x, y \in \mathcal{M}, \gamma \in (0,1)$ and $\alpha \in (0,1)$. Assume WLOG that $V(x) \geq V(y)$. Then, by Mixture-Betweenness, $V(x) \geq V(\alpha x + (1-\alpha)y) \geq V(y)$.

There are four possible cases to consider:
(i) $V(x) \geq V(y) \geq \gamma$. (ii) $\gamma \geq V(x) \geq V(y)$.
(iii) $V(x) \geq V(\alpha x + (1-\alpha)y) \geq \gamma \geq V(y)$. (iv) $V(x) \geq \gamma \geq V(\alpha x + (1-\alpha)y) \geq V(y)$. The proofs of (i) and (ii) are analogous, and the proofs of (iii) and (iv) are analogous. Therefore, we only consider (i) (Lemma A.3) and (iii) (Lemma A.4).

Lemma A.3. $V(x), V(y) \geq \gamma \implies \Phi(\alpha x + (1-\alpha)y, \gamma) = a\Phi(x,\gamma) + (1-a)\Phi(y,\gamma)$.

Proof. First note that if $V(x) = V(y) = \gamma$, then there is nothing to prove. Assume $V(x) > V(y) \geq \gamma$. Then, by Mixture-Betweenness, $V(\alpha x + (1-\alpha)y) > \gamma$. Thus,

$$\begin{aligned}\Phi(\alpha x + (1-\alpha)y, \gamma) &= \frac{\gamma}{\lambda(\alpha x + (1-\alpha)y, \gamma)} = \frac{\gamma}{\frac{\lambda(x,\gamma)\lambda(y,\gamma)}{\alpha\lambda(y,\gamma)+(1-\alpha)\lambda(x,\gamma)}} \\ &= \alpha\Phi(x,\gamma) + (1-\alpha)\Phi(y,\gamma),\end{aligned}$$

where the second equality follows from Part 1 of Lemma A.2. ☐

Lemma A.4. $V(x) > \gamma > V(y)$ *and* $V(\alpha x + (1-\alpha)y) \geq \gamma \implies \Phi(\alpha x + (1-\alpha)y, \gamma) = \alpha\Phi(x,\gamma) + (1-\alpha)\Phi(y,\gamma)$.

Proof. The proof of this lemma is done in two steps.

Step 1: Calculate $\Phi(\alpha x + (1-\alpha)y, \gamma)$

Let $\beta \in (0,1)$ be such that $\beta x + (1-\beta)y \sim x_\gamma$. Then $\alpha x + (1-\alpha)y \succeq x_\gamma$ implies $\alpha \geq \beta$. Let $c = \frac{\alpha-\beta}{1-\beta} \leq 1$. Then $\alpha x + (1-\alpha)y = cx + (1-c)(\beta x + (1-\beta)y)$. Since $V(x), V(\beta x + (1-\beta)y) \geq \gamma$, by Part 1 of Lemma A.2,

$$\begin{aligned}\lambda(cx + (1-c)(\beta x + (1-\beta)y)), \gamma) &= \frac{\lambda(x,\gamma)}{c + (1-c)\lambda(x,\gamma)} \\ &= (1-\beta)\frac{\lambda(x,\gamma)}{(\alpha-\beta) + (1-\alpha)\lambda(x,\gamma)}.\end{aligned}$$

Therefore,

$$\Phi(\alpha x+(1-\alpha)y,\gamma)=\Phi(cx+(1-c)(\beta x+(1-\beta)y),\gamma)$$
$$=\gamma\frac{1}{\lambda(cx+(1-c)(\beta x+(1-\beta)y),\gamma)}$$
$$=\gamma\frac{\alpha-\beta+(1-\alpha)\lambda(x,\gamma)}{(1-\beta)\lambda(x,\gamma)}.$$

Step 2: Calculate $\alpha\Phi(x,\gamma)+(1-\alpha)\Phi(y,\gamma)$ and show it is equal to $\Phi(\alpha x+(1-\alpha)y,\gamma)$

Observe that

$$\alpha\Phi(x,\gamma)+(1-\alpha)\Phi(y,\gamma)=\alpha\frac{\gamma}{\lambda(x,\gamma)}+(1-\alpha)(1-\frac{1-\gamma}{\lambda(y,\gamma)}). \tag{7}$$

We will show that

$$\lambda(y,\gamma)=\frac{(1-\beta)\lambda(x,\gamma)(\gamma-1)}{\lambda(x,\gamma)(\gamma-1)-\beta(\gamma-\lambda(x,\gamma))}. \tag{8}$$

Substituting (8) into (7) yields the desired result.

To prove (8), we need to distinguish between two cases: (i) $\lambda(x,\gamma)>\lambda(\overline{x},\gamma)$ and (ii) $\lambda(x,\gamma)\leq\lambda(\overline{x},\gamma)$. The proof of both cases is analogous. Thus, we only consider (i).

Assume $\lambda(x,\gamma)>\lambda(\overline{x},\gamma)$. By Part 3 of Lemma A.2 there exists a unique $d\in(0,1)$ such that $\lambda(dy+(1-d)\overline{x},\gamma)=\lambda(x,\gamma)$. Moreover, by Part 5 of Lemma A.2, $\beta(dy+(1-d)\overline{x}))+(1-\beta)y)\sim x_\gamma$. Since

$$\beta(dy+(1-d)\overline{x}))+(1-\beta)y)=(1-\beta(1-d))y+\beta(1-d)\overline{x},$$

we have that $\lambda(y,\gamma)=1-\beta(1-d)$.

Finally, let $e=\frac{\gamma-\lambda(x,\gamma)}{\lambda(x,\gamma)(\gamma-1)}$ and observe that

$$dy+(1-d)\overline{x}=e(\lambda(y,\gamma)y+(1-\lambda(y,\gamma))\overline{x})+(1-e)\overline{x}$$
$$=e\lambda(y,\gamma)y+(e(1-\lambda(y,\gamma))+(1-\lambda(y,e))\overline{x}.$$

Hence by the uniqueness in Part 3 of Lemma A.2, $d=e\lambda(y,\gamma)$. Thus,

$$\lambda(y,\gamma)=1-b(1-e\lambda(y,\gamma))=\frac{1-b}{1-be}=\frac{(1-b)\lambda(x,\gamma)(\gamma-1)}{\lambda(x,\gamma)(\gamma-1)-b(\gamma-\lambda(x,\gamma))}. \quad\square$$

A.1.2. $\Phi(\cdot,1)$ and $\Phi(\cdot,0)$

Finally, we need to construct $\Phi(\cdot,1),\Phi(\cdot,0)$. Because the construction of $\Phi(\cdot,1)$ and $\Phi(\cdot,0)$ are analogous. We only show the construction of $\Phi(\cdot,1)$.[24]

Let $\mathcal{M}_{\overline{x},\underline{x}}=\{z\in\mathcal{M}|\exists x\sim\overline{x},\alpha\in[0,1]:z=\alpha x+(1-\alpha)\underline{x}\}$. Then, $\mathcal{M}_{\overline{x},\underline{x}}$ is convex. Define $\Phi(x,1)=1$ for all x such that $x\sim\overline{x}$, $\Phi(\underline{x},1)=0$, and for any $z=\alpha x+(1-\alpha)\underline{x}$ and $x\sim\overline{x}$ define $\Phi(z,1)=\alpha$. Since $\mathcal{M}$ is a convex subset of a linear space, this function is well defined.

Suppose that $\Phi(\cdot,1)$ has been extended from $\mathcal{M}_{\overline{x},\underline{x}}$ to a convex set $\mathcal{N}$ such that $\mathcal{N}\subseteq\mathcal{M}$ in such a way that $\Phi(\cdot,1)$ is mixture-linear and $\Phi(z,1)<1$ for all $z\in\mathcal{N}\backslash\mathcal{M}_{\overline{x},\underline{x}}$. Suppose there exists $x\in\mathcal{M}\backslash\mathcal{N}$. Define

$$\mathcal{N}_x=\{z\in\mathcal{M}|\exists\alpha\in[0,1],y\in\mathcal{N}:z=\alpha x+(1-\alpha)y\}.$$

Then, $\mathcal{N}_x$ is convex. Define $\Phi(x,1)=0$, and for any $z=\alpha x+(1-\alpha)y$ define $\Phi(z,1)=(1-\alpha)\Phi(y,1)$. To see that $\Phi(\cdot,1)$ is well defined note that if $z=\alpha x+(1-\alpha)y=\beta x+(1-\beta)y'$ for some $y,y'\in\mathcal{N}$, then either $y=y'$ and $\alpha=\beta$, or $y\neq y'$ and $\alpha\neq\beta$. Assume $y\neq y'$ and $\beta>\alpha$. Then,

$$y=(\frac{\beta-\alpha}{1-\alpha})x+(1-\frac{\beta-\alpha}{1-\alpha})y'.$$

Hence, $\Phi(z,1)$ is the same whichever of the two decompositions is chosen. Further, $\Phi(\cdot,1)$ is mixture linear on $\mathcal{N}_x$. Hence, $\Phi(\cdot,1)$ can be extended to $\mathcal{N}_x$ in such a way that it is mixture-linear and $\Phi(z,1)<1$ for all $z\in\mathcal{N}_x$.

Consider all the convex subsets S of $\mathcal{M}$ such that $\mathcal{M}_{\overline{x},\underline{x}}\subset S$ and $\Phi(\cdot,1)$ has been extended to S in such a way that $\Phi(z,1)<1$ for all $z\notin\mathcal{M}_{\overline{x},\underline{x}}$. Partially order the set P of pairs $(S,\Phi_S(.,1))$ in the following way:

$$(S,\Phi_S(.,1))\leq(S',\Phi_{S'}(.,1)) \text{ iff } S\subseteq S' \text{ and } \Phi_{S'}(.,1) \text{ extends } \Phi_S(.,1).$$

Take any totally ordered chain of P, $(S_i,\Phi_{S_i}(.,1))_{i\in I}$. It has an upperbound $(\cup_{i\in I},S_i,\Phi^*(\cdot,1))$ with $\Phi^*(\cdot,1)$ defined by: $\Phi^*(x,1)=\Phi_{S_i}(x,1)$ if $x\in S_i$. Thus, by Zorn's Lemma (Aliprantis and Border (2006) Theorem 1.7) P has a maximal element for $\geq$. Denote it by $(S^*,\Phi^*(\cdot,1))$.

[24] Our construction is based on the construction of the u function in the proof of Proposition 1 in Mongin (2001).

Assume towards a contradiction that $\mathcal{M} \neq S^*$. Then we can take $x \in \mathcal{M}\backslash S^*$, construct $\mathcal{N}_x$ as in the second part of the construction to get $(T, \Phi_T(\cdot, 1))$ such that $S^* \subseteq T$ and $\Phi_T(\cdot, 1)$ extends $\Phi^*(\cdot, 1)$. Hence, $(S^*, \Phi^*(\cdot, 1)) \leq (T, \Phi_T(\cdot, 1))$ and $T \neq S^*$, a contradiction. Hence, $S^* = \mathcal{M}$ and $\Phi^*(\cdot, 1)$ extends $\Phi(\cdot, 1)$. Thus, $\Phi^*(x, 1) = 1$ if and only if $x \sim \overline{x}$.

A.2. Necessity

The proof of necessity of Weak Order and Continuity is routine. Mixture-Betweenness follows from the following lemma and the unique solution property.

Lemma A.5. *Assume Φ represents $\succeq$. Then for any x such that $V(x) \in (0,1)$, $\Phi(x,\gamma) \leq \gamma \iff V(x) \leq \gamma$*

Proof. Assume towards a contradiction that there exists γ and x such that $V(x) \geq \gamma$ and $\Phi(x,\gamma) < \gamma$. If $V(x) = \gamma$, then by the unique solution property $\Phi(x,\gamma) = \gamma$, a contradiction. Thus, $V(x) > \gamma$. Since $\Phi(x,\gamma) < \gamma < 1$, there exists $a \in (0,1)$ such that $a\Phi(x,\gamma) + (1-a) = \gamma$. Hence, $V(\alpha x + (1-\alpha)\overline{x}) = \gamma < V(x) < 1$. Since V is mixture-continuous, there exists $b \in (0,1)$ such that $V(bx + (1-b)\overline{x}) = V(x)$. Hence, $V(x) = b\Phi(x, V(x)) + (1-b) = bV(x) + (1-b)$, a contradiction.

An analogous argument implies that $\Phi(x,\gamma) > \gamma$ and $V(x,\gamma) \geq \gamma$ leads to a contradiction. The only difference is that one needs to use $\underline{x}$ instead of $\overline{x}$ to derive the contradiction. □

A.3. Uniqueness

We prove a stronger result:

Theorem A.1. *Φ and Φ' represent $\succeq$ if and only if there exist continuous functions $a, b : (0,1) \to \mathbb{R}$ such that $a(\gamma) > 0$ for all $\gamma \in (0,1)$, $\phi(\gamma) = a(\gamma)\gamma + b(\gamma)$ is continuous and strictly increasing, and $\Phi'(\cdot, \phi(\gamma)) = a(\gamma)\Phi(\cdot,\gamma) + b(\gamma)$ for all $\gamma \in (0,1)$.*

Necessity: Assume Φ, Φ' represent $\succeq$ and let $\overline{x}, \underline{x}, \overline{x}', \underline{x}'$ be such that $\Phi(\overline{x},\gamma) = 1, \Phi(\underline{x},\gamma) = 0$, $\Phi'(\overline{x}',\gamma) = 1, \Phi'(\underline{x}',\gamma) = 0$ for all γ. Then $V(x)$ and $V'(x)$ are the unique scalars $\alpha, \beta \in [0,1]$ such that $x \sim \alpha\overline{x} + (1-\alpha)\underline{x}$ and $x \sim \beta\overline{x}' + (1-\beta)\underline{x}'$ respectively.

Let $\phi(\gamma)$ be the unique $\alpha \in [0,1]$ such that $\alpha\overline{x}' + (1-\alpha)\underline{x}' \sim \gamma\overline{x} + (1-\gamma)\underline{x}$. Then, ϕ is continuous and strictly increasing. Observe that $\Phi(\cdot,\gamma)$ and $\Phi'(\cdot,\phi(\gamma))$ share an indifference curve: $\{x | \Phi'(x,\phi(\gamma)) = \phi(\gamma)\} = \{x | \Phi(x,\gamma) = \gamma\}$ and so, $\Phi(\cdot,\gamma)$ and $\Phi'(\cdot,\phi(\gamma))$ are positive affine transformation of each other: there exists $a, b : (0,1) \to \mathbb{R}$ such that $a(\gamma) > 0$ and $\Phi'(\cdot,\phi(\gamma)) = a(\gamma)\Phi(\cdot,\gamma) + b(\gamma)$. Hence, $\phi(\gamma) = \Phi'(\gamma\overline{x} + (1-\gamma)\underline{x}, \phi(\gamma)) = a(\gamma)\Phi(\gamma\overline{x} + (1-\gamma)\underline{x}, \gamma) + b(\gamma) = a(\gamma)\gamma + b(\gamma)$.[25]

Sufficiency: Assume $V(x) \geq V(y)$. Then $\phi(V(x)) \geq \phi(V(y))$ and so $V'(x) = \Phi'(x, \phi(V(x))) \geq \Phi'(y, \phi(V(y))) = y$.

Appendix B. Uncertainty

This section contains the proofs for the results in Section 4. We only show sufficiency of the axioms.

B.1. Proof of Corollary 4.1

Continuity and Mixture-Betweenness ensures the existence of a best ($\overline{\delta}$) and worst ($\underline{\delta}$) degenerate constant acts. Applying Theorem 3.1, Theorem A.1 and Proposition 7.4 of Kreps (2018) concludes the proof.

B.2. Proof of Theorem 4.1

Let U and V be the functions given by Corollary 4.1. Observe that, by Lemma A.5,

$$\sum_{\omega} U(f,\gamma,\omega) \geq \gamma \iff V(f) \geq \gamma \tag{9}$$

First we show that if ω is null, then $U(\cdot,\gamma,\omega)$ is constant for all $\gamma \in (0,1)$. By definition of U, $V(\gamma\overline{\delta} + (1-\gamma)\underline{\delta}) = \gamma$. Let $g_{\omega,p}(\omega) = p, g_{\omega,q}(\omega) = q$ and $g_{\omega,p}(\omega') = g_{\omega,q}(\omega') = \gamma\overline{\delta} + (1-\gamma)\underline{\delta}$ for all $\omega' \neq \omega$. Then, since ω is null, $V(g_{\omega,p}) = V(g_{\omega,q}) = \gamma$ for all p, q. Hence, $U(p,\gamma,\omega) = U(q,\gamma,\omega)$ for all p, q.

Next, fix non null states $\omega, \omega^* \in \Omega$, $p, q \in \Delta(X)\backslash\{\overline{\delta}, \underline{\delta}\}$ and $\gamma \in (0,1)$ and assume that $U(p,\gamma,\omega) \geq U(q,\gamma,\omega)$. We will show that it must be the case that $U(p,\gamma,\omega^*) \geq U(q,\gamma,\omega^*)$. Consider the act f_{ω^*} such that $f_{\omega^*}(\omega^*) = q$ and $f_{\omega^*}(\omega') = p$ for all $\omega' \neq \omega$. There are two cases: (1) $V(p) \leq \gamma$ and (2) $V(p) > \gamma$.

Case (1): Since $V(\beta p + (1-\beta)\overline{\delta})$ is continuous in β, there exists α such that $V(\alpha p + (1-\alpha)\overline{\delta}) = \gamma$. Assume towards a contradiction that $U(p,\gamma,\omega^*) < U(q,\gamma,\omega^*)$. Then, $\sum_{\omega} U(f_{\omega_*},\gamma,\omega) > \gamma$ so by (9), $\alpha f_{\omega^*} + (1-\alpha)\overline{\delta} \succ \alpha p + (1-\alpha)\overline{\delta}$. Thus, by Weak State-Independence,

[25] Under the assumptions of Theorem 3.2, $a(\gamma) = 1$ and $b(\gamma) = 0$ for all $\gamma \in (0,1)$ since Φ and Φ' share the normalization.

$\alpha f_\omega + (1-\alpha)\overline{\delta} \succ \alpha p + (1-\alpha)\overline{\delta}$. Hence, by (9), $\sum_\omega U(f_\omega, \gamma, \omega) > \gamma = \sum_\omega U(\alpha p + (1-\alpha)\overline{\delta}, \gamma, \omega) > \gamma$. Since U is mixture linear in its first argument, this implies $U(q,\gamma,\omega) > U(p,\gamma,\omega)$, a contradiction.

To prove that case (2) also yields a contradiction, one first needs to consider $\beta p + (1-\beta)\underline{\delta}$ to show there is an α such that $V(\alpha p + (1-\alpha)\underline{\delta}) = \gamma$. The rest is analogous.

By the standard uniqueness result for vNM representation and continuity of U there exist $a, b : \Omega \times [0,1] \to \mathbb{R}$ such that a, b are continuous on their second argument on $(0,1)$, $a(\omega,\gamma) > 0$ for all non-null ω, $a(\omega,\gamma) = 0$ for all null ω, and $\sum_\omega U(f,\gamma,\omega) = \sum_{\omega\in\Omega} u(f(\omega),\gamma)a(\omega,\gamma) + b(\omega,\gamma)$ where $u(\cdot) = U(\cdot,\gamma,\omega)$ for some $\omega \in \Omega$. Therefore, $V(\gamma\overline{\delta} + (1-\gamma)\underline{\delta}) = \gamma$ implies that $V(f)$ is the unique γ that solves

$$\begin{aligned}&\gamma u(\overline{\delta},\gamma)\sum_{\omega\in\Omega} a(\omega,\gamma) + (1-\gamma)u(\underline{\delta},\gamma)\sum_{\omega\in\Omega} a(\omega,\gamma) + \sum_{\omega\in\Omega} b(\omega,\gamma)\\ &= \sum_{\omega\in\Omega} u(f(\omega),\gamma)a(\omega,\gamma) + \sum_{\omega\in\Omega} b(\omega,\gamma).\end{aligned}$$

Hence, $V(f)$ is the unique γ that solves $\gamma u(\overline{\delta},\gamma) + (1-\gamma)u(\underline{\delta},\gamma) = \sum_{\omega\in\Omega} u(f(\omega),\gamma)\mu(\omega,\gamma)$ where $\mu(\omega,\gamma) = \frac{a(\omega,\gamma)}{\sum_{\omega\in\Omega} a(\omega,\gamma)}$.

To conclude, Independence over Lotteries implies that $u(\cdot,\gamma)$ induces the same preferences over lotteries as $u(\cdot,\gamma')$ for all γ', so there exist u such that $u(\cdot) = u(\cdot,\gamma')$ for all $\gamma \in [0,1]$, $u(\overline{\delta}) = 1$ and $u(\underline{\delta}) = 0$.

Appendix C. Preference for commitment

This section contains the proofs for the results in Section 5. The section is divided into three subsections: the proof of the sufficiency part of Theorem 5.1, the necessity part, and the uniqueness part.

C.1. *Sufficiency*

$\Delta(X)$ compact implies $\mathcal{X}$ is compact (Aliprantis and Border (2006), Theorem 3.71(3)). Let $\mathcal{K}(\Delta(X))$ denote the set of all closed and convex menus of lotteries. By the Blaschke Selection Theorem (Schneider (2014) Theorem 1.8.5), $\mathcal{K}(\Delta(X))$ is compact.

By Lemmas S.2 and S.3 in the supplemental material Dekel et al. (2007), there exists a mixture preserving bijection from $\mathcal{K}(\Delta(X))$ to a convex subset of a linear space. Therefore, $\mathcal{K}(\Delta(X))$ can be treated as a compact and convex space.

Let $\overline{p}, \underline{p} \in \Delta(X)$ denote a fixed pair of lotteries such that $\{\overline{p}\} \succeq x \succeq \{\underline{p}\}$ for all $x \in \mathcal{X}$. Given our axioms, such lotteries always exist. For each $\gamma \in (0,1)$, let $\{p_\gamma\}$ denote $\gamma\{\overline{p}\} + (1-\gamma)\{\underline{p}\}$.

By Theorem 3.1 there exists Φ is mixture linear for all $\gamma \in (0,1)$ and, $\Phi(\{\overline{p}\},\gamma) = 1$ and $\Phi(\{\underline{p}\},\gamma) = 0$ for all $\gamma \in [0,1]$. Further, for all x such that $\{\overline{p}\} \succ x \succ \{\underline{p}\}$, $V(x) = \gamma$ is the unique solution of $\gamma = \Phi(x,\gamma)$.

In what follows we will use $\Phi(\cdot,\gamma)$ to construct the representation in Theorem 5.1 for the case in which $\gamma \in (0,1)$. Afterwards we construct the representation for the case in which $\gamma = 1$ and $\gamma = 0$.

Step 1: Extend V and $\Phi(\cdot,\gamma)$ to $\mathcal{X}$.

For each menu $x \in \mathcal{X}$, let $ch(x)$ denote its convex hull. Extend V by letting $V(x) = V(ch(x))$ for all $x \in \mathcal{X}\backslash\mathcal{K}(\Delta(X))$. We claim that V represents $\succeq$. To prove this, it is enough to show that our axioms imply $x \sim ch(x)$ for all $x \in \mathcal{X}$.

Lemma C.1. *Let $\succeq$ be a binary relation over $\mathcal{X}$ that satisfies Weak Order, Continuity and Mixture-Betweenness. Then $x \sim ch(x)$ for all $x \in \mathcal{X}$.*

Proof. Let $K = |X|$ and assume, by way of contradiction, that there exists x such that $x \nsim ch(x)$. Then, by Mixture-Betweenness, $\alpha x + (1-\alpha)ch(x) \nsim ch(x)$ for all $\alpha \in (0,1)$. This is a contradiction. In particular, Lemma S.6 in the supplemental material of Dekel et al. (2007) shows that $\alpha x + (1-\alpha)ch(x) = ch(x)$ for all $\alpha \in [0, \frac{1}{K}]$. $\square$

Extend Φ by letting $\Phi(x,\gamma) = \Phi(ch(x),\gamma)$ for all $x \in \mathcal{X}\backslash\mathcal{K}(\Delta(X))$. Since $ch(x+y) = ch(x) + ch(y)$ for all x, y, the extension of Φ is also mixture linear in its first argument and continuous in the second.

Finally, Continuity on the first argument follows from the fact that for all $x, y \in \mathcal{X}$, $d_h(ch(x), ch(y)) \leq d_h(x,y)$.

Step 2: Show that $\Phi(\cdot,\gamma)$ satisfies Set-Betweenness.

Fix $x, y \in \mathcal{X}$ and $\gamma \in (0,1)$. Assume WLOG that $\Phi(x,\gamma) \geq \Phi(y,\gamma)$. Then either there exists $\alpha \in (0,1]$ such that $\alpha\Phi(x,\gamma) + (1-\alpha)\Phi(\underline{x},\gamma) = \gamma \geq \alpha\Phi(y,\gamma) + (1-\alpha)\Phi(\underline{x},\gamma)$ or $\alpha\Phi(x,\gamma) + (1-\alpha)\Phi(\overline{x},\gamma) \geq \gamma = \alpha\Phi(y,\gamma) + (1-\alpha)\Phi(\overline{x},\gamma)$. The proof of both cases is analogous so we only consider the former.

Assume there exists $\alpha \in (0,1]$ such that $\alpha\Phi(x,\gamma) = \gamma \geq \alpha\Phi(y,\gamma)$. Thus, by Lemma A.5, $\alpha x + (1-\alpha)\underline{p} \succeq \alpha y + (1-\alpha)\underline{p}$ so by Set-Betweenness,

$$\begin{aligned}\alpha x + (1-\alpha)\underline{p} &\succeq \alpha x + (1-\alpha)\underline{p} \cup \alpha y + (1-\alpha)\underline{p}\\ &= \alpha x \cup y + (1-\alpha)\underline{p} \succeq \alpha y + (1-\alpha)\underline{p}.\end{aligned}$$

Finally, by Lemma A.5, $\Phi(\alpha x + (1-\alpha)\underline{p},\gamma) \geq \Phi(\alpha x \cup y + (1-\alpha)\underline{p}) \geq \Phi(\alpha y + (1-\alpha)\underline{p},\gamma)$ which implies

$\Phi(x,\gamma) \geq \Phi(x \cup y) \geq \Phi(y,\gamma)$.

Step 3: Show that there exists $u(\cdot)$ and $v(\cdot,\gamma)$ such that for all $\gamma \in (0,1)$ and $x \in \mathcal{X}$,

$$\Phi(x,\gamma) = \max_{p\in x}\{u(p) + v(p,\gamma) - \max_{q\in x} v(q,\gamma)\}.$$

Restrict $\succeq$ to $\Delta(X)$, then by Commitment Independence, there exists a unique vNM utility $u : \Delta(X) \to \mathbb{R}$ such that $u(\overline{p}) = 1$, $u(\underline{p}) = 0$ and $V(\{p\}) = u(p)$ is the unique $\gamma \in [0,1]$ that solves $\gamma = u(p)$. Hence, $\Phi(\{p\},\gamma) = u(p)$ for all $p \in \Delta(X)$ and $\gamma \in (0,1)$.

By Lemmas 2, 4 and 5 in GP, there exists a vNM function $v(\cdot,\gamma)$ such that

$$\Phi(x,\gamma) = \max_{p\in x}\{u(p) + v(p,\gamma) - \max_{q\in x} v(q,\gamma)\}$$

for all $x \in \mathcal{X}$.

Step 4: Construct $v(\cdot,1)$ and $v(\cdot,0)$.

The construction of $v(\cdot,0)$ and $v(\cdot,1)$ are analogous. Thus, we only show the latter and specify how to adapt the argument for the former. Since $\succeq$ satisfies Set-Betweenness, we will restrict our attention to binary menus.[26]

Let $\mathcal{P} = \{p|\{p\} \succeq \{q\}$ for all $q\}$. There are two possible cases: (i) $\{p\} \sim \{p,q\}$ for all $p \in \mathcal{P}, q \in \Delta(X)$ and (ii) there exist $p \in \mathcal{P}$ and $q \in \Delta(X)$ such that $\{p\} \succ \{p,q\}$.

(i) $\{p\} \sim \{p,q\}$ for all $p \in \mathcal{P}, q \in \Delta(X)$

Let $v(\cdot,1) = 0$. Then, $\max_{p'\in\{p,q\}} u(p,1) = 1$ if and only if $V(\{p,q\}) = 1$.

(ii) There exist $p \in \mathcal{P}$ and $q \in \Delta(X)$ such that $\{p\} \succ \{p,q\}$

Define $\succeq^T$ over $\Delta(X)$ as follows

$p \succeq^T q$ if and only if $\{p\} \sim \{p,q\} \succ \{q\}$ and $p \in \mathcal{P}$

$q \succ^T p$ if and only if $\{p\} \succ \{p,q\}$ and $p \in \mathcal{P}$.

Let $\overline{0}$ denote the zero vector and $\mathcal{T} = cl(\{\sum_{i=1}^n \lambda_i(p_i - q_i)|n \in \mathbb{N}, \lambda_i > 0$ and $p_i \succ q_i$ or $p_i \succeq^T q_i$ for $i = 1,...,n\})$. By Lemma 2 in Fishburn (1975), there exists $v(\cdot,1)$ such that $p \succ^T q$ implies $v(p,1) > v(q,1)$ and $p \succeq^T q$ implies $v(p,1) \geq v(q,1)$ if $\overline{0} \notin \mathcal{T}$. We prove this in three steps.

Step 4.1: Show that if $\{p_t\} \sim \{p_t,q_t\}$ for $t = 1,...,n$ and $p_t \in \mathcal{P}$ for all t, then $\{\sum_t^n \lambda_t p_t\} \sim \{\sum_t^n \lambda_t p_t, \sum_t^n \lambda_t q_t\}$ for all $\lambda \in \Delta(\{1,...,n\})$.

The proof is by induction. Fix $p_1,p_2 \in \mathcal{P}$ such that $\{p_1\} \sim \{p_1,q_1\}$ and $\{p_2\} \sim \{p_2,q_2\}$. Observe that

$$\begin{aligned}\lambda\{p_1,q_1\} + (1-\lambda)\{p_2,q_2\} = &\{\lambda p_1 + (1-\lambda)p_2, \lambda q_1 + (1-\lambda)q_2\}\\ &\cup \lambda\{q_1\} + (1-\lambda)\{p_2,q_2\}\\ &\cup (1-\lambda)\{q_2\} + \lambda\{p_1,q_1\}.\end{aligned}$$

Thus, by Set-Betweenness, $\{\lambda p_1 + (1-\lambda)p_2\} \succ \{\lambda p_1 + (1-\lambda)p_2, \lambda q_1 + (1-\lambda)q_2\}$ implies $V(\lambda\{p_1,q_1\} + (1-\lambda)\{p_2,q_2\}) < 1$, a contradiction.

Induction step: Suppose the result is true for n. We will now show it holds for $n+1$.

Fix $p_t \in \mathcal{P}$ such that $p_t \sim \{p_t,q_t\}$ for all $t = 1,...,n+1$. By the induction hypothesis $\{\sum_{t=1}^n \frac{\lambda_t}{\sum_t^n \lambda_t} p_t\} \sim \{\sum_{t=1}^n \frac{\lambda_t}{\sum_t^n \lambda_t} p_t, \sum_{t=1}^n \frac{\lambda_t}{\sum_t^n \lambda_t} q_t\}$. Hence, by the base case, $\{\sum_{t=1}^{n+1} \lambda_t p_t\} \sim \{\sum_{t=1}^{n+1} \lambda_t p_t, \sum_t \lambda_t q_t\}$.

Step 4.2: Show that if $\{p_t\} \succ \{p_t,q_t\}$ $t = 1,...,n$ and $p_t \in \mathcal{P}$ for all t, then $\{\sum_{t=1}^n \lambda_t p_t\} \succ \{\sum_{t=1}^n \lambda_t p_t, \sum_t \lambda_t q_t\}$ for all $\lambda \in \Delta(\{1,....,n\})$.

The proof is by induction. Fix $p_1,p_2 \in \mathcal{P}$ and $\lambda \in [0,1]$ such that $\{p_1\} \succ \{p_1,q_1\}$ and $\{p_2\} \succ \{p_2,q_2\}$. By Mixture-Betweenness, $\{\lambda p_1 + (1-\lambda)p_2\} \succeq \{\lambda p_1 + (1-\lambda)p_2, \lambda q_1 + (1-\lambda)q_2\}$. Assume towards a contradiction that $\{\lambda p_1 + (1-\lambda)p_2\} \sim \{\lambda p_1 + (1-\lambda)p_2, \lambda q_1 + (1-\lambda)q_2\}$. By the previous step

$$\{\lambda p_1 + (1-\lambda)p_2\} \succ \{\lambda p_1 + (1-\lambda)p_2, q_1\}$$

$$\{\lambda p_1 + (1-\lambda)p_2\} \succ \{\lambda p_1 + (1-\lambda)p_2, q_2\}.$$

Let $p = \lambda p_1 + (1-\lambda)p_2$. By Continuity, there exists $\alpha > \lambda > \beta$ such that

$$\{p\} \succ \{p, \alpha q_1 + (1-\alpha)q_2\} \sim \{p, \beta q_1 + (1-\beta)q_2\}.$$

Let $\nu = \frac{\lambda-\beta}{\alpha-\beta}$. Then, by Mixture-Betweenness,

[26] Our construction of $v(\cdot,1)$ is similar to the construction in the proof of Theorem 2 of Noor and Takeoka (2015).

$$\{p, \alpha q_1 + (1-\alpha)q_2\} \sim \nu\{p, \alpha q_1 + (1-\alpha)q_2\} + (1-\nu)\{p, \beta q_1 + (1-\beta)q_2\}.$$
$$= \{p, \lambda q_1 + (1-\lambda)q_2\} \cup [\nu\{p\} + (1-\nu)\{p, \beta q_1 + (1-\beta)q_2\}]$$
$$\cup [(1-\nu)\{p\} + \nu\{p, \alpha q_1 + (1-\alpha)q_2\}].$$

However

$$\{p, \lambda q_1 + (1-\lambda)q_2\}, \nu\{p\} + (1-\nu)\{p, \beta q_1 + (1-\beta)q_2\}, \succ \{p, \beta q_1 + (1-\beta)q_2\}$$
$$(1-\nu)\{p\} + \nu\{p, \alpha q_1 + (1-\alpha)q_2\} \succ \{p, \alpha q_1 + (1-\alpha)q_2\}.$$

So by Set-Betweenness,

$$\nu\{p, \alpha q_1 + (1-\alpha)q_2\} + (1-\nu)\{p, \beta q_1 + (1-\beta)q_2\} \succ \{p, \alpha q_1 + (1-\alpha)q_2\}$$
$$\sim \{p, \beta q_1 + (1-\beta)q_2\},$$

a contradiction.

Induction step: Suppose the result is true for n. We will show it is true for $n+1$.

Fix $p_t \in \mathcal{P}$ such that $\{p_t\} \succ \{p_t, q_t\}$ for $t = 1, ..., n+1$. By the induction hypothesis $\{\sum_{t=1}^{n} \frac{\lambda_t}{\sum_t^n \lambda_t} p_t\} \succ \{\sum_{t=1}^{n} \frac{\lambda_t}{\sum_t^n \lambda_t} p_t, \sum_{t=1}^{n} \frac{\lambda_t}{\sum_t^n \lambda_t} q_t\}$. Hence, by the base case, $\{\sum_t \lambda_t p_t\} \succ \{\sum_t \lambda_t p_t, \sum_t \lambda_t q_t\}$.

Step 4.3: show that $0 \notin \mathcal{T}$.

We only show that $0 \notin int(\mathcal{T})$ because Continuity implies that if $0 \notin int(\mathcal{T})$, then $0 \notin \mathcal{T}$.

Assume towards a contradiction $w = \overline{0} \in \mathcal{T}$. Then, there exists $p_t \in \mathcal{P}, \{p_t\} \sim \{p_t, q_t\}$ for $t = 1, ..., n$ and $p'_t \in \mathcal{P}, \{p'_t\} \succ \{p'_t, q'_t\}$ for $t = n+1, ..., N$, and $\lambda_t > 0$ for all t such that $\sum_{t=1}^{n} \lambda_t q_t + \sum_{t=n+1}^{N} \lambda_t p'_t = \sum_{t=1}^{n} \lambda_t p_t + \sum_{t=n+1}^{N} \lambda_t q'_t$.

Let $\lambda^* = \sum_{t=1}^{n} \frac{\lambda_t}{\sum_{t=1}^{N} \lambda_t}$, $p = \sum_t \frac{\lambda_t}{\sum_{t=1}^{N} \lambda_t} p_t, p' = \sum_{t=n+1}^{N} \frac{\lambda_t}{\sum_{t=1}^{N} \lambda_t} p'_t, q = \sum_{t=1}^{n} \frac{\lambda_t}{\sum_{t=1}^{N} \lambda_t} q_t$ and $q' = \sum_{t=n+1}^{N} \frac{\lambda_t}{\sum_{t=1}^{N} \lambda_t} q'_t$. Then, $\lambda^* p + (1-\lambda^*)q' = \lambda^* q + (1-\lambda_*)p'$. By steps 4.1 and 4.2, $\{p\} \sim \{p, q\}$ and $\{p'\} \succ \{p', q'\}$. Hence, by Mixture-Betweenness,

$$\{\lambda^* p + (1-\lambda^*)p'\} \succ \lambda^*\{p\} + (1-\lambda^*)\{p', q'\} = \{\lambda^* p + (1-\lambda^*)p', \lambda^* q + (1-\lambda^*)p'\}$$
$$= \lambda^*\{p, q\} + (1-\lambda^*)\{p'\} \sim \{\lambda^* p + (1-\lambda^*)p'\},$$

a contradiction.

To conclude the proof we outline the construction of $v(\cdot, 0)$. Let $\mathcal{Q} = \{q \in \Delta(X) | \{p\} \succeq \{q\} \text{ for all } p \in \Delta(X)\}$. There are two possible cases: (i) $\{p, q\} \sim \{q\}$ for all $p \in \Delta(X)$ and $q \in \mathcal{Q}$, and (ii) there exist $p \in \Delta(X)$ and $q \in \mathcal{Q}$ such that $\{p, q\} \succ \{q\}$.

(i) $\{p, q\} \sim \{q\}$ for all $p \in \Delta(X)$ and $q \in \mathcal{Q}$ Let $v(\cdot, 0) = -u(\cdot, 0)$. Then,

$$\min_{p' \in \{p,q\}} u(p', 0) = 0 \iff p \in \mathcal{Q} \text{ or } q \in \mathcal{Q} \iff V(\{p, q\}) = 0.$$

(ii) There exist $p \in \Delta(X)$ and $q \in \mathcal{Q}$ such that $\{p, q\} \succ \{q\}$

Define $\succeq_T$ over $\Delta(X)$ as follows:

$$q \succeq_T p \text{ if and only if } \{p\} \succ \{p, q\} \sim \{q\} \text{ and } q \in \mathcal{Q}$$
$$p \succ_T q \text{ if and only if } \{p, q\} \succ \{q\} \text{ and } q \in \mathcal{Q}.$$

An identical argument to the one in the construction of $v(\cdot, 1)$ shows there exists v' such that $p \succeq_T q$ implies $v'(p) \geq v'(q)$, $p \succ_T q$ implies $v'(p) > v(q)$. Let $v(\cdot, 0) = -u(\cdot, 0) + v'$.

C.2. Necessity

We only show that the representation satisfies Set-Betweenness.

Assume (u, v) represent $\succeq$. Fix x, y such that $V(x) \geq V(y)$ and let

$$\Phi(x, \gamma) = \max_{p \in x}\{u(p) + v(p, \gamma)\} - \max_{q \in x} v(q, \gamma).$$

By Lemma A.5, $\Phi(x, V(x)) \geq \Phi(y, V(x))$. Moreover, $\Phi(\cdot, \gamma)$ satisfies Set-Betweenness. Therefore, $\Phi(x, V(x)) \geq \Phi(x \cup y, V(x)) \geq \Phi(y, V(x))$. Hence, by Lemma A.5, $V(x \cup y) \leq V(x)$. An analogous argument establishes that $V(y) \leq V(x \cup y)$.

C.3. Uniqueness

Sufficiency is provided in the text. Here we prove necessity.

Assume (u, v) and (u', v') represent $\succeq$. Fix $\gamma \in (0, 1)$ and let $\Phi(\cdot, \gamma)$ and $\Phi(\cdot, \gamma')$ denote the GP functionals induced by $(u(\cdot), v(\cdot, \gamma))$ and $(u(\cdot, \gamma'), v(\cdot, \gamma'))$ respectively.

By Theorem 3.2, $\Phi(x,\gamma)=\Phi'(x,\gamma)$ for all $x\in\mathcal{K}(\Delta(X))$ and $\gamma\in(0,1)$. In particular, $\Phi(\{p\},\gamma)=\Phi'(\{p\},\gamma)$ for all $p\in\Delta(X)$ and $\gamma\in(0,1)$. Hence, $u=u'$. Fix $\gamma\in(0,1)$. If $v(\cdot,\gamma)$ is a constant or a positive affine transformation of $u(\cdot)$. Then, $\Phi'(x,\gamma)=\max_{p\in x}\{u(p)\}$ for all $x\in X$. Thus, by GP (p. 1414), either $v'(\cdot,\gamma)$ is a constant or a positive affine transformation of $u(\cdot)$.

If $v(\cdot,\gamma)$ is not a constant or a positive affine transformation of $u(\cdot)$. Then, by GP (p.1414), there are two cases: either there exist p,q such that $\Phi(\{p\},\gamma)>\Phi(\{p,q\},\gamma)>\Phi(\{q\},\gamma)$ or $\Phi(\{p\},\gamma)>\Phi(\{p,q\},\gamma)=\Phi(\{q\},\gamma)$ for all p,q such that $\Phi(\{p\},\gamma)>\Phi(\{q\},\gamma)$. If there exist p,q such that $\Phi(\{p\},\gamma)>\Phi(\{p,q\},\gamma)>\Phi(\{q\},\gamma)$, then since $u(\cdot)$ is unique, by GP's Theorem 4, $v'(\cdot,\gamma)=v(\cdot,\gamma)+b_\gamma$. If $\Phi(\{p\},\gamma)>\Phi(\{p,q\},\gamma)=\Phi(\{q\},\gamma)$ for all p,q such that $\Phi(\{p\},\gamma)>\Phi(\{q\},\gamma)$, then by GP (p.1414), $v(\cdot,\gamma)$ and $v'(\cdot,\gamma)$ are negative affine transformation of $u(\cdot)$. Hence, we only need to rule out the case in which $v(\cdot,\gamma)=-a_\gamma u(\cdot)+b_\gamma$ and $a_\gamma\in(0,1)$. To see that this is impossible let p_γ be such that $\gamma=u(p_\gamma,\gamma)$. Then, $u(\overline{p},\gamma)=1>u(p_\gamma,\gamma)$. Hence, $\Phi(\{p_\gamma,\overline{p}\},\gamma)=\gamma$. However, if $a_\gamma<1$, then

$$u(\overline{p},\gamma)-a_\gamma u(\overline{p},\gamma)=1-a_\gamma>u(p_\gamma,\gamma)-a_\gamma u(p_\gamma,\gamma)=\gamma-a_\gamma\gamma.$$

Hence, $\Phi(\{p_\gamma,\overline{p}\},\gamma)=1-a_\gamma+a_\gamma\gamma\neq\gamma$ and $a_\gamma\geq 1$.

Data availability

No data was used for the research described in the article.

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